School Mathematics 1

CHARLES R. FLEENOR

ROBERT E. EICHOLZ

PHARES O'DAFFER

METRIC

ext

Text

School Mathematics 1

1974 Edition

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Illustrations by Betty Delphey

TO THE STUDENT

Welcome to another year of mathematics!

This mathematics book was written in a manner that may be different from other mathematics that you have used. You will have the chance to take an active part in learning mathematics. You will be given the opportunity to carry out interesting investigations of mathematical ideas. You will discuss your results with your classmates. You will solve problems and develop your skills in mathematics. You will find interesting mathematical recreations, challenging Think problems and special projects.

While it is important for you to develop computational skills in mathematics, it is also important for you to know why various rules or procedures work and when to use them. Therefore, you will need to study carefully, work out examples for yourself, and ask questions when you do not understand.

We hope that you find your study of mathematics this year, interesting and rewarding.

Good Luck

The Authors

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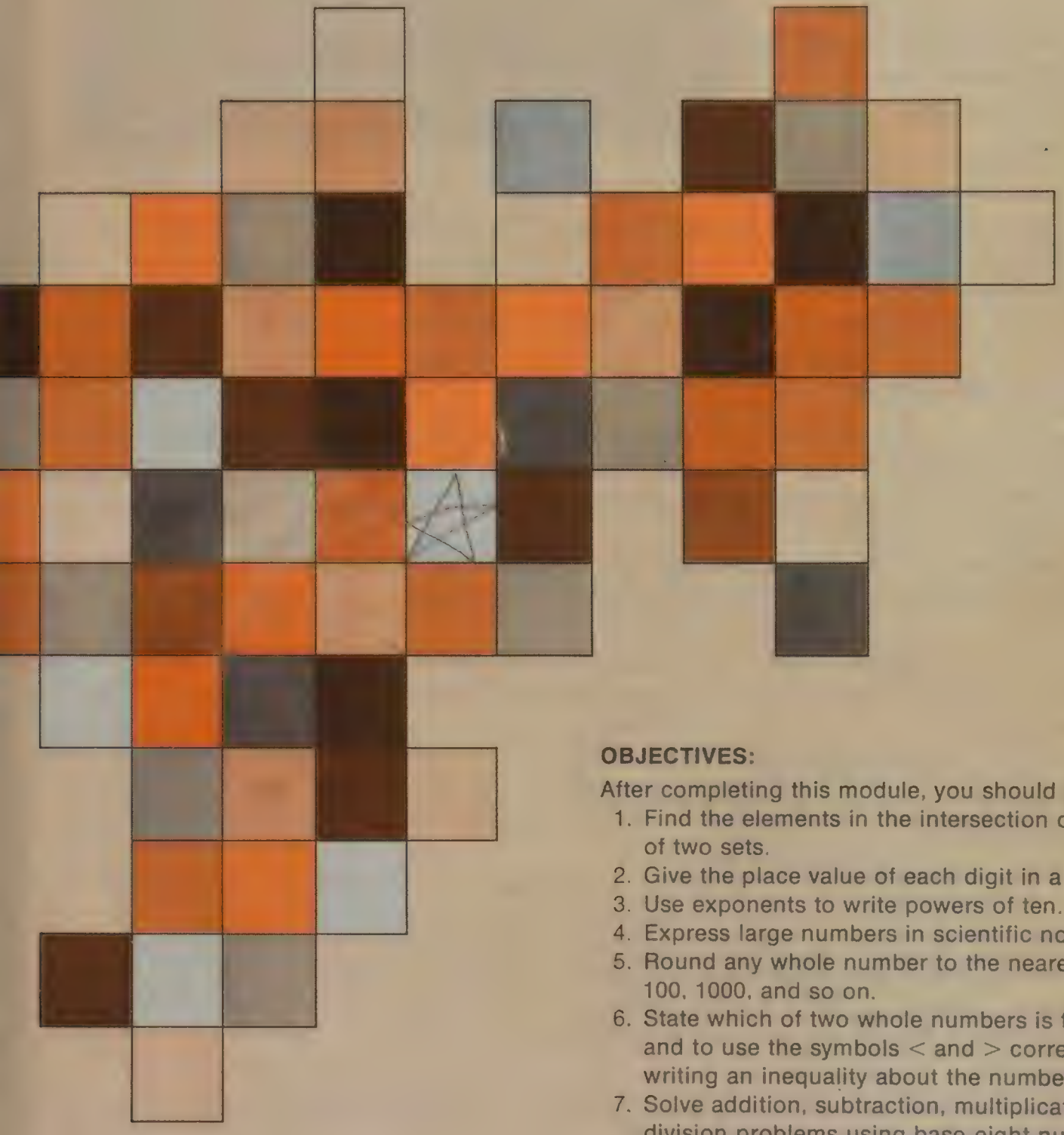
UNIT A: *The Arithmetic of Whole Numbers*

MODULE 1: *Numbers and Numerals*



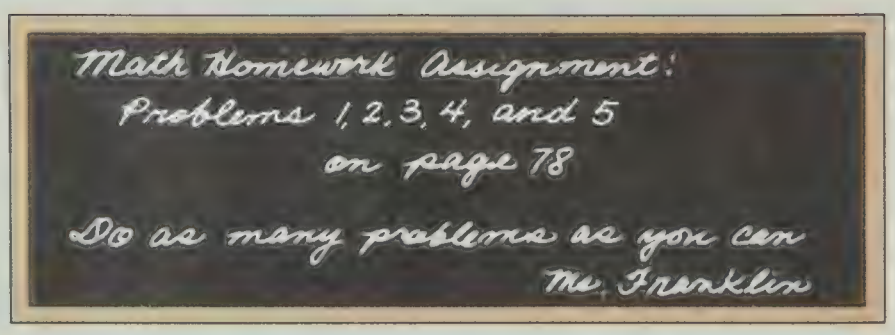
OBJECTIVES:

After completing this module, you should be able to:

1. Find the elements in the intersection or union of two sets.
2. Give the place value of each digit in a numeral.
3. Use exponents to write powers of ten.
4. Express large numbers in scientific notation.
5. Round any whole number to the nearest 10, 100, 1000, and so on.
6. State which of two whole numbers is the larger and to use the symbols $<$ and $>$ correctly when writing an inequality about the numbers.
7. Solve addition, subtraction, multiplication, and division problems using base-eight numerals.
8. Write a decimal numeral for a Roman numeral and write a Roman numeral for a decimal numeral.

Investigating the Ideas

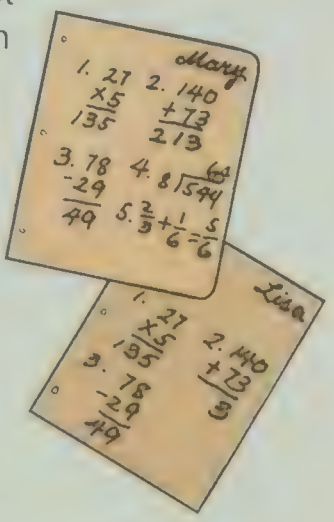
When Ms Franklin's class turned in their homework papers, she found that **no two students had completed exactly the same set** of homework problems.



Can you determine the largest possible number of students that could be in Ms Franklin's class?	List the different sets of problems that the students might have completed.

Discussing the Ideas

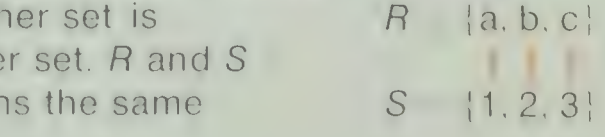
1. Mary completed all five of the homework problems. The set of numbers designating these problems could be written in **set notation** as $\{1, 2, 3, 4, 5\}$.
   Lisa completed only problems 1 and 3. Use set notation to show which problems she completed.



2. Bill did not complete any of the problems. The set of problems that he completed is the **empty set**. This set is denoted by $\{\ \}$ or ϕ. The set of men who have walked on the sun is the empty set.
   - **A** Is the set of men who have walked on the moon the empty set?
   - **B** Give an example of the empty set.

3. The sets $\{1, 2\}$, $\{2, 3, 4\}$, $\{5\}$, $\{\ \}$, $\{1, 3, 4, 5\}$, and many others are **subsets** of the set $\{1, 2, 3, 4, 5\}$. However, the set $\{1, 3, 6\}$ is not a subset of $\{1, 2, 3, 4, 5\}$. How can you decide when one set is a subset of another set?

4. The double-headed arrows show a **one-to-one correspondence** between set R and set S. Each element of either set is matched with exactly one element of the other set. R and S are **equivalent sets.** That is, each one contains the same number of elements as the other.
   Give another example of equivalent sets.



1. List all the subsets of the set of letters {d, e, f}.
   Do not forget to list the set itself and the empty set.

2. Use set notation to show the elements of each set described below.
   - **A** The set of whole numbers greater than zero and less than eight.
   - **B** The set of whole numbers between 4998 and 5004.
   - **C** The set of the last seven letters of the alphabet.
   - **D** The set of months whose names begin with the letter *M*.
   - **E** The set of whole numbers that are divisors of 12.

3. For each set described below, indicate whether or not it is the empty set. If it is not empty, list the elements in the set.
   - **A** The set of students in your mathematics class who are less than six years old.
   - **B** The set of months of the year which have less than 30 days.
   - **C** The set of days of the week that begin with the letter *C*.
   - **D** The set of odd numbers that are exactly divisible by 2.

4. Suppose set *X* contains 23 elements and there is a one-to-one correspondence between set *X* and another set *Y*.
   - **A** How many elements must set *Y* contain?
   - **B** Is set *X* equivalent to set *Y*?

5. Set $R = \{a, b, c\}$ contains exactly the same elements as set $T = \{b, c, a\}$ except that the order in which they are listed is different. We say that set *R* **equals** set *T* and we write $R = T$.
   Suppose set *A* consists of the letters in the word triangle and set *B* consists of the letters in the word altering
   - **A** Is there a one-to-one correspondence between sets *A* and *B*?
   - **B** Is $A = B$?

think

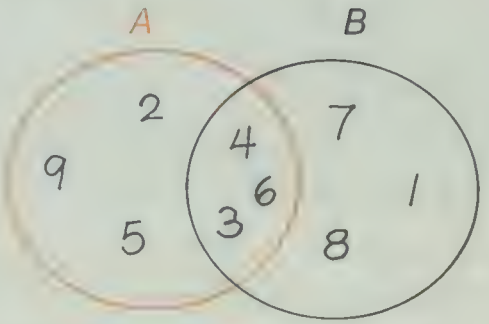
How many different words can you find in the word ARITHMETIC if the letters in the words occur in the same order as in ARITHMETIC?
EXAMPLE: ART is in ARITHMETIC but TAR is not.

Investigating the Ideas

Think of the colored ring as containing the elements of set A and the black ring as containing the elements of set B.

The union of sets A and B is the set containing the elements in A or in B, or in both A and B.
We write: $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

The intersection of sets A and B is the set containing the elements in both A and B.
We write: $A \cap B = \{3, 4, 6\}$.

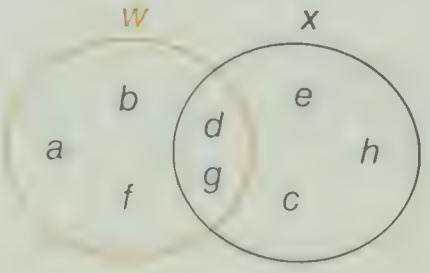


Can you place the numbers 1 through 9 into two sets like A and B so that the sum of the numbers in one set is the same as the sum of the numbers in the other set and the sum of the numbers in the intersection is 15?

Discussing the Ideas

1. Refer to sets A and B above to answer the following questions:
   - **A** What number in $A \cap B$ is an odd number?
   - **B** What is the largest number in set B?
   - **C** What is the number in set A that is not in set B and is less than 4?

2. Refer to the sets at the right.
   - **A** What elements are in $W \cup X$?
   - **B** What elements are in $W \cap X$?



3. If $C = \{1, 2, 3, 4\}$ and $D = \{3, 4, 5, 6, 7\}$, what are the elements in the following sets?
   - **A** $C \cup D$ **B** $C \cap D$

4. If $R = \{m, n, o\}$ and $S = \{x, y, z\}$, you can see that sets R and S have no common elements and $R \cap S = \phi$. What elements are in $R \cup S$?

5. Let $M = \{1, 2, 3, 4\}$ and $N = \{2, 3, 4\}$.
   - **A** What is $M \cup N$? What do you notice?
   - **B** What is $M \cap N$? What do you notice?

1. Let $A = \{a, c, d, e\}$, $B = \{b, c, d, e\}$, $C = \{e, f\}$, and $D = \{b, f, g\}$. List the elements in each of the following sets.

   A $A \cup B$ **C** $A \cup D$ **E** $D \cap B$ **G** $A \cup C$ **I** $C \cup D$

   B $B \cup C$ **D** $A \cap B$ **F** $C \cap D$ **H** $A \cap C$ **J** $B \cap C$

2. Let $R = \{0, 1, 2, 3, 4, 5\}$, $S = \{0, 2, 4\}$, and $T = \{1, 3, 5\}$. Which statements are true and which are false?

   A $R \cup S = R$ **C** $S \cup T = R$ **E** $R \cup T = T$ **G** $R \cap T = R$

   B $R \cap S = R$ **D** $T \cap S = \phi$ **F** $S \cap T = R$ **H** $S \cap T = T$

3. **A** If A and B are any sets, is $A \cup B$ the same as $B \cup A$?

   B Is $A \cap B$ the same as $B \cap A$?

4. If $M = \{1, 2, 3, 4, 5\}$, what set is

   A $M \cup M$? **B** $M \cap M$? **C** $M \cup \phi$? **D** $M \cap \phi$?

5. Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$, and $C = \{3, 4, 5, 6\}$. Find the elements in each of the following sets. Consider the sets in the parentheses first.

   A $(A \cup B) \cup C$ **D** $(A \cap B) \cup C$ **G** $(A \cap B) \cup (A \cap C)$

   B $(A \cap B) \cap C$ **E** $A \cup (B \cap C)$ **H** $A \cap (B \cap C)$

   C $A \cap (B \cup C)$ **F** $A \cup (B \cup C)$ **I** $(A \cup B) \cap (A \cup C)$

6. Which parts of Exercise 5 are sets which have the same elements?

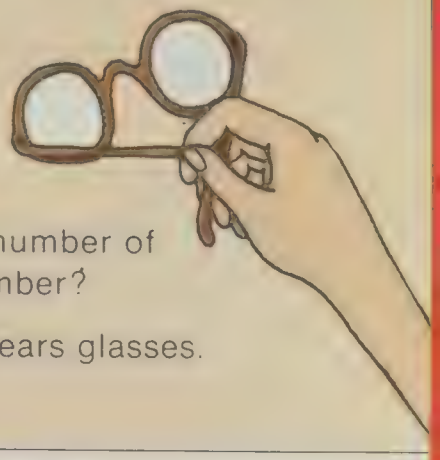
7. Suppose set A contains 5 elements and set B contains 4 elements.

   A What is the greatest possible number of elements in $A \cup B$?

   B What is the fewest possible number of elements in $A \cup B$?

think

In a set of 24 students, 12 of them wear glasses.



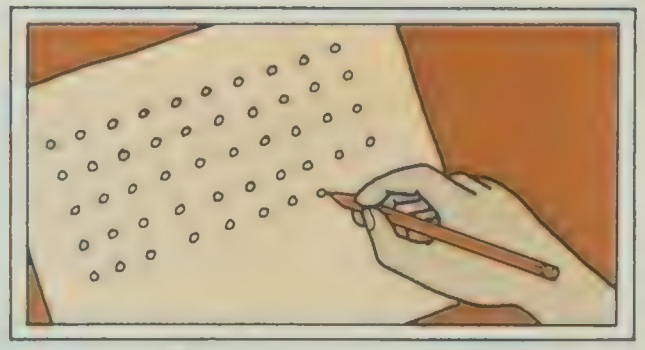
1. If you know that none of the boys wear glasses, what is the greatest number of boys possible in the set?
2. If there are 9 boys in the set, what is the fewest possible number of girls who wear glasses? What is the greatest possible number?
3. Now suppose there are 2 boys for every girl and no girl wears glasses. How many boys do not wear glasses?

More practice, page S-2, Set 1

The Decimal Numeration System

Investigating the Ideas

Mark one hundred dots on a piece of paper so that they are easy to count. Time yourself to see how long it takes you to mark all of the dots.



How long do you think it would take you to mark a thousand dots if you worked at the same rate? Could you ever mark a million dots?

Discussing the Ideas

1. How many students would be needed to combine their papers in order to show a thousand dots?

2. Our numeration system is called a **decimal system** because we group by tens. The decimal system uses **place value** because each digit in a numeral, depending upon its place, tells how many ones, tens, hundreds, and so on.

   6543 → The 3 tells how many ones. ——→ (3×1)

   6543 → The 4 tells how many tens. ——→ (4×10)

   6543 → The 5 tells how many hundreds. ⟶ (5×100)

   6543 → The 6 tells how many thousands. → (6×1000)

   In the numeral **7839**, give the place value of each digit.

3. The numeral [illegible] can be written as [illegible] using **expanded notation**. How would you write [illegible] using expanded notation?

4. To write numerals with several digits, we separate the digits into groups of three. Each group is called a **period**. The diagram shows the periods and place-value names through the billions.

PERIOD NAMES	BILLIONS	MILLIONS	THOUSANDS	UNITS
PLACE-VALUE NAMES	hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones
NUMERAL	5	4	7	2	0	8	7	9	3	1	6	5

   A What digit is in the hundred millions' place? ten thousands' place?

   B How would you read the numeral above?

1. Each sentence is about a digit in the numeral **321 896 574**.
Copy the sentences. Give the missing words and numbers.

A 7 in the [illegible] place means 7 · [illegible].
B 9 in the ? place means 9 · [illegible].
C 3 in the ? place means 3 · [illegible].
D 5 in the ? place means 5 · [illegible].
E 6 in the ? place means 6 · [illegible].
F 2 in the ? place means 2 · [illegible].

2. In the example, the number is written using **expanded notation.**
Use expanded notation to represent each number.

EXAMPLE: $7423 \cdot (7 \times 1000) + (4 \times 100) + (2 \times 10) + 3$

A 358
B 4562
C 53 729
D 70 364
E 458 297
F 6 200 876
G 24 364 200
H 647 870 000

3. Each number is written using expanded notation. Give the usual numeral for each number.

A $(9 \times 100) + (7 \times 10) + 4$
B $(8 \times 1000) + (6 \times 100) + (7 \times 10) + 5$
C $(5 \times 10\,000) + (3 \times 1000) + (4 \times 100) + (2 \times 10) + 7$
D $(3 \times 100\,000) + (5 \times 1000) + (8 \times 100) + (9 \times 10) + 4$

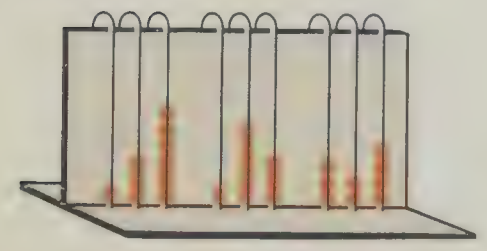
4. The diagram shows the names of the periods through the sextillions. Use it to help you write the numeral mentioned in each of the statements below.

SEXTILLIONS	QUINTILLIONS	QUADRILLIONS	TRILLIONS	BILLIONS	MILLIONS	THOUSANDS	UNITS

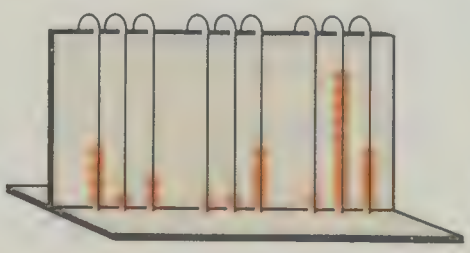
A The speed of light is about three hundred million metres per second.
B The Earth's mass is about seven sextillion tonnes.
C The estimated age of the sun is about one hundred forty quadrillion seconds.

5. An **abacus** is a simple computing device that uses place value.
Write the numeral shown by each abacus.

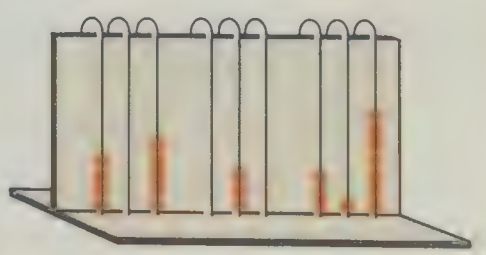
A



B



C



Investigating the Ideas

You know that $10 \times 10 = 100$,
$10 \times 10 \times 10 = 1000$,
and $10 \times 10 \times 10 \times 10 = 10\,000$.
Look at the numeral at the right.

This number is called a **googol**:
10 000 000 000 000 000 000 000 000 000
000 000 000 000 000 000 000 000 000
000 000 000 000 000 000 000 000 000
000 000 000 000 000 000.

Can you find how many tens would have to be multiplied together in order to make a googol?

Discussing the Ideas

1. How many times does the digit 0 appear in the numeral for a googol? How can you use this fact to help you answer the Investigation question?

2. When the same number is used as factor several times, you can use **exponents** to simplify the writing of a numeral.
(NOTE: The middle dot " · " indicates multiplication.)
The first power of ten, 10^1, is usually written as 10.
10^2 is read as "10 squared" or "ten to the second power."

$10^3 = 10 \cdot 10 \cdot 10$
10^3 is the third **power of ten.**
10^3 is read "10 cubed" or "ten to the third power."
10 is called the **base.**
3 is called the **exponent.**

Give the missing words and numbers.

A $10^{?}$ means 10 used as a factor ? times → $10^{?} = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = $ ▒▒▒.

B $10^{?}$ means 10 used as a factor ? times → $10^{?} = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = $ ▒▒▒

C $10^{?}$ means 10 used as a factor ? times → $10^{?} = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = $ ▒▒▒.

3. Read and complete each statement.
   - A One hundred is ten to the ? power.
   - B One thousand is ten to the ? power.
   - C Ten thousand is ten to the ? power.
   - D A googol is ten to the ? power.

1. Write each of the following by using 10 with an exponent.
   - **A** $10 \cdot 10 \cdot 10$
   - **B** $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$
   - **C** $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

2. Give the correct number of zeros for each blank.
   - **A** To write 10^3, we can write a 1 followed by ? zeros.
   - **B** To write 10^6, we can write a 1 followed by ? zeros.
   - **C** To write 10^{19}, we can write a 1 followed by ? zeros.
   - **D** To write 10^{100}, we can write a 1 followed by ? zeros.

3. Write each of the following using 10 with an exponent.
   - **A** 100 **B** 1000 **C** 10 000 **D** 100 000 **E** 1 000 000
   - **F** Light can travel about 10 000 000 000 000 kilometres per year.
   - **G** Some rocks from the moon are more than 100 000 000 000 000 000 seconds old.
   - **H** The approximate mass of Uranus is 100 000 000 000 000 000 000 000 tonnes.

4. Give the number for each ▒.
   - **A** $10^2 =$ ▒ **C** $10^4 =$ ▒ **E** $10^6 =$ ▒ **G** $10^8 =$ ▒ **I** $10^{10} =$ ▒
   - **B** $10^3 =$ ▒ **D** $10^5 =$ ▒ **F** $10^7 =$ ▒ **H** $10^9 =$ ▒ **J** $10^{11} =$ ▒

5. For each part, give the product **P**. Then give the missing power of ten for each ▒.
   - **A** $10 \cdot 100 = \mathbf{P}$; $10^1 \cdot 10^2 =$ ▒
   - **B** $100 \cdot 100 = \mathbf{P}$; $10^2 \cdot 10^2 =$ ▒
   - **C** $1000 \cdot 100 = \mathbf{P}$; $10^3 \cdot 10^2 =$ ▒

6. Find the products.
   - **A** $10^3 \cdot 10^1$ **C** $10^2 \cdot 10^4$ **E** $9 \cdot 10^2$ **G** $87 \cdot 10^4$ **I** $3 \cdot 10^2 \cdot 10^3$
   - **B** $10^3 \cdot 10^3$ **D** $10^3 \cdot 10^5$ **F** $42 \cdot 10^3$ **H** $68 \cdot 10^5$ **J** $47 \cdot 10^3 \cdot 10^4$

7. For each part, give the quotient **Q**. Then give the missing power of ten for each ▒.
   - **A** $100 \div 10 = \mathbf{Q}$; $10^2 \div 10^1 =$ ▒
   - **B** $10\,000 \div 10 = \mathbf{Q}$; $10^4 \div 10^1 =$ ▒
   - **C** $100\,000 \div 100 = \mathbf{Q}$; $10^5 \div 10^2 =$ ▒

8. Observe that $10^1 \cdot 10^1 = 10^{1+1} = 10^2$ and $10^2 \cdot 10^3 = 10^{2+3} = 10^5$. Therefore, $10^2 \cdot 10^0 = 10^{2+0} = 10^2$. According to the example, does the number 10^0 represent **1**, **10**, or **0**?

More practice, page S-2, Set 2

Investigating the Ideas

Study sets A and B.

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$B = \{10^0, 10^1, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8, 10^9, 10^{10}\}$

Can you express each of the numbers below as a product of two numbers such that one number is from set A and the other is from set B?

1. 6000
2. 9.000 000
3. 200 000
4. 70 000
5. 30 000 000 000
6. 8

Discussing the Ideas

1. Larry and Carol did not agree on the way to represent 6000. Who do you think is correct? How can you tell?

Larry: $6 \cdot 10^4 = 6000$

Carol: $6 \cdot 10^3 = 6000$

2. Jerry did not think that it was possible to represent 8 using one number from set A and the other from set B. So Max tried to help him by writing $8 \cdot \square\square\square\square 8$. What power of ten from set B is correct for the $\square\square\square\square$?

3. A number is represented in scientific notation when it is written as **a number from 1 through 9 times a power of ten.**

EXAMPLES:
$300 = 3 \cdot 100 = 3 \cdot 10^2$
$700\,000 = 7 \cdot 100\,000 = 7 \cdot 10^5$
$1000 = 1 \cdot 1000 = 1 \cdot 10^3$

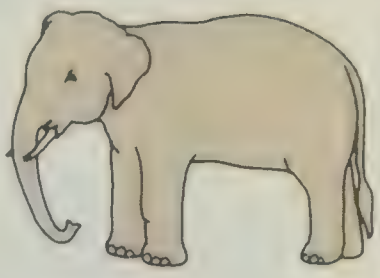
How would you express these numbers in scientific notation?

A 9000 B 80 C 2 000 000 D 300 000

4. What number does each of these represent?

A $6 \cdot 10^4$ B $9 \cdot 10^3$ C $7 \cdot 10^6$ D $4 \cdot 10^0$

1. Express the number in each statement in scientific notation.
   - **A** A jet airliner can travel 900 kilometres (km) per hour (km/h).
   - **B** A large telescope lens has a 500-centimetre (cm) diameter.
   - **C** A large elephant weighs about 4000 kilograms (kg).
   - **D** An artificial satellite travels 32 000 km/h.
   - **E** A whale weighs about 90 000 kilograms.
   - **F** The diameter of the Earth is over 10 000 000 metres (m).
   - **G** The radius of the sun is about 700 000 000 metres.
   - **H** Light travels at about 30 000 000 000 cm/s.
   - **I** The sun's mass is about 8 000 000 000 000 000 000 000 000 000 kg.



2. Each number below is represented in scientific notation. Write a usual numeral for each of them.

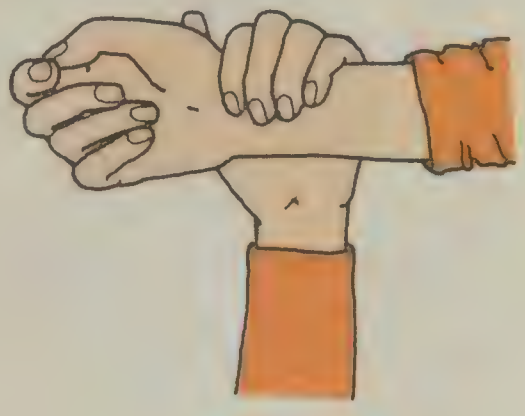
A $6 \cdot 10^4$	**D** $7 \cdot 10^5$	**G** $2 \cdot 10^6$	**J** $8 \cdot 10^9$
B $3 \cdot 10^7$	**E** $1 \cdot 10^8$	**H** $9 \cdot 10^2$	**K** $6 \cdot 10^{15}$
C $4 \cdot 10^{10}$	**F** $6 \cdot 10^0$	**I** $5 \cdot 10^3$	**L** $3 \cdot 10^{23}$

3. Use scientific notation to represent each number name.

A three million	**D** five quadrillion	**G** nine quintillion
B six trillion	**E** two sextillion	**H** six quadrillion
C fifty	**F** four billion	**I** two hundred million

think

1. Check your pulse rate to find how many times your heart beats in one minute.
2. **Estimate** the answer to each question below. Then calculate the answer and compare it with your estimate.
   - **A** If your heartbeat rate remains the same, how many days (to the nearest day) will it take for your heart to beat one million times?
   - **B** How many years (to the nearest year) will it take for your heart to beat one billion times?



More practice, page S-3, Set 3

Investigating the Ideas

The 1970 population of four small towns is shown in the table.

TOWN	POPULATION
Brookings	2720
Burns	3293
Canby	3813
Central Point	4004

Can you match each town with the multiple of 1000 that is nearest to the population of that town?

A 2000 **B** 3000 **C** 4000 **D** 5000

Discussing the Ideas

1. When you chose the multiple of 1000 that was nearest to the population of a town, you were **rounding the number to the nearest thousand.** Explain how you made your choice for each of the numbers in the Investigation.

2. To round 274 793 **to the nearest hundred thousand,** you must decide whether the number is nearer to 200 000 or 300 000. What is 274 793 rounded to the nearest hundred thousand?

3. When a number is halfway between two multiples, we agree to **round up.** What is 4500 rounded to the nearest thousand?

4. The population of Centerville is between 5000 and 6000. How can you decide which number is closer to the actual population even though two digits of the numeral are hidden?

EARTH FACTS

1. Earth's average diameter (distance through) is about 12 755 kilometres (km). Round this number to the
   - **A** nearest hundred.
   - **B** nearest thousand.

2. Earth's circumference (distance around) at the equator is about 40 074 kilometres. Round this number to the
   - **A** nearest thousand.
   - **B** nearest ten thousand.

3. The average distance from Earth to the moon is 384 393 kilometres. Round this number to the
   - **A** nearest hundred.
   - **B** nearest thousand.
   - **C** nearest hundred thousand.

4. The average distance from Earth to the sun is 149 593 690 kilometres. Round this number to the
   - **A** nearest hundred.
   - **B** nearest hundred thousand.
   - **C** nearest million.

5. The surface area of Earth is 410 171 500 square kilometres (km^2). Round this area to the nearest hundred million.

6. Mount Everest, the highest mountain on Earth, is 8848 metres tall. Round this height to the nearest thousand metres.

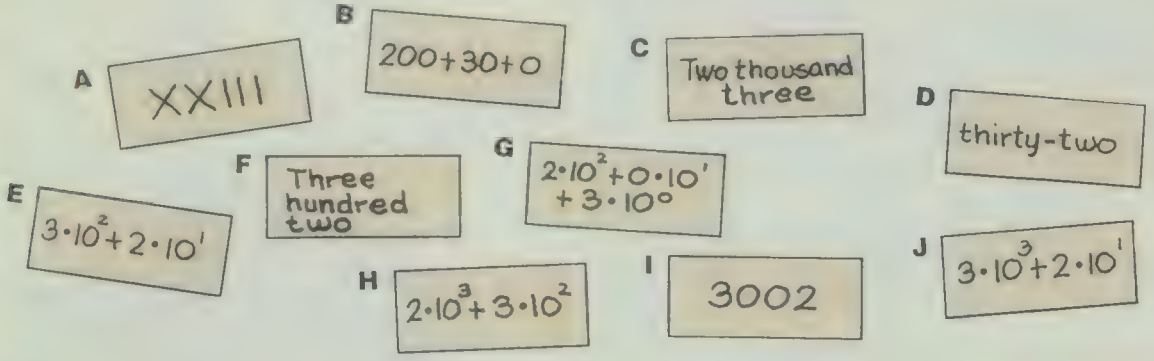
7. The Marianas Trench in the Pacific Ocean is 11 033 metres deep.
   - **A** Round this depth to the nearest thousand metres.
   - **B** Round this depth to the nearest ten thousand metres.

★8. The diameter of the sun is about 109 times the diameter of Earth. (See Exercise 1.)
   - **A** Find the diameter of the sun and round it to the nearest hundred thousand kilometres.
   - **B** Express this number in scientific notation.

★9. The volume of a ball can be measured approximately by finding the third power of the diameter (d^3 or $d \cdot d \cdot d$) and then dividing this number by 2.
   - **A** Using your answer to Exercise 1**b**, find the approximate volume of the earth.
   - **B** Using your answer to Exercise 8**a**, find the approximate volume of the sun.
   - **C** About how many "earths" would it take to fill as much space as the sun fills?

nvestigating the Ideas

A symbol for a number is printed on each card.

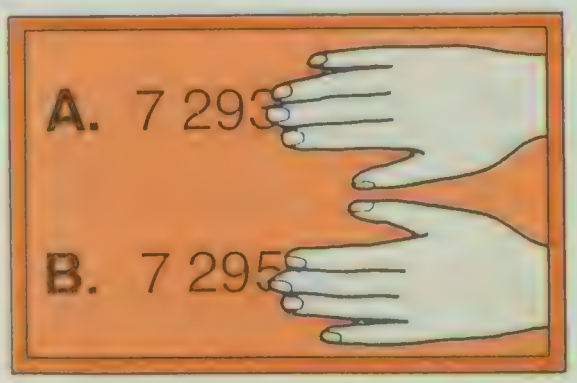


Can you arrange the numbers shown by the cards in order from the smallest to the largest?

)iscussing the Ideas

1. In order to show that 23 is less than 32, we write $23 < 32$. We also can say that 32 is greater than 23 and write $32 > 23$. Can you think of some ways to help you remember which symbol, $<$ or $>$, means "is less than" or means "is greater than"?

2. The same number of digits are covered in both A and B. How can you use your understanding of place value to decide which is the larger number?



3. What is the largest number that can be represented by a 3-digit numeral?

4. What is the smallest number that can be represented by a 3-digit numeral?

5. Which symbol ($<$ or $>$) would you put in the ▢ in order to make a true statement.

A 76 ▢ 84 C 3248 ▢ 3748 E 72 865 ▢ 92 765

B 219 ▢ 209 D 7036 ▢ 7063 F 3 030 300 ▢ 3 003 003

1. Give the correct symbol ($<$, $>$, or $=$) for each ▒.

A 68 ▒ 64	D 6758 ▒ 6578	G 49 753 ▒ 94 753
B 89 ▒ 98	E 5999 ▒ 6001	H 748 932 ▒ 784 923
C 349 ▒ 359	F 3989 ▒ 3899	I 6 004 400 ▒ 6 040 040

2. Copy each pair of numerals placing the correct inequality symbol between. EXAMPLE: For 29, 92, write $29 < 92$.

A 73, 75	D 3574, 3534	G 4672, 4592
B 39, 93	E 8878, 8888	H 1000, 999
C 452, 442	F 6491, 6492	I 0, 364

3. Give the correct symbol ($<$, $>$, or $=$) for each ▒.
Try to do this without finding the sums.

A $24 + 37$ ▒ $25 + 37$	D $343 + 68$ ▒ $86 + 343$
B $62 + 93$ ▒ $72 + 83$	E $487 + 352$ ▒ $497 + 342$
C $249 + 356$ ▒ $239 + 346$	F $6941 + 7213$ ▒ $6841 + 7513$

4. Write the numeral for each number indicated.

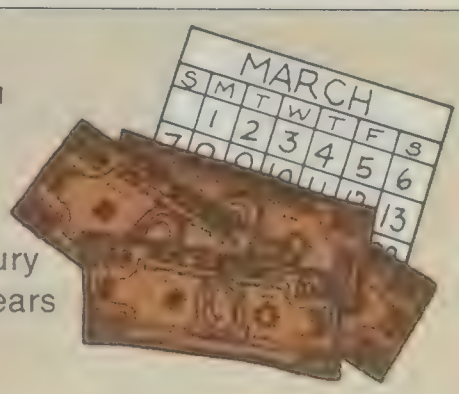
A 100 less than one million	G 52 million more than 846
B 1000 less than one million	H 8 billion more than 59
C 100 000 less than one million	I A half million more than 6 587 346
D 10 000 less than one million	J A half million less than 456 972 863
E 101 less than one billion	K The amount that must be added to 456 972 863 to give 457 000 000
F Half as much as one billion	

★ 5. How old, to the nearest year, would a person be who had lived one million hours? Is he older or younger than a person who has lived one billion seconds?

think

1. Suppose you had **one billion** dollars to give away. If you give away a $100 bill each minute, how long will it take you to give it all away? Guess which answer is closest to the correct answer.

A A day	C A month	E 10 years	G A century
B A week	D A year	F 20 years	H 2000 years



2. Compute the correct answer to the nearest year.

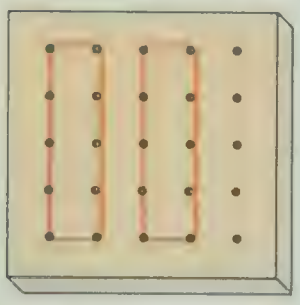
More practice, page S-3, Set 4

Other Number Bases

Investigating the Ideas

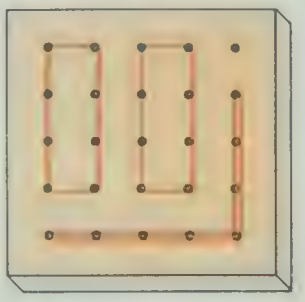
How many nails are on each geoboard? The number is the same for each of them, but the numeral we write depends upon the **base** or **grouping** that we use. In our decimal system, the base is ten. However, we can use other bases to write numerals much the same as we do in base ten.

BASE TEN



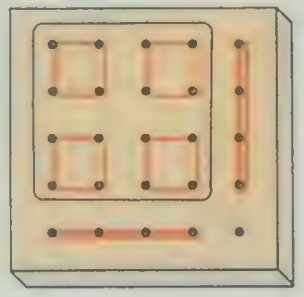
2 tens and 5
We write: $25_{(ten)}$

BASE EIGHT



3 eights and 1
We write: $31_{(8)}$

BASE FOUR



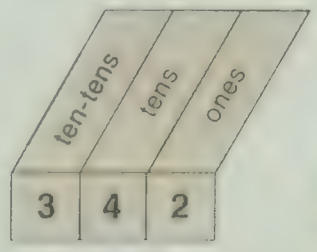
1 four-fours, 2 fours, and 1
We write: $121_{(4)}$

Can you show the groupings and write the numerals for the number of nails on a geoboard using base-seven, base-six, and base-five numeals?

Discussing the Ideas

1. In the base-ten numeration system, the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are used with place value to write any numeral. In base eight, only the eight digits 0, 1, 2, 3, 4, 5, 6, and 7 are used. What digits do you think are used in base seven?

2. The names of the first three places are shown for a base-ten and a base-eight numeral. What are the names of the first three places for a base-six numeral?



Base ten

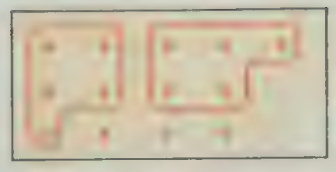
Base eight

3. Complete the statement.
   2 groups of eight and 3 ones in base eight is the same as ? groups of ten and ? ones in base ten.

4. Use your understanding of place value to explain what each numeral shows. EXAMPLE: $23_{(5)}$ means 2 fives and 3
   A $14_{(6)}$ B $27_{(8)}$ C $30_{(5)}$ D $101_{(ten)}$ C $121_{(3)}$

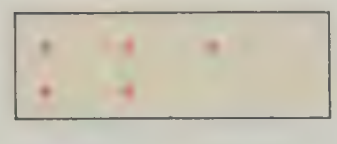
1. Think about grouping the following sets of dots as indicated by the given base. Write a numeral which indicates the number of dots in each box.

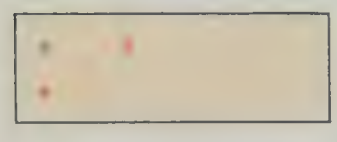
EXAMPLE:

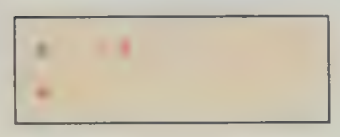


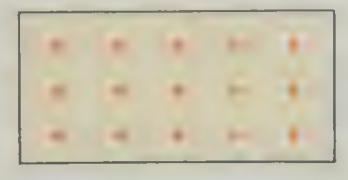
A (4)

B (ten)

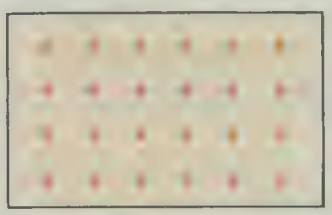
C  (8)

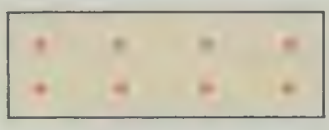
D  (2)

E  (5)

F  (6)

G (4)

H  (9)

I  (8)

J (3)

2. Give the missing base-eight numerals for the number of dots shown in each block.

$0_{(8)}$ $1_{(8)}$ $2_{(8)}$ $3_{(8)}$ A B C $7_{(8)}$ $10_{(8)}$ $11_{(8)}$ D E F

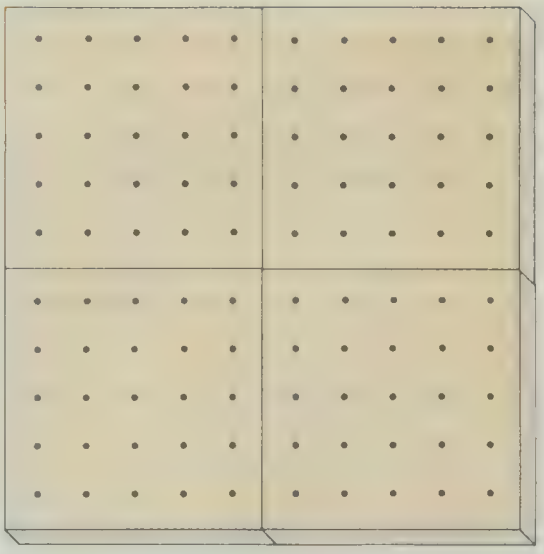
3. For each part, write a numeral using the indicated base.
   - A 3 eights and 2
   - B 1 six-sixes, 3 sixes, and 5
   - C 5 eight-eights, 7 eights and 1
   - D 2 four-fours, 3 fours, and 3
   - E 3 five-fives, 0 fives, and 2
   - F 1 two-twos, 1 two, and 0

★4. "$312_{(4)}$" is a base four numeral that represents a certain whole number. Draw $312_{(4)}$ dots. (**Hint:** Think of the place-value scheme that we use in base ten.)

A Base-Eight Numeration System

Investigating the Ideas

When four geoboards are placed together, the base-ten numeral for the number of nails is $100_{(ten)}$.



What is the base-eight numeral for the number of nails on the geoboards?

Discussing the Ideas

1. The diagram at the right compares the place-value names for base ten and for base eight.

   A In a base-ten numeral, what is the name of the next place to the left of the thousands place?

   B In a base-eight numeral, what would be the name of the next place to the left of the five-hundred-twelves place?

BASE-TEN PLACE-VALUE NAMES		thousands	hundreds	tens	ones
NUMERAL		3	1	2	5
BASE-EIGHT PLACE-VALUE NAMES		five-hundred-twelves	sixty-fours	eights	ones

2. By answering each question below, you can see how to rewrite the base-ten numeral $89_{(ten)}$ as a base-eight numeral.

   A How many sixty-fours in $89_{(ten)}$?

   B How many eights in $25_{(ten)}$?

   C How many ones left over?

   D What is the base-eight numeral for $89_{(ten)}$?

$$\begin{array}{rl} 89 & \\ -64 & \leftarrow \text{(1) sixty-fours} \\ \hline 25 & \\ -24 & \text{(3) eights} \\ \hline 1 & \leftarrow \text{(1) ones} \end{array}$$

3. You can express base-eight numerals in expanded notation in much the same way as in the base-ten system. However, you use powers of eight rather than powers of ten. How would you express each of these base-eight numerals in expanded notation?

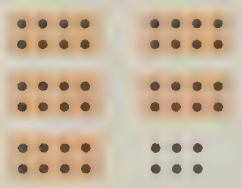
A $245_{(ten)}$ means $(2 \cdot 10^2) + (4 \cdot 10^1) + 5$.

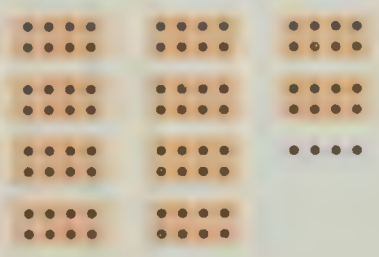
B $245_{(8)}$ means $(2 \cdot 8^2) + (4 \cdot 8^1) + 5$, or $165_{(ten)}$.

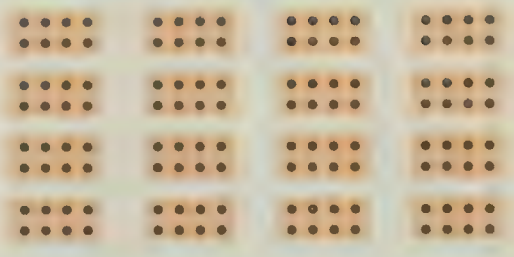
Note that just as 10^2 means $10 \cdot 10$, 8^2 means $8 \cdot 8$.

A $37_{(8)}$ **B** $137_{(8)}$ **C** $416_{(8)}$ **D** $506_{(8)}$

1. Give the base-eight numeral for the number of dots shown in each part.

A 

B 

C 

2. Complete each sentence with the correct place-value name.
   - A $276_{(8)}$: The 6 tells how many ? .
   - B $276_{(8)}$: The 7 tells how many ? .
   - C $276_{(8)}$: The 2 tells how many ? .

3. Solve the equations.
   - A $19_{(ten)} = (2 \cdot 8) + n$
   - B $30_{(ten)} = (n \cdot 8) + 6$
   - C $58_{(ten)} = (n \cdot 8) + 2$
   - D $62_{(ten)} = (7 \cdot 8) + n$
   - E $45_{(ten)} = (n \cdot 8) + 5$
   - F $48_{(ten)} = (6 \cdot 8) + n$

4. Write a 2-digit base-eight numeral for each base-ten numeral in Exercise 3.

5. Write a 3-digit base-eight numeral for each base-ten numeral.
   - A $77_{(ten)}$
   - B $102_{(ten)}$
   - C $99_{(ten)}$
   - D $149_{(ten)}$
   - E $215_{(ten)}$
   - F $511_{(ten)}$

6. Write each numeral in base-eight expanded notation.
   EXAMPLE: $137_{(8)} = 1 \cdot 8^2 + 3 \cdot 8^1 + 7$
   - A $35_{(8)}$
   - B $125_{(8)}$
   - C $740_{(8)}$
   - D $346_{(8)}$
   - E $407_{(8)}$
   - F $623_{(8)}$

7. Write a base-ten numeral for each base-eight numeral in Exercise 6. EXAMPLE: $137_{(8)} = 1 \cdot 8^2 + 3 \cdot 8^1 + 7$
   $= 64 + 24 + 7 = 95_{(ten)}$

★8. What number comes immediately after each of these numbers? Use base-eight numerals.
   - A $57_{(8)}$
   - B $127_{(8)}$
   - C $277_{(8)}$
   - D $7077_{(8)}$
   - E $100777_{(8)}$

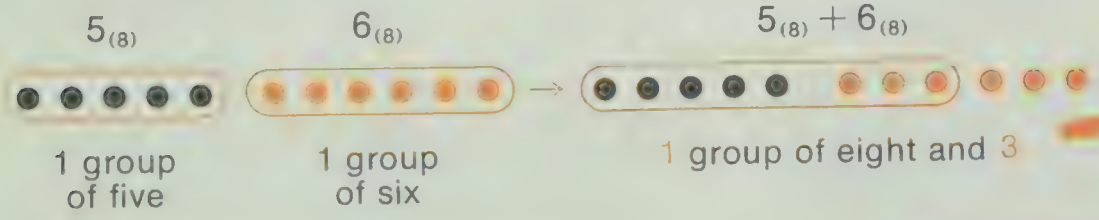
★9. In base-eight notation, $3412_{(8)}$ means $(3 \cdot 8^3) + (4 \cdot 8^2) + (1 \cdot 8^1) + 2$
Copy each exercise and give the base-ten numeral for each ▨.
   - A $3412_{(8)} =$ ▨ $_{(ten)}$
   - B $5632_{(8)} =$ ▨ $_{(ten)}$
   - C $4273_{(8)} =$ ▨ $_{(ten)}$
   - D $2541_{(8)} =$ ▨ $_{(ten)}$
   - E $7675_{(8)} =$ ▨ $_{(ten)}$
   - F $7777_{(8)} =$ ▨ $_{(ten)}$

Adding and Subtracting with Base-Eight Numerals

Investigating the Ideas

Addition and subtraction with base-eight numerals may be performed in much the same way as with base-ten numerals. The example shows how you can use sets to think about addition in base eight.

	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								



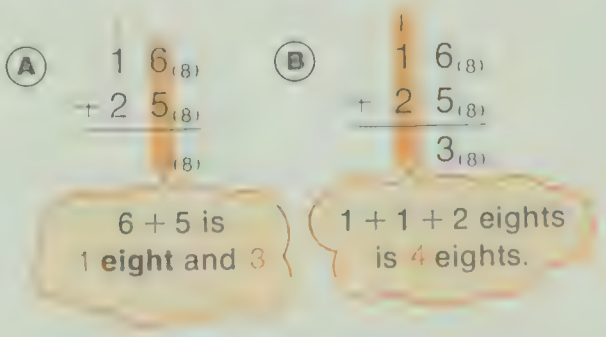
Can you fill in the addition table like the one above using base-eight numerals?

Discussing the Ideas

1. Explain how to find $7_{(8)} + 6_{(8)}$. Use sets of dots, if necessary.

2. Study the steps for finding a sum using base-eight numerals. Can you explain how to find the sums below?

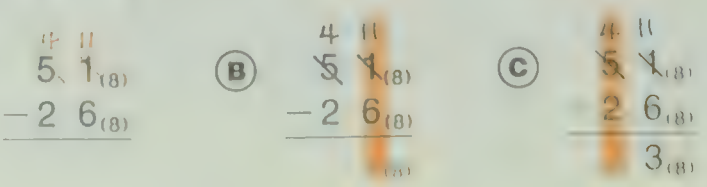
   A $27_{(8)} + 46_{(8)}$ **B** $53_{(8)} + 35_{(8)}$



3. How can the base-eight addition table be used to find these differences?

   A $13_{(8)} - 5_{(8)}$ **B** $16_{(8)} - 7_{(8)}$ **C** $10_{(8)} - 7_{(8)}$ **D** $11_{(8)} - 3_{(8)}$

4. Explain the steps in this subtraction problem.



1. Find the sums. Use the addition table completed in the Investigation, if necessary.

A $15_{(8)} + 2_{(8)}$	**B** $15_{(8)} + 3_{(8)}$	**C** $16_{(8)} + 4_{(8)}$	**D** $26_{(8)} + 1_{(8)}$	**E** $26_{(8)} + 2_{(8)}$	**F** $26_{(8)} + 3_{(8)}$
G $24_{(8)} + 33_{(8)}$	**H** $24_{(8)} + 34_{(8)}$	**I** $24_{(8)} + 37_{(8)}$	**J** $54_{(8)} + 23_{(8)}$	**K** $44_{(8)} + 36_{(8)}$	**L** $54_{(8)} + 24_{(8)}$
M $67_{(8)} + 21_{(8)}$	**N** $135_{(8)} + 216_{(8)}$	**O** $344_{(8)} + 125_{(8)}$	★ **P** $626_{(8)} + 547_{(8)}$	★ **Q** $316_{(8} + 712_{(8)}$	★ **R** $1735_{(8)} + 2276_{(8)}$

2. Find the sums. Use base-eight numerals.

A $2_{(8)}$ $3_{(8)}$ $+4_{(8)}$	**B** $6_{(8)}$ $5_{(8)}$ $+4_{(8)}$	**C** $3_{(8)}$ $7_{(8)}$ $+6_{(8)}$	**D** $6_{(8)}$ $6_{(8)}$ $6_{(8)}$ $+6_{(8)}$	**E** $7_{(8)}$ $1_{(8)}$ $4_{(8)}$ $+4_{(8)}$	**F** $5_{(8)}$ $7_{(8)}$ $6_{(8)}$ $+4_{(8)}$

3. Find the differences. Use the addition table, if necessary.

A $13_{(8)} - 4_{(8)}$	**B** $7_{(8)} - 2_{(8)}$	**C** $10_{(8)} - 5_{(8)}$	**D** $12_{(8)} - 6_{(8)}$
E $16_{(8)} - 6_{(8)}$	**F** $15_{(8)} - 4_{(8)}$	**G** $15_{(8)} - 6_{(8)}$	**H** $14_{(8)} - 5_{(8)}$
I $13_{(8)} - 5_{(8)}$	**J** $15_{(8)} - 7_{(8)}$	**K** $10_{(8)} - 7_{(8)}$	**L** $16_{(8)} - 7_{(8)}$

4. Find the differences. Use base-eight numerals.

A $26_{(8)} - 13_{(8)}$	**B** $60_{(8)} - 27_{(8)}$
C $41_{(8)} - 5_{(8)}$	**D** $74_{(8)} - 47_{(8)}$
E $55_{(8)} - 16_{(8)}$	**F** $100_{(8)} - 77_{(8)}$
★ **G** $205_{(8)} - 173_{(8)}$	★ **H** $431_{(8)} - 163_{(8)}$

think

Place base-eight numerals in the empty spaces of the square to form a **magic square**. The sum in each row, column, and both diagonals must be the same.

$15_{(8)}$		$13_{(8)}$
	$12_{(8)}$	
		$7_{(8)}$

Investigating the Ideas

You can multiply and divide using base-eight numerals in much the same way as you do using base-ten numerals. The example below shows how you can use sets to think about multiplication in base eight.

3 groups of six

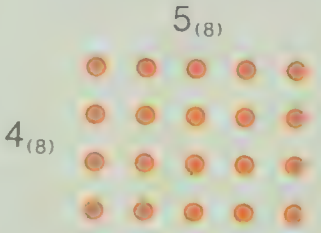
2 groups of eight and 2

×	0	1	2	3	4	5	6	7
0								
1								
2							14	
3							22	
4								
5								
6								
7								

Can you fill in a multiplication table like the one above using base-eight numerals?

Discussing the Ideas

1. A What multiplication fact does the array of dots suggest?
   B Write the fact using base-eight numerals.



2. Explain how to find $4_{(8)} \cdot 7_{(8)}$. Use an array of dots, if necessary.

3. Explain the steps in the multiplication problem.

A
$$\begin{array}{r} {\scriptstyle 2}\\ 3\ 4_{(8)} \\ \times\ 5_{(8)} \\ \hline 4_{(8)} \end{array}$$

B
$$\begin{array}{r} {\scriptstyle 2}\\ 3\ 4_{(8)} \\ \times\ 5_{(8)} \\ \hline 2\ 1\ 4_{(8)} \end{array}$$

4. A Since $3_{(8)} \cdot 6_{(8)} = 22_{(8)}$, what is $22_{(8)} \div 6_{(8)}$?
   B What is $22_{(8)} \div 3_{(8)}$?

5. Explain how the base-eight multiplication table can be used to find these quotients.

   A $17_{(8)} \div 3_{(8)}$ B $34_{(8)} \div 7_{(8)}$ C $10_{(8)} \div 2_{(8)}$ D $52_{(8)} \div 7_{(8)}$

1. Find the products. Use the base-eight multiplication table in Investigation.

A $12_{(8)} \times 3_{(8)}$ B $12_{(8)} \times 4_{(8)}$ C $12_{(8)} \times 5_{(8)}$ D $23_{(8)} \times 3_{(8)}$ E $34_{(8)} \times 2_{(8)}$ F $17_{(8)} \times 6_{(8)}$

G $15_{(8)} \times 7_{(8)}$ H $42_{(8)} \times 5_{(8)}$ I $36_{(8)} \times 2_{(8)}$ J $33_{(8)} \times 3_{(8)}$ K $57_{(8)} \times 6_{(8)}$ L $66_{(8)} \times 5_{(8)}$

2. Find the quotients. Use the multiplication table, if necessary.

A $6_{(8)} \div 2_{(8)}$ B $10_{(8)} \div 2_{(8)}$ C $17_{(8)} \div 3_{(8)}$ D $16_{(8)} \div 2_{(8)}$

E $30_{(8)} \div 6_{(8)}$ F $44_{(8)} \div 6_{(8)}$ G $43_{(8)} \div 7_{(8)}$ H $36_{(8)} \div 5_{(8)}$

3. Use base-eight numerals to continue the doubling process until you reach $10000_{(8)}$.

$1_{(8)}$, $2_{(8)}$, $4_{(8)}$, $10_{(8)}$, $20_{(8)}$, ▮, ▮, ▮, . . .

4. Is $10000_{(8)}$ greater than or less than $5000_{(ten)}$?

★ 5. Find the products. Use base-eight numerals.

A $23_{(8)} \times 15_{(8)}$ B $34_{(8)} \times 26_{(8)}$

C $304_{(8)} \times 12_{(8)}$ D $537_{(8)} \times 46_{(8)}$

★ 6. Find the quotients. Use base-eight numerals.

A $6_{(8)}\overline{)556_{(8)}}$

B $5_{(8)}\overline{)1572_{(8)}}$

C $23_{(8)}\overline{)503_{(8)}}$

D $34_{(8)}\overline{)2444_{(8)}}$

think

Numbo was in grade 22 and was 110 years old. He slept 101 hours each night, awoke at 21 o'clock, and ran 1212 metres in 22 seconds. About all the history he knew was that Columbus discovered North America in the year 2001021.

1. The numbers in this story are represented using a number base other than ten. Find what number base is used and recopy the story using the correct base ten numerals.
2. Write a story using a number base other than base ten. Give the story to a classmate to figure out.

ROMAN NUMERALS

The **Roman numeration system** does not use place value. When two Roman symbols are placed side by side, we add or subtract to determine the number represented. We **add** when the smaller number is written on the right.

XI means 10 + 1

We **subtract** when the smaller number is written on the left.

IX means 10 − 1

The seven basic Roman numerals	
I = 1	C = 100
V = 5	D = 500
X = 10	M = 1000
L = 50	

1. Write a decimal numeral for each Roman numeral. Each numeral is an "addition numeral."

A VI	**C** XV	**E** LX	**G** CX	**I** DC	**K** MC
B LI	**D** XVI	**F** CL	**H** CV	**J** DXI	**L** MD

2. Write a decimal numeral for each of the six basic "subtraction numerals" shown below.

A IV	**B** IX	**C** XL	**D** XC	**E** CD	**F** CM

3. Write a Roman numeral for each decimal numeral.

EXAMPLES: 7 = VII 24 = XXIV

A 3	**C** 11	**E** 38	**G** 49	**I** 529	**K** 1372
B 6	**D** 34	**F** 19	**H** 126	**J** 340	**L** 2968

4. Write a decimal numeral for each Roman numeral.

A CCLXII	**C** CDXLIV	**E** CMLIX	**G** MCDXCII
B DXLV	**D** CDXXXVII	**F** MMLI	**H** MCMLXXVI

5. Write the Roman numerals for the indicated sets of numbers.

A 1 through 10	**C** 40 through 50	**E** 395 through 400
B 15 through 20	**D** 120 through 130	**F** 990 through 1000

6. Add each of the following without changing to decimal numerals.

A CXI + XVI	**B** CM + D	**C** LXVII + CLXVI

7. Subtract each of the following without changing to decimal numerals.

A XXXIII − XII	**B** MDCCXXXVIII − MCCXXI	**C** MDXI − MCX

8. Find the product of CLVI and XV without changing to decimal numerals.

OTHER ANCIENT NUMERALS

Since very early times, people have needed numbers. Throughout the ages **numbers have not changed, but many different numerals have been used to represent them.**

The **Egyptian numeration system** did not use place value. To show larger numbers, they wrote symbols over again and **added** the value of each symbol.

For example, 𓍢𓎆𓎆𓎆𓏺𓏺 is a symbol for 132

Egyptian numerals 1700 B.C.		Our numerals
𓏺	=	1
𓎆	=	10
𓍢	=	100
𓆼	=	1000
𓂭	=	10 000
𓆐	=	100 000
𓁨	=	1 000 000

1. Write a decimal numeral for each Egyptian numeral.

A 𓏺𓏺𓏺 **B** 𓎆𓎆𓎆𓎆𓏺𓏺 **C** 𓍢𓎆𓏺

D 𓆼𓆼𓍢𓍢𓍢𓎆𓎆𓏺𓏺𓏺𓏺𓏺 **E** 𓂭𓂭𓂭𓆼𓍢𓍢𓎆𓎆𓎆

F 𓁨𓁨𓁨𓆐𓆐𓆐𓂭𓂭𓍢𓍢𓍢 **G** 𓁨𓂭𓍢𓏺

H 𓆐𓆼𓆼𓆼𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺𓏺𓏺𓏺

2. Write an Egyptian numeral for each decimal numeral.

A 18	**C** 426	**E** 1 000 010	**G** 1968	**I** 2 432 324
B 42	**D** 100 100	**F** 1 111 111	**H** 10 001	**J** 3 005 418

The **Babylonian numeration system** used the idea of place value. It was a **base sixty** numeration system! The Babylonians used a piece of wood 𓏤 called a **stylus** to write in clay. They wrote numerals for numbers up to 59, using combinations of the two symbols shown in the Babylonian numeral table.

Babylonian numerals 1700 B.C.		Our numerals
𒐕	=	1
𒌋	=	10

For example, 𒐕𒐕𒐕 𒐕𒐕𒐕 is a symbol for 6 and 𒌋𒌋𒌋𒌋 𒐕𒐕 is a symbol for 44

3. Write a decimal numeral for each Babylonian numeral.

A 𒐕𒐕𒐕 𒐕𒐕𒐕 **B** 𒌋𒌋𒐕𒐕 𒐕𒐕 **C** 𒌋𒌋𒐕𒐕𒐕𒐕 𒌋𒌋𒐕𒐕𒐕𒐕 **D** 𒌋𒌋 𒌋𒐕𒐕𒐕 𒌋𒌋 𒐕𒐕𒐕

4. Write a Babylonian numeral for each decimal numeral.

A 7 **B** 16 **C** 36 **D** 40 **E** 55 **F** 11

The ancient **Mayas** of Yucatan used a numeration system with a base of 20. For example, the Mayan numeral 𝍠 is a symbol for 12, and 𝍡 would represent $1 \cdot 20 + 12$ or 32

Mayan numerals		Our numerals
𝋠	=	0
•	=	1
—	=	5
𝍠	=	7
𝍡	=	27

5. Write a decimal numeral for each Mayan numeral.

A 𝍣 **B** 𝍡 **C** 𝍤 **D** ∴ **E** 𝍥

REVIEWING THE IDEAS

1. Let $R = \{1, 2, 3, 4, 5\}$ and $S = \{0, 2, 4, 6, 8\}$.
   - A List the elements in $R \cup S$.
   - B List the elements in $R \cap S$.

2. Write 7369 using expanded notation.

3. In the numeral **563 412 989**, what digit is in each of these places?
   - A ones'
   - B thousands'
   - C millions'
   - D ten millions'
   - E hundreds'
   - F hundred thousands'

4. Write the usual numeral for $9 \cdot 10\,000 + 8 \cdot 1000 + 5 \cdot 100 + 3 \cdot 10 + 7$.

5. Which expression is not a numeral for ten?
   - A $10 \cdot 1$
   - B 10^1
   - C $10 + 0$
   - D $10 \div 1$
   - E 10^0
   - F $10^3 \div 10^2$

6. A 10^4 means 10 used as a factor ? times.
   B $10^4 =$ ▒

7. Write 100 000 using 10 with an exponent.

8. Give the product **P** and the missing power of ten.

$$\begin{array}{ccccc} 100 & \cdot & 1000 & = & \mathbf{P} \\ \downarrow & & \downarrow & & \downarrow \\ 10^2 & \cdot & 10^3 & = & \text{▒} \end{array}$$

9. Write the usual numeral for each.
   - A $4 \cdot 10^3$
   - B $7 \cdot 10^9$
   - C $5 \cdot 10^0$

10. Express 400 000 in scientific notation.

11. Round 7 856 467 to the nearest
    - A hundred.
    - B thousand.
    - C hundred thousand.
    - D million.

12. Give the correct symbol ($<$, $>$, or $=$) for each ▒.
    - A 347 ▒ 374
    - B $637 + 146$ ▒ $627 + 156$
    - C 34 843 ▒ 34 749

13. What is one less than one million?

14. Write a numeral telling how many dots there are using
    - A base-ten.
    - B base-eight.
    - C base-five.

15. A How many eights in 43?
    B How many ones left over?
    C Write the base-eight numeral for $43_{(ten)}$.

16. Find the sums and differences.
    - A $5_{(8)} + 5_{(8)} =$ ▒
    - B $11_{(8)} - 5_{(8)} =$ ▒
    - C $\begin{array}{r} 44_{(8)} \\ +\,16_{(8)} \\ \hline \end{array}$
    - D $\begin{array}{r} 21_{(8)} \\ -\,14_{(8)} \\ \hline \end{array}$

17. Find the products and quotients.
    - A $4_{(8)} \cdot 3_{(8)} =$ ▒
    - B $24_{(8)} \div 5_{(8)} =$ ▒
    - C $\begin{array}{r} 23_{(8)} \\ \times\,5_{(8)} \\ \hline \end{array}$
    - D $7_{(8)}\overline{)430_{(8)}}$

18. Write a decimal numeral for each Roman numeral.
    - A CLXVII
    - B MDCCIX

19. Write a Roman numeral for each decimal numeral.
    - A 629
    - B 1984

TEST YOURSELF

1. Let $X = \{0, 1, 3, 4, 6\}$ and $Y = \{2, 3, 4, 5, 7\}$.
   - A List the elements in $X \cup Y$.
   - B List the elements in $X \cap Y$.

2. In the numeral **3 459 726**, which digit is in the
   - A thousands' place?
   - B hundred thousands' place?

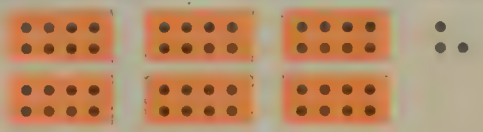
3. What is the exponent of 10 in the equation $1\,000\,000 = 10^{\square}$?

4. Express 700 000 in scientific notation.

5. Round 3 456 726 to the nearest
   - A thousand.
   - B hundred thousand.

6. Give the correct symbol ($<$ or $>$) for each ▨.
   - A 649 ▨ 694
   - B 21 749 ▨ 21 726
   - C 23 388 635 ▨ 23 386 835

7. Write the base-eight numeral for the number of dots shown below.



8. Find the sum and difference.
   - A $5_{(8)} + 7_{(8)}$
   - B $13_{(8)} - 6_{(8)}$

9. Find the product and quotient.
   - A $5_{(8)} \cdot 7_{(8)}$
   - B $30_{(8)} \div 4_{(8)}$

10. Write a decimal numeral for MDLXIV.

RESEARCH PROJECTS

A Do you know how large a decillion is? A quintillion? Find the names of the periods of very large numbers and make a chart to show your findings. (See *Lore of Large Numbers* by Philip Davis, New Mathematics Library; Syracuse, New York: Singer Company, 1961 available from Random House of Canada Ltd.)

B Read and report about the **abacus**. There are several different kinds. What people have used the abacus? What people still use the abacus today (See *I Can Learn About Calculators and Computers* by Raymond Kenyon; Evanston, Illinois: Harper and Row, 1961 available from Fitzhenry and Whiteside Ltd.)

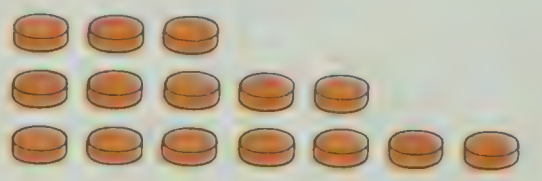
C Make a model of a Babylonian clay tablet using modeling clay. Use a triangular stylus to mark Babylonian numerals on the tablet. (See *History of Mathematics* by David E. Smith; New York: Dover Publications, 1953 available from General Publishing, Ltd.)

D Make a calender with numerals for the days of the month in a base other than base ten.

E Find out about the ancient Greek numeration system. Make a poster showing how their numerals compare to ours. (See *A History of Mathematics* by Carl B. Boyer; New York: John Wiley & Sons, Canada, Ltd. 1968)

THE GAME OF NIM

The game Nim is played by two persons using three rows of counters, 3 in the first row, 5 in the second, and 7 in the third. Each player in turn takes one or more counters from just one row. The player who is forced to remove the last of the 15 counters loses the game.



For example, the first player might take all 7 counters in the third row, and the second might remove 2 from the second row. This leaves two rows of 3.

The second player can now win the game regardless of how many counters the first player removes if he thinks carefully.

Play the game with a classmate. Try to develop some strategies that will help you win the game.

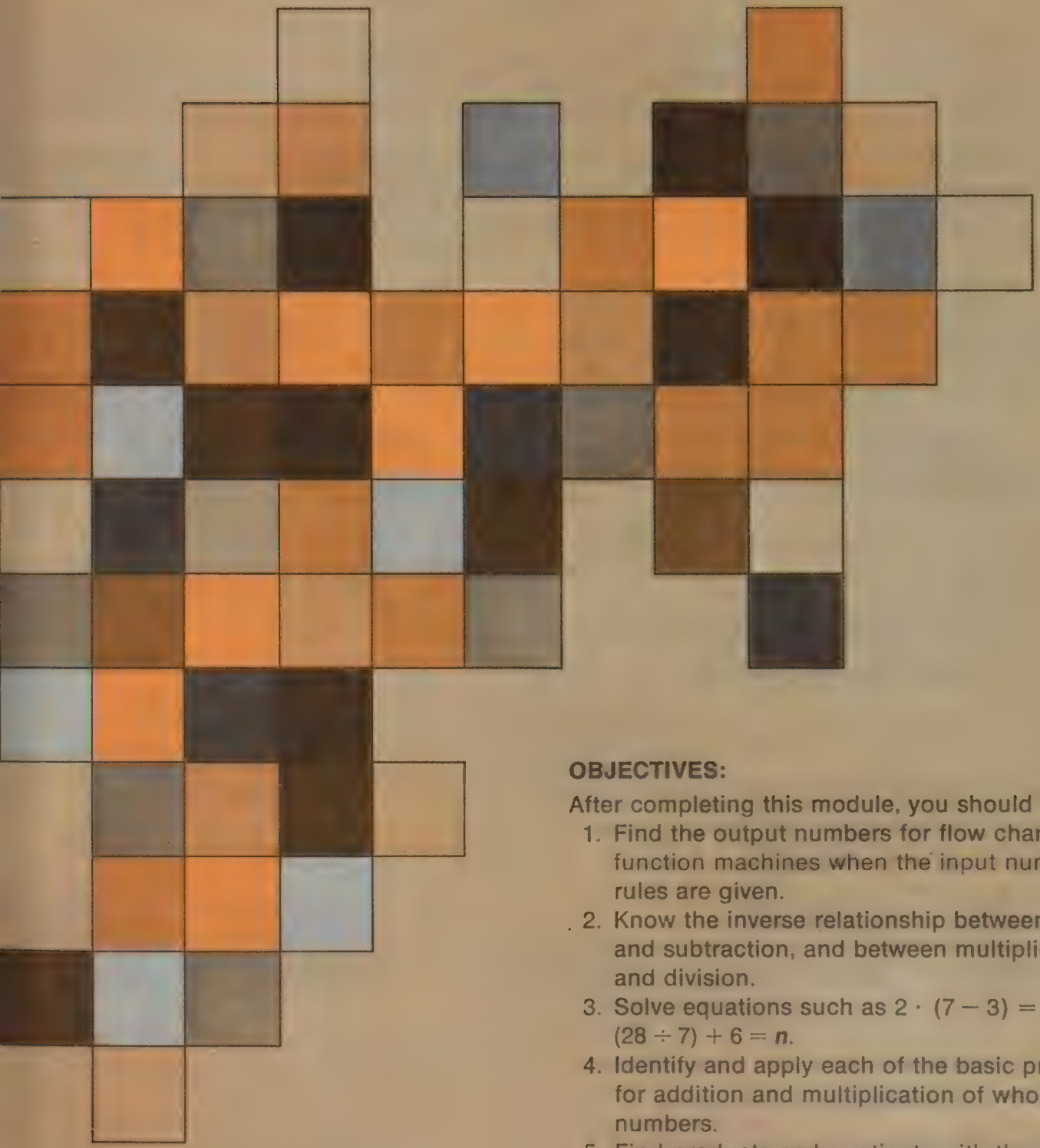
MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

1. A $\{0, 1, 2, 3, 4, 5, 6, 7\}$; B $\{3, 4\}$ **2.** A 9; B 4 **3.** 6
4. $7 \cdot 10^{9}$ **5.** A 3 457 000; B 3 500 000 **6.** A $<$; B $>$; C $>$
7. $63_{(8)}$ **8.** A $14_{(8)}$; B $5_{(8)}$ **9.** A $43_{(8)}$; B $6_{(8)}$ **10.** 1564

UNIT A: *The Arithmetic of Whole Numbers*

MODULE 2: *Whole Numbers—Operations and Computation*



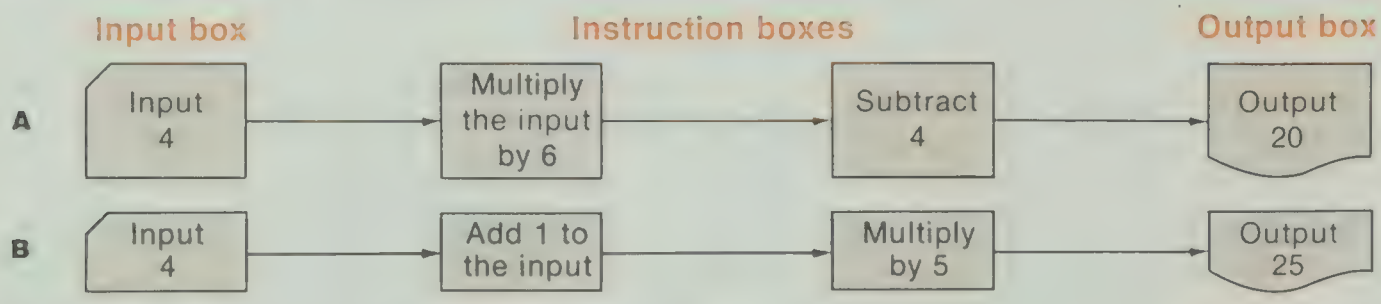
OBJECTIVES:

After completing this module, you should be able to:

1. Find the output numbers for flow charts and function machines when the input numbers and rules are given.
2. Know the inverse relationship between addition and subtraction, and between multiplication and division.
3. Solve equations such as $2 \cdot (7 - 3) = n$ or $(28 \div 7) + 6 = n$.
4. Identify and apply each of the basic principles for addition and multiplication of whole numbers.
5. Find products and quotients with the aid of multiples.
6. Add, subtract, multiply, and divide whole numbers.

Investigating the Ideas

A **flow chart** is a diagram that gives instructions arranged in a logical order. Study flow charts A and B below.

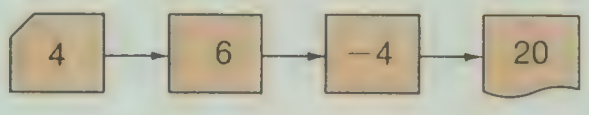


Choose other input numbers for each flow chart and then find corresponding outputs.

Can you find an input number for both flow charts that will give the same number as an output?

Discussing the Ideas

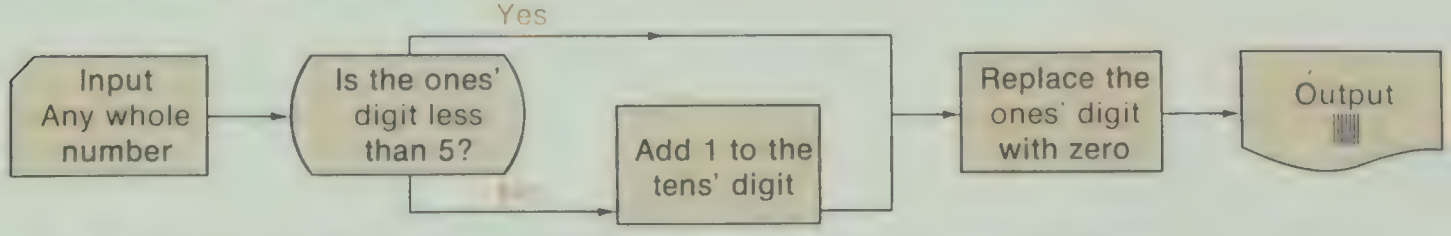
1. The flow chart below is a simplified version of flow chart A.



   A What does the instruction "·6" tell you to do?
   B What does the instruction "−4" mean?

2. How would you simplify flow chart B?

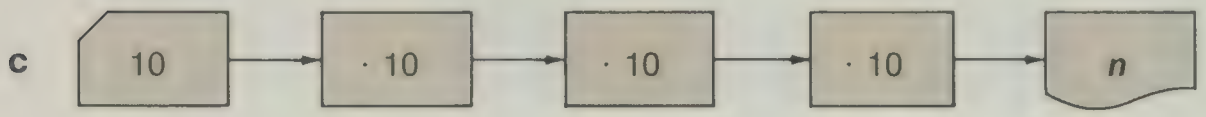
3. Some flow charts have **question boxes**. Flow chart questions must be answered with either "yes" or "no."

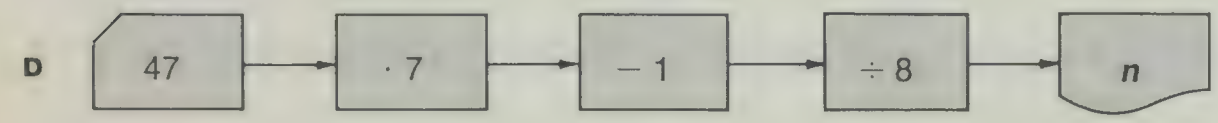


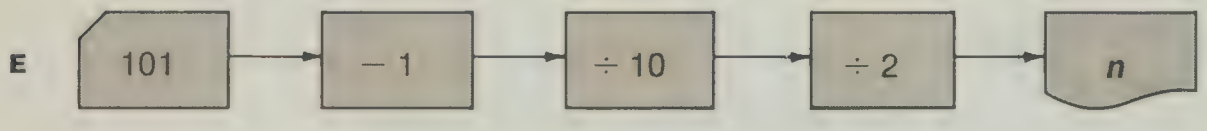
   A Explain how the flow chart works if the input number is 243. What is the output number?
   B Explain how the flow chart works if the input number is 517. What is the output number?
   C Describe in your own words what the flow chart does to each input number.

1. Find the output number n in each flow chart.
Part A is worked as an example.

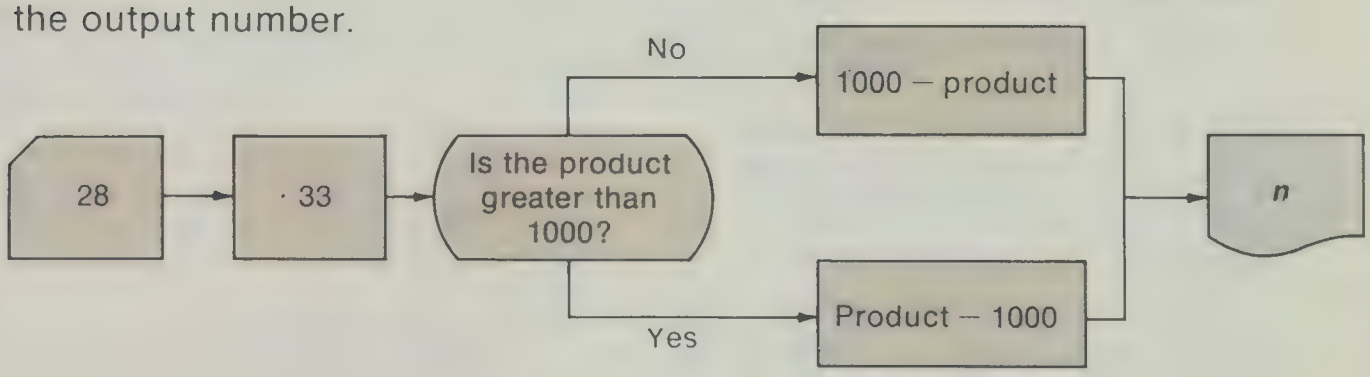
ANSWER: $n = 75$







2. Find the output number.

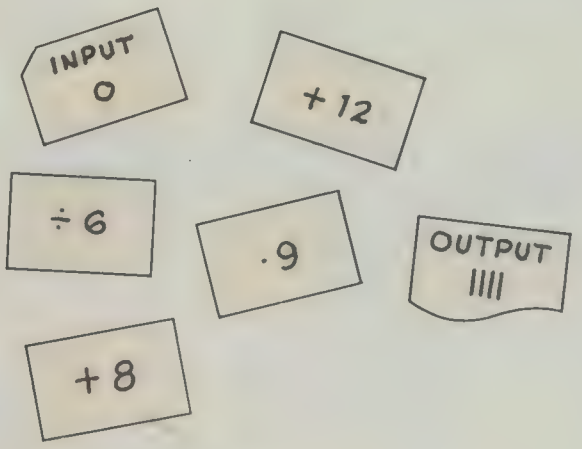


★3. Arrange these cards into flow charts so that they will have the following outputs.

A 14 D 10

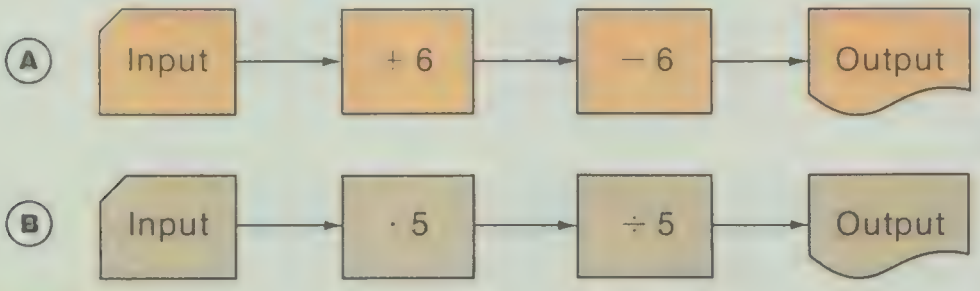
B 26 E 24

C 30 F 180



Investigating the Ideas

Study flow charts A and B below. Try various input numbers for each flow chart.



What do you notice about the output numbers in the two flow charts?

Discussing the Ideas

1. Adding and subtracting the same number are opposite in effect and are called **inverse operations**.
Are the operations multiplying and dividing by the same number inverse operations?

2. The table shows how you can think of subtraction as finding a missing addend. In each equation, tell whether the missing number represents an addend or a sum. Then give the missing number.

   A $7 + 3 = \square$ C $7 - 3 = \square$

   B $6 + \square = 10$ D $8 - \square = 3$

When you find this addend, A A S $n + 6 = 9$

you have found this difference. $9 - 6 = \square$

3. Study the table at the right Tell whether the missing number in each equation is a factor or a product. Then give the missing number.

   A $9 \cdot \square = 36$ C $24 \div \square = 4$

   B $5 \cdot 7 = \square$ D $\square \div 8 = 7$

When you find this factor. $\square \cdot 4 = 28$

you have found this quotient. $28 \div 4 = \square$

1. Solve the equations. EXAMPLE: $6 + 5 = a$, $a = 11$

A $4 + 5 = a$	D $8 + 4 = m$	G $y + 0 = 8$	J $6 + 6 = n$
B $9 + n = 10$	E $d + 9 = 17$	H $6 + 9 = q$	K $9 + d = 12$
C $b + 6 = 15$	F $0 + r = 13$	I $5 + f = 5$	L $r + 3 = 11$

2. Think about finding the missing addend. Then solve each subtraction equation.

A $? + 6 = 13$ $13 - 6 = r$	B $? + 5 = 11$ $11 - 5 = a$	C $? + 7 = 16$ $16 - 7 = n$
D $12 - 4 = x$	E $8 - 0 = c$	F $14 - 7 = t$
G $10 - 9 = q$	H $7 - 1 = m$	I $16 - 9 = s$

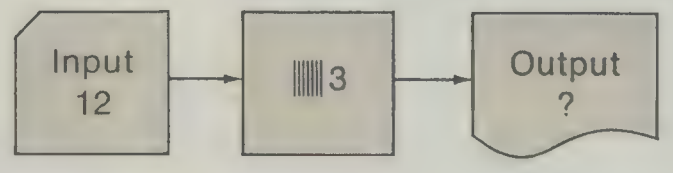
3. Solve the equations.

A $2 \cdot 6 = a$	D $c \cdot 4 = 28$	G $7 \cdot y = 21$	J $t \cdot 4 = 36$
B $8 \cdot n = 40$	E $6 \cdot d = 36$	H $3 \cdot 5 = r$	K $8 \cdot 1 = p$
C $9 \cdot 3 = b$	F $x \cdot 8 = 56$	I $8 \cdot s = 16$	L $7 \cdot m = 49$

4. Think about finding the missing factor. Then solve each division equation.

A $? \cdot 4 = 24$ $24 \div 4 = n$	B $? \cdot 7 = 28$ $28 \div 7 = a$	C $? \cdot 5 = 30$ $30 \div 5 = r$
D $3 \div 3 = t$	E $42 \div 6 = q$	F $40 \div 8 = s$
G $18 \div 2 = h$	H $0 \div 9 = b$	I $7 \div 7 = d$

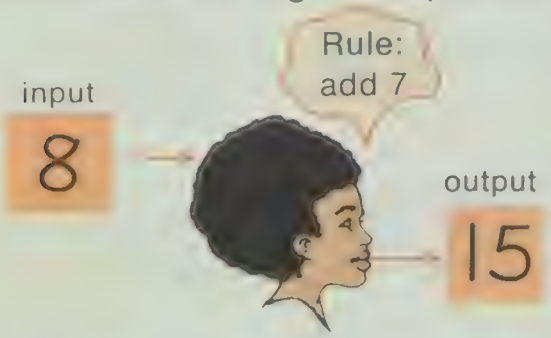
5. Find the output number for the flow chart if the missing operation in the instruction box is

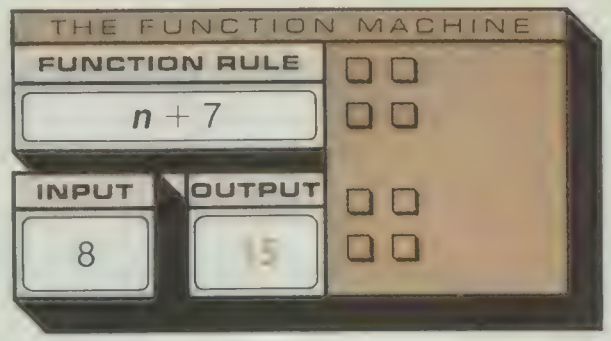


A $+$ B $-$
C $\cdot$ D $\div$

Discussing the Ideas

The picture below can help you understand the **function machine.** The function rule on the function machine tells how to find the output number for a given input number.





For the example above, you can describe the function rule as

output number = input number + 7.

If n denotes the input number and $f(n)$ denotes the output number, we can write $f(n) = n + 7$.

For the input number 8, we write $f(8) = 15$ and we read "f of 8 equals 15."

1. Using the function rule above, tell what number each of these outputs represents.

   A $f(4)$ **B** $f(9)$ **C** $f(6)$ **D** $f(0)$ **E** $f(10)$

2. The input and output numbers for a function rule can be arranged in a **function table.** Give the missing numbers for the table.

Function Rule: $n + 7$

	Input n	Output $f(n)$
A	4	
B	3	
C	7	
D	0	
E	5	

3. Suppose the function rule was $7 - n$. How would you complete the function table for Exercise 2?

4. Explain why the function tables for rules A and B would be the same.

   A $f(n) = 3 \cdot n$ **B** $f(n) = n + n + n$

5. In Exercise 4 above, you can see how repeatedly adding the same number is an operation related to multiplication. Explain how subtraction and division are related in a similar way.

1. Using the function rule $f(n) = n \cdot 9$, give the output number for each of the following input numbers.

A 5 **B** 3 **C** 7 **D** 1 **E** 10 **F** 2 **G** 0 **H** 100

In Exercises 2 through 4, give the missing numbers for each table.

2. Function Rule

	Input n	Output $f(n)$
	2	8
	3	12
A	4	
B	1	
C	6	

3. Function Rule

	Input n	Output $f(n)$
	5	9
A	3	
B	0	
C	6	
D	9	

4. Function Rule

	Input n	Output $f(n)$
A	24	
B	6	
C		3
D		20
E	15	

For Exercises 5 through 8, study each table and invent a suitable function rule.

5.

input n	6	1	4	0	10
output $f(n)$	10	5	8	4	14

6.

input n	3	5	0	8	10
output	6	10	0	16	20

7.

input n	28	70	21	42	7
output	4	10	3	6	1

8.

input n	2	5	1	7	4
output	6	30	2	56	20

think

Copy the triangular array of numbers and try to give the missing numbers. Find the sum of the numbers in each row. Do you see a pattern? Now find the next row.

Investigating the Ideas

When more than one operation is used in the same problem, you can use **parentheses** to show which operations to perform first.

EXAMPLES: $(8 + 3) \cdot 2 = 22$

$8 + (3 \cdot 2) = 14$

Can you place parentheses in the equations so that each equation will be a true statement?

1. $16 - 7 + 8 = 1$
2. $16 - 7 + 8 = 17$
3. $12 \div 2 + 4 = 2$
4. $12 \div 2 + 4 = 10$
5. $3 \cdot 5 - 4 = 11$
6. $3 \cdot 5 - 4 = 3$
7. $18 \div 6 \cdot 3 = 1$
8. $18 \div 6 \cdot 3 = 9$

Discussing the Ideas

1. Show how the expression $9 - 3 + 2$ can be made to represent two different numbers depending upon how parentheses are placed in the expression.

2. Can you place parentheses in the expression $8 + 5 + 7$ so that it will represent two different numbers?

3. Can you place parentheses in the expression $2 \cdot 3 \cdot 4$ so that it will represent two different numbers?

4. When a combination of **multiplication and addition** or of **multiplication and subtraction** is used in an expression without parentheses, we agree to **multiply adjoining numbers first** and then do the adding or subtracting.

   For example, $3 \cdot 7 + 4 = 25$ and $6 + 5 \cdot 2 = 16$.

   Tell what number each of these expressions represents.

   A $9 \cdot 2 + 3$ C $18 - 3 \cdot 5$ E $3 \cdot 5 + 2 \cdot 6$

   B $7 \cdot 3 - 10$ D $12 + 4 \cdot 3$ F $4 \cdot 2 \cdot 3 - 5$

1. Tell what number each expression represents.
Perform the operation in the parentheses first.

A $12 + (3 \cdot 4)$ **C** $(16 \div 4) + 2$ **E** $20 - (8 \div 4)$
B $8 - (5 + 2)$ **D** $(6 \cdot 5) - 15$ **F** $(20 - 8) \div 4$

2. The examples illustrate how to use parentheses in equations.
Solve the equations.

A $(9 - 3) + 2 = n$
$6 + 2 = n,\ n = 8$

B $16 \div (6 + 2) = m$
$16 \div 8 = m,\ m = 2$

A $(14 - 9) + 7 = x$ **G** $(3 \cdot 6) \div 2 = n$ **M** $6 \cdot (6 \div 6) = u$
B $9 + (10 - 2) = n$ **H** $5 + (8 - 3) = a$ **N** $6 \div (6 \div 6) = v$
C $(12 + 4) - 9 = a$ **I** $4 \cdot (7 + 2) = c$ **O** $(6 \cdot 6) \div 6 = w$
D $16 - (8 + 1) = d$ **J** $(21 \div 3) + 6 = f$ **P** $6 + (6 \div 6) = x$
E $10 + (6 - 2) = q$ **K** $18 - 6 \cdot 3 = b$ **Q** $(6 + 6) \div 6 = y$
F $(17 - 8) + 4 = h$ **L** $(24 \div 6) \cdot 8 = m$ **R** $6 - (6 \div 6) = z$

3. Solve the equations.

A $7 \cdot 4 + 5 = r$ **C** $8 \cdot 3 - 7 \cdot 2 = x$ **E** $9 \cdot 8 - 9 \cdot 7 + 9 \cdot 6 = n$
B $12 - 2 \cdot 4 = t$ **D** $2 \cdot 3 \cdot 5 + 1 = y$ **F** $(10 \cdot 9 - 14) \div 2 = s$

4. Give the missing numbers in each function table.

A

Function Rule: $n \cdot 3 - 2$	
n	$f(n)$
2	
5	
8	
10	
7	

F

Function Rule: $20 - 2 \cdot n$	
n	$f(n)$
1	
3	
6	
7	
10	

K

Function Rule: $(n \div 3) - 1$	
n	$f(n)$
15	
9	
21	
30	
18	

★ **5.** Solve the equations.

A $2 \cdot x + 3 = 13$ **C** $7 + 2 \cdot n = 27$ **E** $2 \cdot b + 2 \cdot b = 20$
B $3 \cdot n + 4 = 10$ **D** $10 - 5 \cdot a = 0$ **F** $3 \cdot d + 2 \cdot d = 35$

More practice, page S-5, Set 7

Discussing the Ideas

1. Study the statements concerning the commutative principles.
   - **A** Can you tell, in your own words, what the commutative principle means?
   - **B** Illustrate the commutative principle for addition when $a = 12$ and $b = 34$.

Commutative ( ) principles
If a and b are any whole numbers, then $a + b = b + a$ and $a \cdot b = b \cdot a$

   - **C** Illustrate the commutative principle for multiplication when $a = 9$ and $b = 12$.

2. Study the associative principles.
   - **A** Is $289 + (746 + 868)$ the same sum as $(289 + 746) + 868$?
   - **B** Illustrate the associative principle for multiplication using $a = 5$, $b = 4$, and $c = 2$.

Associative ( ) principles
If a, b and c are any whole numbers, then [illegible] and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

3. Together, the commutative and associative principles assure us that **we can arrange three or more addends or factors in any convenient way and still get the same sum or product.**

   EXAMPLES: $(7 + 9) + 3 = (7 + 3) + 9$ $\quad 4 \cdot (25 \cdot 9) = (25 \cdot 4) \cdot 9$

   - **A** Explain how the commutative and associative principles for multiplication can help you quickly find the product $5 \cdot 39 \cdot 20$.

4. Study the zero and one principles. Use the inverse relation between addition and subtraction and the **zero principle** to explain why

   $a - 0 = a$ and $a - a = 0$

5. Use the inverse relation between multiplication and division and the **one principle** to explain why

   $a \div 1 = a$ and $a \div a = 1$

The zero principle
Zero is the **identity element** for addition. This means that for each whole number a, $a + 0 = a$

The one principle
The number 1 is the **identity element** for multiplication. This means that for each whole number a, $a \cdot 1 = a$

6. Use the inverse relation between multiplication and division to explain the following statements:
   - (A) We can divide zero by every whole number except zero.
   - (B) We never divide any number by zero.

1. Tell whether each equation illustrates the commutative principle, the associative principle, or both principles.

   A $8 + 7 = 7 + 8$
   B $5 + (6 + 2) = (6 + 2) + 5$
   C $(4 + 3) + 1 = 4 + (3 + 1)$
   D $8 \cdot 7 = 7 \cdot 8$
   E $5 \cdot (6 \cdot 2) = (6 \cdot 2) \cdot 5$
   F $(3 \cdot 4) + 6 = 6 + (3 \cdot 4)$

2. Solve the equations. Think of the basic principles to help you solve them quickly.

   A $37 + 26 = n + 37$
   B $51 \cdot 24 = 24 \cdot a$
   C $(56 + 2) + 13 = 56 + (d + 13)$
   D $8 \cdot (7 \cdot 6) = (b \cdot 7) \cdot 6$
   E $(4 \cdot 3264) \cdot 7 = 4 \cdot (s \cdot 7)$
   F $6 + (156 + 38) = (n + 156) + 38$

3. Use the commutative and associative principles to find each sum or product.

   A $99 + (47 + 1)$
   B $96 + (79 + 4)$
   C $999 + (356 + 1)$
   D $(200 + 653) + 800$
   E $5 \cdot (13 \cdot 2)$
   F $(25 \cdot 41) \cdot 4$
   G $25 \cdot (37 \cdot 4)$
   H $(50 \cdot 97) \cdot 2$
   I $(125 \cdot 93) \cdot 8$

4. Solve the equations.

   A $67 + x = 67$
   B $92 - x = 0$
   C $74 - x = 74$
   D $67 \cdot a = 67$
   E $92 \div a = 1$
   F $74 \div a = 74$
   G $87 + (x - 12) = 87$
   H $87 \cdot (a \div 12) = 87$
   I $96 - (x + 96) = 0$

5. Both addition and multiplication are associative. Do you think that subtraction is associative? That is, is it true that if the subtractions can be performed for all numbers a, b, and c, then $a - (b - c) = (a - b) - c$?

   Answer the same question for division. Is it true that for all numbers a, b, and c, then $a \div (b \div c) = (a \div b) \div c$?

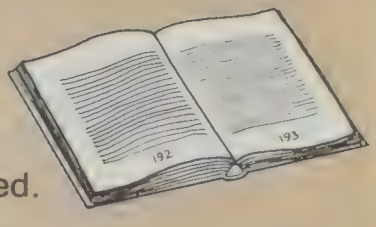
6. Which of the expressions do not represent numbers?

   A $0 \div 6$ B $6 \div 0$ C $0 \div 0$

★7. Suppose two students both choose a whole number. What can you say about the numbers if

   A their product is zero?
   B their product is 1?

think



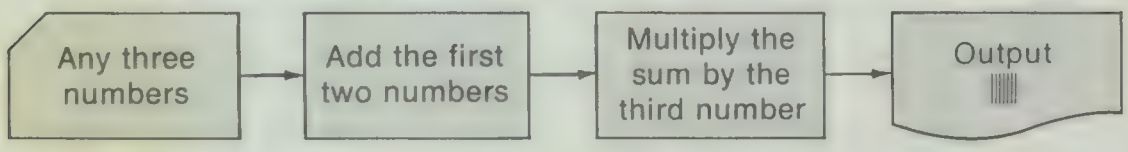
In numbering the pages of a book 495 digits were used.

How many pages were there?
How many 9's were used?

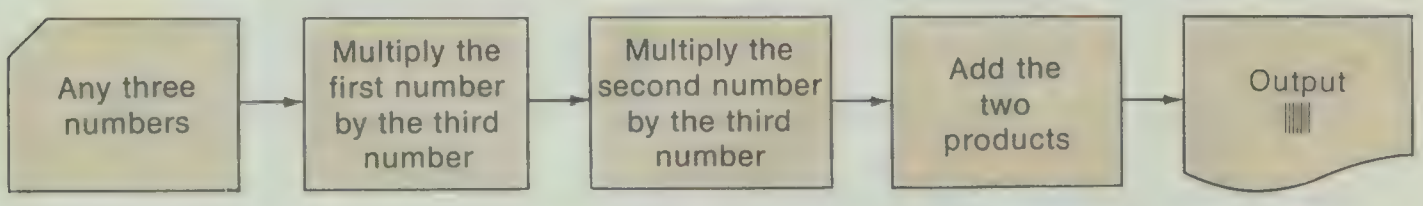
The Distributive Principle

Investigating the Ideas

Study the flow charts and then find the output for each one.



27, 73, and 10 $\quad$ $27 + 73$ $\quad$ $10 \cdot (27 + 73)$ $\quad$?


Any three numbers
Multiply the first number by the third number
Multiply the second number by the third number
Add the two products
Output

27, 73, and 10 $\quad$ $10 \cdot 27$ $\quad$ $10 \cdot 73$ $\quad$ $10 \cdot 27 + 10 \cdot 73$ $\quad$?

Choose three more input numbers. Then find the output.

What can you say about the two output numbers in the flow charts above for any three numbers?

Discussing the Ideas

1. The flow charts illustrate the **distributive principle for multiplication with respect to addition.**
The flow charts show that
$10 \cdot (27 + 73) = (10 \cdot 27) + (10 \cdot 73)$.
Illustrate the distributive principle using $a = 20$, $b = 34$, and $c = 9$.

Distributive principle
If a, b, and c are any whole numbers, then
$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$

2. The sets below may help you to better understand the distributive principle. Describe and explain each display and then give the number for n.

$4 \cdot 5$ $\quad = \quad$ $4 \cdot (2 + 3)$ $\quad = \quad$ $(n \cdot 2) + (4 \cdot 3)$

3. Explain how to find the number for n in each equation.

A $5 \cdot 6 + 5 \cdot 3 = 5(6 + 3) = 5 \cdot n$ $\qquad$ B $3 \cdot 7 + 3 \cdot 6 = 3(n + 6) = 3 \cdot 13$

1. Complete each equation in the left column by choosing the correct expression from the right column.

A	$8 \cdot (7 + 5) =$ ▨	A	$7 \cdot (8 + 5)$
B	$(6 \cdot 2) + (9 \cdot 2) =$ ▨	B	$(9 \cdot 2) + (9 \cdot 6)$
C	$(7 \cdot 8) + (7 \cdot 5) =$ ▨	C	$(8 \cdot 7) + (8 \cdot 5)$
D	$5 \cdot (7 + 8) =$ ▨	D	$(6 + 9) \cdot 2$
E	$9 \cdot (2 + 6) =$ ▨	E	$(9 + 2) \cdot 6$
F	$(9 \cdot 6) + (2 \cdot 6) =$ ▨	F	$(5 \cdot 7) + (5 \cdot 8)$

2. Solve the equations.

A $5 \cdot 8 = 5 \cdot (6 + a) \rightarrow 5 \cdot 8 = (5 \cdot 6) + (5 \cdot a)$

B $6 \cdot 4 = 6 \cdot (n + 1) \rightarrow 6 \cdot 4 = (6 \cdot n) + (6 \cdot 1)$

C $4 \cdot 9 = 4 \cdot (3 + b) \rightarrow 4 \cdot 9 = (4 \cdot 3) + (4 \cdot b)$

3. Solve the equations.

A $(6 \cdot 5) + (6 \cdot 1) = 6 \cdot a$

B $(5 \cdot 3) + (5 \cdot 4) = 5 \cdot m$

C $(8 \cdot 10) + (8 \cdot 3) = 8 \cdot n$

D $(5 \cdot 10) + (5 \cdot 7) = 5 \cdot s$

E $(6 \cdot 30) + (6 \cdot 5) = 6 \cdot y$

F $(4 \cdot 70) + (4 \cdot 8) = 4 \cdot r$

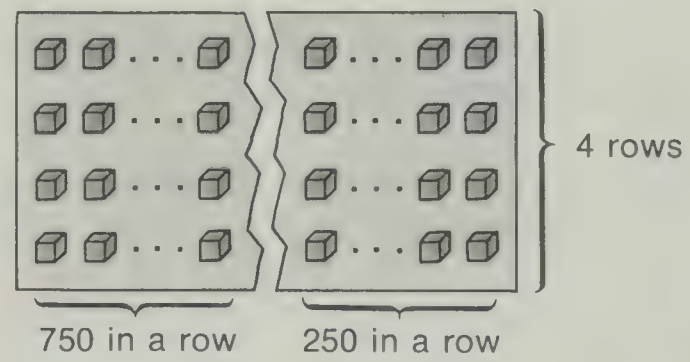
G $(7 \cdot 100) + (7 \cdot 50) = 7 \cdot a$

H $(6 \cdot 600) + (6 \cdot 10) + (6 \cdot 9) = 6 \cdot c$

I $(8 \cdot 400) + (8 \cdot 30) + (8 \cdot 8) = 8 \cdot m$

J $(5 \cdot 900) + (5 \cdot 90) + (5 \cdot 9) = 5 \cdot q$

4. The small black dots are to indicate that many cubes have been left out. Give the total number of cubes, without pencil-and-paper computation.



5. Copy and complete the equations.

A $a \cdot (b + c) = (a \cdot b) +$ ▨

B $s \cdot (r + t) =$ ▨ $+ (s \cdot t)$

C ▨ $\cdot (c + d) = (n \cdot c) + (n \cdot d)$

D $b \cdot ($▨ $+ n) = (b \cdot m) + (b \cdot n)$

E $r \cdot (a +$ ▨$) = (r \cdot a) + (r \cdot y)$

F $f \cdot (g + h) =$ ▨ $+$ ▨

G ▨ $= (a \cdot n) + (a \cdot m)$

H ▨ $= (b \cdot s) + (b \cdot s)$

Investigating the Ideas

Some of the digits are missing in the addition and subtraction problems shown below.

1.
$$\begin{array}{r} \square 937 \\ 8\square \\ 109 \\ +4\square 85 \\ \hline 85\square 7 \end{array}$$

2.
$$\begin{array}{r} 5373\square \\ \square 1059 \\ \square 006 \\ +\quad 69 \\ \hline \square 31\square\square 1 \end{array}$$

3.
$$\begin{array}{r} 7\square 25 \\ -384\square \\ \hline \square 7\square 3 \end{array}$$

4.
$$\begin{array}{r} 6\square 02 \\ -435\square \\ \hline 162\square \end{array}$$

Can you find the missing digits in each of the problems?

Discussing the Ideas

In Examples A, B, C, and D, some of the digits are covered by screens.

1. Explain why the tens' digit in the sum for Example A can be either 6 or 7.

2. Explain why the hundreds' digit in the sum for Example B can be 1, 2, or 3.

3. Explain why the number of tens in the difference for Example C is either 3 or 2.

4. What are the possible digits for the ▪ in Example D?

5. What are some methods you can use to help you find sums rapidly and accurately?

6. A How can you check your work in an addition problem?
   B How can you check your work in a subtraction problem?

A

$$\begin{array}{r} \square 7\square \\ +\square 9\square \\ \hline \square\blacksquare\square \end{array}$$

B

$$\begin{array}{r} \square 8\square\square \\ \square 5\square\square \\ +\square 8\square\square \\ \hline \square\square\square\square \end{array}$$

C

$$\begin{array}{r} \square 7\square \\ -\square 4\square \\ \hline \square\blacksquare\square \end{array}$$

D

$$\begin{array}{r} \square 5\square \\ -\square 8\square \\ \hline \square\square\square \end{array}$$

1. Find the sums.

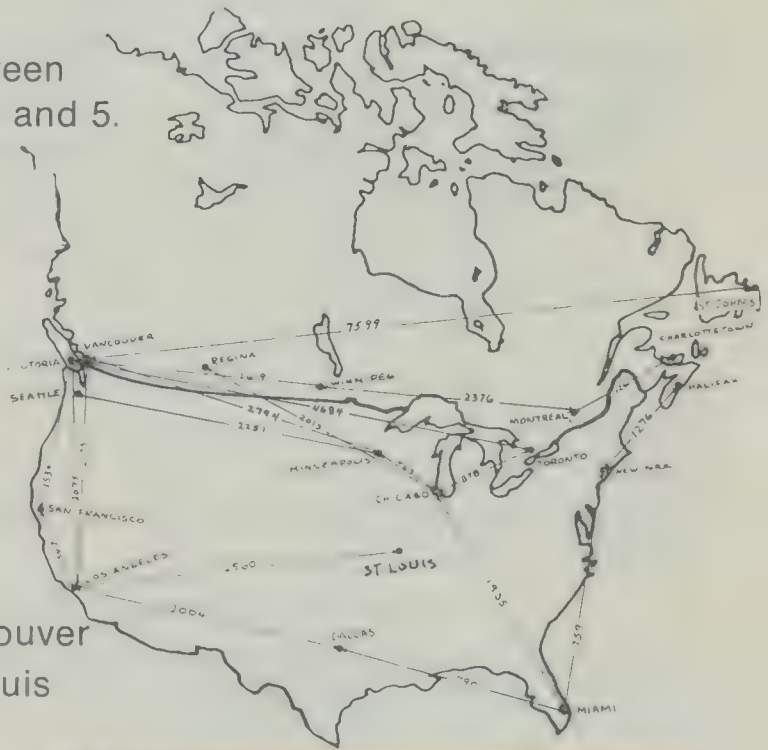
A 37 + 52	**B** 86 + 37	**C** 534 + 256	**D** 744 + 257	**E** 2898 + 7265
F 217 + 314 + 253	**G** 328 + 426 + 153 + 867	**H** 2347 + 981 + 49 + 6	**I** 6243 + 1527 + 4639 + 7254	**J** 28 705 + 44 192 + 89 763 + 17 541

K $57 + 41 + 96$ **L** $123 + 174 + 6209$ **M** $2338 + 5620 + 23\,746$

2. Find the differences.

A 87 − 35	**B** 92 − 68	**C** 131 − 49	**D** 862 − 777	**E** 1207 − 849
F 5200 − 2637	**G** 7891 − 4987	**H** 9451 − 5789	**I** 23 742 − 19 888	**J** 987 654 − 321 098

The map shows airline distances (km) between some cities. Use the map for Exercises 3, 4, and 5.



3. Give the total distance for each trip.

- **A** St. John's–Vancouver–Los Angeles
- **B** Regina–Chicago–Miami
- **C** Seattle–Minneapolis–Chicago
- **D** Miami–New York–Halifax

4. Which of the two trips is the shorter? How much shorter is it?

- **A** Toronto–Chicago–Minneapolis–Vancouver
- **B** Toronto–Victoria–Los Angeles–St. Louis

5. What is the total airline distance covered on a trip from Charlottetown to Montreal to Winnipeg to Victoria to San Francisco to Los Angeles to Dallas to Miami to New York?

Think

The sum of two numbers is 56.
Their difference is 18.
What are these two numbers?

$a + b = 56$
$a - b = 18$

More practice, pages S-5 and 6, Sets 8 and 9

NORTH AMERICA

REGION	AREA (km²)	POPULATION (1970 est.)
British Honduras	22 965	130 000
Canada	9 974 890	21 681 000
Canal Zone	1432	45 000
Costa Rica	50 699	1 800 000
Cuba	109 345	8 553 000
El Salvador	21 393	3 534 000
Guatemala	108 889	5 189 000
Honduras	112 087	2 582 000
Mexico	1 963 891	48 313 000
Nicaragua	139 699	1 984 000
Panama	75 649	1 464 000
United States	9 346 460	204 766 000

1. How much larger in area is Canada than the United States?

2. What is the difference in the areas of Nicaragua and Cuba?

3. How many more people live in Mexico than in Canada?

4. **A** Find the total population of Guatemala and Honduras.
   B Find the difference between the total in **A** and the population of Cuba.

5. Find the total area of North America.

6. Find the total population of North America.

7. How much more is the population of the United States than all the remaining North American regions together?

Money Problems

1. What is the total cost of these groceries?

2. Janet had $10.00. She bought a blouse for $5.69. How much change did she get back?

3. A sports car has a list price of $3096. With "optional equipment," the price is $3435. How much extra does the "optional equipment" cost?

4. Mr. Johnson bought a house for $17 960. He made a down payment of $3592. How much more does he owe on the house?

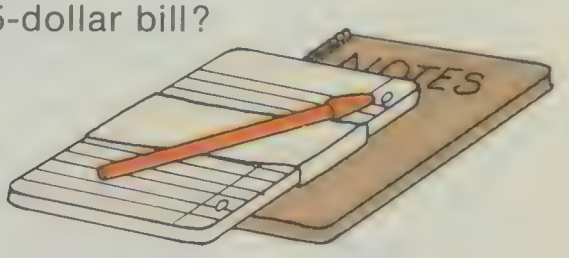
5. The Watson family had these expenses for a certain month:

$265.00	Housing
210.85	Food
36.95	Clothing
46.72	Utilities
30.90	Recreation
79.53	Transportation

What is the total amount of the expenses for the month?

6. Paul saw a 10-speed bicycle advertised for $83.60. He has already saved $25.95 for a bicycle. How much more money must Paul save before he can purchase a bicycle?

7. Lisa bought some school supplies. She paid 69¢ for paper, 98¢ for a ball point pen, and 79¢ for a notebook. How much change should she get back if she paid for the supplies with a 5-dollar bill?



8. Martin earns $10.00 per week doing part time work. One week he spent $3.75 for recreation and $2.60 for snacks. He put the rest in his savings account. How much did Martin save that week?

think

Rick has 44 pennies. Can he put a different number of pennies in each of his pockets if he has a total of ten pockets?

Investigating the Ideas

Study the function tables below.

A

Function Rule [redacted]

n	$f(n)$
2	20
7	70
67	670

B

Function Rule [redacted]

n	$f(n)$
6	600
13	1300
145	14 500

C

Function Rule [redacted]

n	$f(n)$
30	3
150	15
490	49

Can you discover a function rule for each table?

List a few more number pairs for each table.

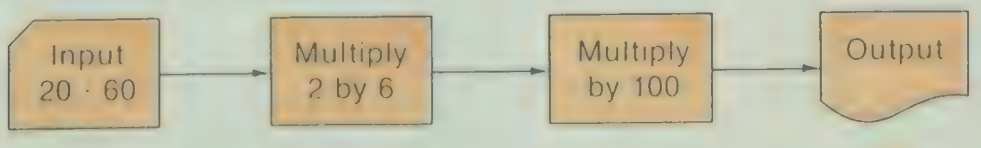
Discussing the Ideas

1. Many special products and quotients involving multiples of 10, 100, and 1000 can easily be found by using the basic multiplication facts, the basic principles, and the inverse relation between multiplication and division.
   - **A** Explain how to find the product of any whole number and 10 mentally.
   - **B** How can you multiply any number by 100? by 1000?

2. The diagram shows some steps in finding the product of 30 and 40. Give the missing number in each step.

$$30 \cdot 40 = (3 \cdot a) \cdot (4 \cdot 10)$$
$$= (3 \cdot 4) \cdot b$$
$$= c$$

3. The flow chart shows a shortcut for multiplying $20 \cdot 60$. What is the product?



$20 \cdot 60 = 2 \cdot 6 \times 100 = $ [redacted]

4. If you know that $20 \cdot 80 = 1600$, explain how you can find these quotients. $1600 \div 20$ and $1600 \div 80$

1. Find the products.

A	$5 \cdot 10$	C	$8 \cdot 100$	E	$100 \cdot 6$	G	$9 \cdot 10$	I	$100 \cdot 7$	K	$9 \cdot 1000$
B	$10 \cdot 7$	D	$2 \cdot 10$	F	$3 \cdot 100$	H	$10 \cdot 4$	J	$5 \cdot 100$	L	$1000 \cdot 40$

2. Find the number for a. Then find the number for b.

A $10 \cdot 10 = a \rightarrow 7 \cdot 10 \cdot 10 = b$

B $7 \cdot 10 \cdot 10 = a \rightarrow 70 \cdot 10 = b$

C $10 \cdot 100 = a \rightarrow 4 \cdot 10 \cdot 100 = b$

D $4 \cdot 10 \cdot 100 = a \rightarrow 40 \cdot 100 = b$

3. Find the products.

A	$40 \cdot 10$	D	$100 \cdot 70$	G	$4 \cdot 300$	J	$120 \cdot 20$	M	$200 \cdot 20$		
B	$10 \cdot 30$	E	$20 \cdot 40$	H	$60 \cdot 40$	K	$60 \cdot 1000$	N	$90 \cdot 300$		
C	$60 \cdot 100$	F	$30 \cdot 50$	I	$50 \cdot 40$	L	$70 \cdot 30$	O	$800 \cdot 60$		

4. Find the quotients.

A Since $60 \cdot 30 = 1800$, we know that $1800 \div 30 = a$.

B Since $40 \cdot 20 = 800$, we know that $800 \div 20 = c$.

C Since $70 \cdot 40 = 2800$, we know that $2800 \div 40 = x$.

D Since $80 \cdot 20 = 1600$, we know that $1600 \div 20 = m$.

E Since $50 \cdot 300 = 15\,000$, we know that $15\,000 \div 300 = r$.

5. Think about the missing factor. Then give the quotient.

A $? \cdot 30 = 600$

$600 \div 30 = n$

B $? \cdot 40 = 2400$

$2400 \div 40 = a$

C $? \cdot 50 = 1500$

$1500 \div 50 = t$

D $r \cdot 60 = 1800$

$1800 \div 60 = r$

E $m \cdot 8 = 1600$

$1600 \div 8 = m$

F $s \cdot 300 = 1200$

$1200 \div 300 = s$

6. Find the quotients.

A	$400 \div 8$	C	$1200 \div 60$	E	$63\,000 \div 700$
B	$6000 \div 3$	D	$2700 \div 90$	F	$280\,000 \div 4000$

7. Find the products and quotients. Write your answers in scientific notation.

A	$3 \cdot 10^3 \cdot 2 \cdot 10^4$	E	$(7 \cdot 10^7) \div 7$	I	$500 \cdot 10^2 \cdot 20 \cdot 10^3$
B	$5 \cdot 10^4 \cdot 4 \cdot 10^5$	F	$50 \cdot 10^3 \cdot 10^2$	J	$3\,000\,000 \div (30 \cdot 10^3)$
C	$4 \cdot 10^6 \div 10^3$	G	$60\,000 \div (2 \cdot 10^2)$	K	$(2 \cdot 10^{10}) \div (10 \cdot 10^2)$
D	$(3 \cdot 10^8) \div 10^8$	H	$100\,000 \div (5 \cdot 10^4)$	L	$10 \cdot 10^4 \cdot 4 \cdot 10^{10}$

More practice, page S-6, Set 10

Investigating the Ideas

Find each product. Then find the sum of the products.

300×6 $\quad$ 20×6 $\quad$ 9×6

Can you find a number such that when multiplied by 6 will have a product equal to the sum above?

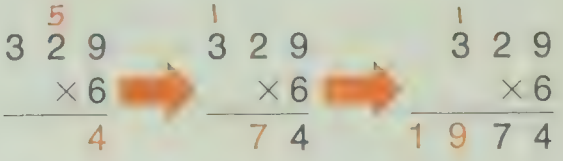
Discussing the Ideas

1. The distributive principle can help you find the number in the Investigation.

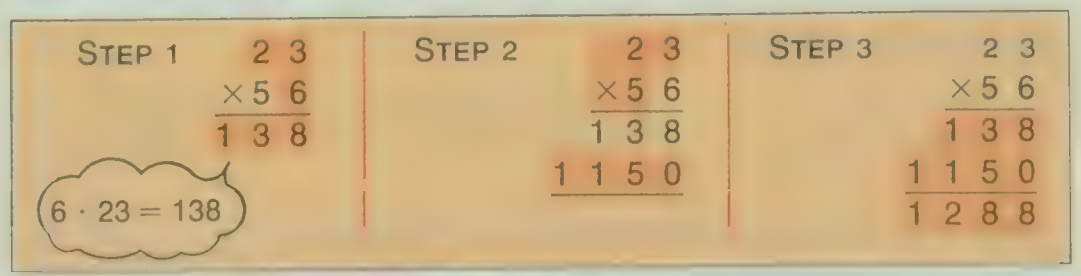
   $6 \cdot 300 + 6 \cdot 20 + 6 \cdot 9 = 6 \cdot (300 + 20 + 9) = 6 \cdot n$

   What is the number for n?

2. Explain the steps in this shortcut method for finding products.



3. The distributive principle is also applied in the shortcut method of multiplying when factors with more than one digit are used. Explain what you do in steps 2 and 3 to complete the work.



4. Tell which two factors give the partial product covered by each row of ▒. Then give this partial product. Find the total product.

THE HUMAN BODY

1. Your heart, under normal conditions, beats about 72 times per minute. About how many times does your heart beat during
   - **A** 1 hour?
   - **B** a 24-hour day?
   - **C** a 365-day year?
   - **D** a 70-year lifetime?

2. We breathe (inhale and exhale) at a rate of about 18 times per minute. About how many times do we breathe in an hour at this rate?

3. An adult man weighs 75 kilograms. An adult whale can weigh 1250 times as much. How many kilograms can an adult whale weigh?

4. A person uses about 275 joules of energy per hour while sleeping. How many joules are used by a person who sleeps from 9:30 P.M. to 7:30 A.M.?

5. A student uses about 550 joules of energy per hour while sitting at a desk in the classroom. How many joules does the student use during a 6-hour school day?

6. The brain of a baby weighs about 340 grams. The weight of a baby is 12 times his brain weight. About how many kilograms does the baby weigh?

7. An adult weighs about 35 times his brain weight. If a man's brain weight is about 2 kilograms, what is his body weight?

8. Find the products.

A 32×43	**B** 32×54	**C** 27×58	**D** 352×46	**E** 257×84	**F** 368×29
G 423×514	**H** 367×425	**I** 378×496	**J** 3524×86	**K** 4763×584	**L** 8732×4658

9. Rewrite the following multiplication problems replacing each ▮ by a digit so that the answers are correct.

A
$$\begin{array}{r} \blacksquare\blacksquare \\ \times\ 5\ 4 \\ \hline \blacksquare\blacksquare\blacksquare \\ \blacksquare\blacksquare\blacksquare \\ \hline 3\ \blacksquare\ 3\ 4 \end{array}$$

B
$$\begin{array}{r} 9\ 9 \\ \times\ \blacksquare \\ \hline \blacksquare\blacksquare\blacksquare \\ \blacksquare\blacksquare\blacksquare \\ \hline 2\ 2\ \blacksquare\blacksquare \end{array}$$

C
$$\begin{array}{r} \blacksquare\blacksquare\blacksquare \\ \times\ 7\ 7 \\ \hline \blacksquare\blacksquare\blacksquare\blacksquare \\ \blacksquare\blacksquare\blacksquare \\ \hline 2\ 7\ 6\ \blacksquare\blacksquare \end{array}$$

D
$$\begin{array}{r} \blacksquare\blacksquare\blacksquare \\ \times\ 9\ 9\ 9 \\ \hline \blacksquare\blacksquare\blacksquare\blacksquare \\ \blacksquare\blacksquare\blacksquare\blacksquare \\ \blacksquare\blacksquare\blacksquare \\ \hline 4\ 7\ \blacksquare\blacksquare\ 4 \end{array}$$

More practice, page S-7, Set 11

Estimating Large Products

Investigating the Ideas

Try one or more of these estimation problems.

A. Estimate your age in seconds.

B. Estimate your height in millimetres.

C. Estimate your total mass in grams.

Can you find a way of checking your estimate?

Discussing the Ideas

1. **A** Explain how you made your estimates.
   B Were your estimates too large or too small?

2. In estimation problems, you can often choose the nearest multiples of 10, 100, or 1000 to represent the numbers in the problems.
   Tell which of the two multiples of 10, 100, or 1000 is nearer to the number given in color.

   A 70, 78, 80 **C** 100, 193, 200 **E** 100, 149, 200 **G** 6000, 6281, 7000
   B 30, 31, 40 **D** 600, 629, 700 **F** 800, 850, 900 **H** 5000, 5706, 6000

3. Study the example for estimating products in the table below.
   Give the missing **estimated** products for parts B and C.

	Product	We think	We say
A	$\begin{array}{r} 78 \\ \times\ 9 \\ \hline \end{array}$	$80 \cdot 9 = 720$	$78 \cdot 9$ is approximately 720
B	$\begin{array}{r} 815 \\ \times\ 31 \\ \hline \end{array}$	$800 \cdot 30 =$ ▒▒	$815 \cdot 31$ is approximately ▒▒
C	$\begin{array}{r} 629 \\ \times\ 193 \\ \hline \end{array}$	$600 \cdot 200 =$ ▒▒	$629 \cdot 193$ is approximately ▒▒

4. Give an estimate for each product.

   A $69 \cdot 7$ **B** $209 \cdot 8$ **C** $72 \cdot 81$ **D** $427 \cdot 29$

1. Round the first factor to the nearest multiple of 10. Then give an estimate of the product.

   A $78 \cdot 4$ **B** $92 \cdot 8$ **C** $31 \cdot 6$ **D** $84 \cdot 5$ **E** $59 \cdot 9$

2. Round the first factor to the nearest multiple of 100. Then give an estimate of the product.

   A $621 \cdot 8$ **B** $978 \cdot 6$ **C** $409 \cdot 7$ **D** $894 \cdot 5$ **E** $719 \cdot 9$

3. Round each factor to the nearest multiple of 10. Then give an estimate of the product.

   A $23 \cdot 41$ **B** $79 \cdot 52$ **C** $67 \cdot 19$ **D** $45 \cdot 81$ **E** $83 \cdot 58$

4. Give an estimate for each part.

   A A man drove his car at an average speed of 78 kilometres per hour (km/h) for 9 hours. About how far did he travel?

   B An airplane travels for 5 hours at an average speed of 921 kilometres per hour. About how far does it travel?

   C Ball point pens cost 39¢. About how much money would 31 students have to pay for ball point pens?

   D Terry had to take 1267 steps to walk one kilometre. About how many steps would she have to take to walk four kilometres?

   E In one section of a football stadium there are 27 rows with 63 seats in each row. About how many seats are in this section?

   F Sound travels about 332 metres per second in air. About how far does it travel in a minute?

5. From the set of numbers {10, 20, 30, 40, 50, 60, 70, 80, 90}, select the largest number that will make the sentence true.

   A $n \cdot 6 < 307$ **C** $n \cdot 9 < 555$ **E** $n \cdot 4 < 281$ **G** $n \cdot 7 < 644$

   B $n \cdot 8 < 658$ **D** $n \cdot 5 < 127$ **F** $n \cdot 3 < 179$ **H** $n \cdot 6 < 444$

6. Find the largest whole number for n that will make the sentence true.

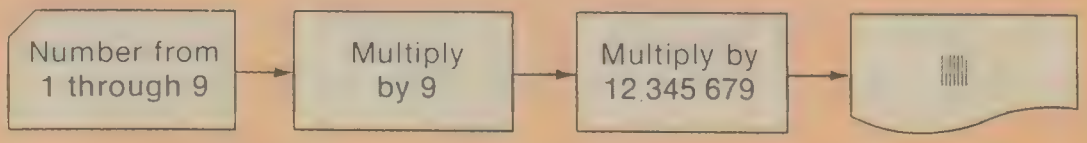
   A $n \cdot 32 < 137$ **D** $n \cdot 83 < 681$ **G** $n \cdot 72 < 584$

   B $n \cdot 57 < 381$ **E** $n \cdot 91 < 508$ ★ **H** $n \cdot 714 < 2347$

   C $n \cdot 23 < 118$ **F** $n \cdot 48 < 380$ ★ **I** $n \cdot 579 < 3827$

Flash

Find the output number for this flow chart.



Investigating the Ideas

How well can you divide when the divisor has only one digit?

1. $3\overline{)87}$	2. $7\overline{)126}$	3. $8\overline{)234}$
4. $9\overline{)4621}$	5. $6\overline{)7041}$	6. $5\overline{)8003}$

Can you find the quotients and remainders without getting more than one incorrect answer?

Check your answers by using the code below.

CODE: 1 = △, 2 = □, 3 = ⌓, 4 = ⏢, 5 = ☆, 6 = ○, 7 = ●, 8 = ◎, 9 = ◎, 0 = ▯

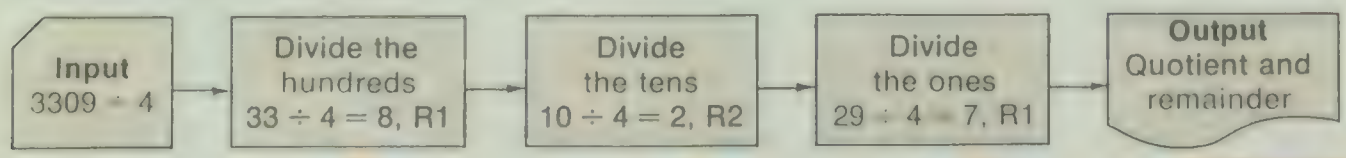
Coded Answers:
1. □◎
2. △◎
3. □◎, R □
4. ☆△⌓, R⏢
5. △△●⌓, R⌓
6. △○▯▯, R⌓

Discussing the Ideas

1. Refer to the example at the right to answer the questions.
   - A How many sevens were subtracted the first time?
   - B How many sevens were subtracted the second time? third time?
   - C All total, how many sevens were subtracted?
   - D How does the remainder compare in size to the divisor?
   - E What was done to check to see if the answer was correct.

Divisor	Quotient		CHECK
	236 ←		
7	1656 Dividend		236
	−1400 ←	200 · 7	× 7
	256		1652
	−210 ←	30 · 7	+ 4
	46		1656
	−42 ←	6 · 7	
	4 Remainder		

2. When the divisor is a number less than 10, you can use a shortcut method called "short division." Study the flow chart and the example given below it.



	8	82	827, R1	827, R1
$4\overline{)3309}$	$4\overline{)3309}$	$4\overline{)3309}$	$4\overline{)3309}$	$4\overline{)3309}$

Try this method on these problems. Check your work.

A $6\overline{)2491}$ B $8\overline{)6213}$ C $9\overline{)4239}$ D $7\overline{)6905}$

1. Use "short division" to find the quotients and remainders.

A $8\overline{)386}$	E $9\overline{)4105}$	I $3\overline{)12\,345}$	M $5\overline{)36\,628}$
B $9\overline{)314}$	F $7\overline{)18\,488}$	J $4\overline{)81\,624}$	N $2\overline{)3\,584\,286}$
C $6\overline{)4455}$	G $9\overline{)61\,407}$	K $9\overline{)2300}$	O $9\overline{)111\,111\,111}$
D $5\overline{)3646}$	H $7\overline{)1001}$	L $4\overline{)70\,771}$	P $3\overline{)186\,001}$

2. On a field trip 4 buses were used to transport 216 students. Each bus contained the same number of students. How many students were on each bus?

3. The Jameson family traveled 865 kilometres in 9 hours. About how many kilometres per hour did they average?

4. John worked in a supermarket. He had to put 9 oranges into each produce bag. How many bags would 1000 oranges fill? How many oranges would be left over?

★5. If the remainder is zero, what is the ones' digit of each dividend? What is the quotient?

A $7\overline{)482\,35\square}$ B $8\overline{)34\,15\square}$ C $6\overline{)31\,22\square}$ D $9\overline{)347\,91\square}$

★6. If the remainder is zero, there are two possibilities for the ones' digit in each dividend. Give both possibilities.

A $6\overline{)31\,51\square}$ B $4\overline{)71\,43\square}$ C $6\overline{)3\,00\square}$ D $7\overline{)68\,71\square}$

★7. What division fact is shown by the repeated subtraction?

5313
−4600
713
−690
23
−23
0

Think

What are the digits in the division problem? Each letter stands for a single digit.

$$\begin{array}{r} b\ b \\ a\overline{)b\ c\ c} \\ a \\ \hline b\ c \\ a \\ \hline b \end{array}$$

More practice, page S-7, Set 12

Discussing the Ideas

1. You can find the quotient for $67\overline{)1594}$ by finding one-digit quotients for $67\overline{)159}$ and $67\overline{)254}$.

   A What are these quotients?

   B Explain how the one-digit quotients are used in the two steps of the division problem at the right.

   C What is the quotient?

   D What is the remainder?

   E What would you do to check the division?

Step 1 Dividing tens	Step 2 Dividing ones
$\begin{array}{r} \quad 2 \\ 67\overline{)1594} \\ -134 \\ \hline 25 \end{array}$	$\begin{array}{r} \quad 23 \\ 67\overline{)1594} \\ -134 \\ \hline 254 \\ -201 \\ \hline 53 \end{array}$

2. The example shows how you can think about rounding a divisor to estimate partial quotients. Complete the division.

Problem	Think	Write
$29\overline{)747}$	$\begin{array}{r} \quad 2 \\ 30\overline{)747} \\ 60 \end{array}$	$\begin{array}{r} \quad 2 \\ 29\overline{)747} \\ 58 \end{array}$

3. Estimate the tens' digit in each quotient.

   A $91\overline{)4186}$ B $37\overline{)1234}$

   C $74\overline{)5143}$ D $27\overline{)821}$

Example: $48\overline{)2762}$

Think: Round 48 to 50. Then 276 ÷ 50 gives 5 as the tens' digit.

4. A Find the quotients in Exercise 3 above.

   B In which parts was your estimate of the tens' digit too large? Why do you think so?

   C In which parts was your estimate too small? Why?

5. Estimate the first digit of the quotient. Round the divisor to the nearest hundred to make your estimate.

   A $832\overline{)190\,851}$ B $419\overline{)322\,498}$

1. Find the quotients and remainders.

A $19\overline{)564}$	**D** $68\,235 \div 47$	**G** $64\overline{)2139}$	**J** $36\overline{)3024}$
B $52\overline{)7922}$	**E** $1147 \div 19$	**H** $77\overline{)61\,256}$	**K** $75\overline{)5101}$
C $86\overline{)2395}$	**F** $49\,221 \div 53$	**I** $91\overline{)8333}$	**L** $43\overline{)1307}$

2. Find the quotients and remainders.

$500\overline{)23\,849}$ $700\overline{)46\,464}$ $900\overline{)38\,574}$

A $512\overline{)23\,849}$	**B** $681\overline{)46\,464}$	**C** $902\overline{)38\,574}$
D $777\overline{)111\,111}$	**E** $832\overline{)190\,851}$	**F** $429\overline{)312\,498}$
G $676\overline{)829\,166}$	**H** $964\overline{)72\,841}$	**I** $2375\overline{)643\,809}$

Short Stories

1. 396 students. 9 school buses. Same number of students in each bus. How many students in each bus?

2. Strawberries cost $10.80 a crate. 24 boxes in a crate. How much for one box?

3. Trip: 747 kilometres. Time: 9 hours. What was the average speed?

4. Three boys on a scale. Total mass: 105 kilograms. What is the average mass?

5. Each lawn mowed for $1.75. Earnings: $14. How many lawns were mowed?

6. A satellite in orbit goes 28 000 kilometres per hour. How far in 15 hours?

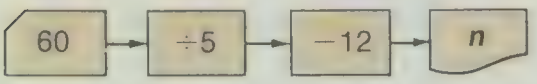
7. 400-metre dash. Time: 44 seconds. About how many metres per second?

8. The Dodgers played 73 baseball games. Scored 366 runs. About how many runs averaged per game?

More practice, page S-8, Set 13

REVIEWING THE IDEAS

1. Find the output number n in the flow chart.



2. In the statement following, give the other subtraction equation: Since $6 + 7 = 13$, we know that $13 - 7 = 6$ and $13 - \blacksquare = \blacksquare$.

3. Write two division equations that are related to the multiplication equation $13 \cdot 8 = 104$.

4. Give the missing numbers for each screen in the function tables.

	Function Rule: $n \cdot 8$	
	n	
A	5	
B	7	
C	9	

	Function Rule: $(5 \cdot n) + 4$	
	n	
D	3	
E	9	
F	0	

5. For each equation, give the number of the basic principle.

A $a + 0 = a$

B $(6 + 7) + 9 = 6 + (7 + 9)$

C $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

D $a + 7 = 7 + a$

E $7 \cdot (n + 6) = (7 \cdot n) + (7 \cdot 6)$

F $586 \cdot 78 = 78 \cdot 586$

G $5280 \cdot 1 = 5280$

1 Commutative principle (+)

2 Commutative principle (×)

3 Associative principle (+)

4 Associative principle (×)

5 Zero principle

6 One principle

7 Distributive principle

6. Solve the equations.

A $8 \cdot (7 + 12) = x$

B $(63 \div 7) \cdot 9 = y$

C $(8 \cdot 4) + (8 \cdot 3) = z$

7. Find the sums.

A
$$\begin{array}{r} 829 \\ 642 \\ +\,189 \\ \hline \end{array}$$

B
$$\begin{array}{r} 86\,288 \\ 127\,429 \\ 5\,766 \\ 62\,981 \\ \hline \end{array}$$

8. Find the differences.

A
$$\begin{array}{r} 2801 \\ -\,1977 \\ \hline \end{array}$$

B
$$\begin{array}{r} 51\,742 \\ -\,36\,955 \\ \hline \end{array}$$

9. Find the products and quotients.

A $7 \cdot 20$

B $50 \cdot 60$

C $360 \div 6$

D $7200 \div 80$

10. Choose the best estimate for each product.

A
$$\begin{array}{r} 82 \\ \times\,37 \\ \hline \end{array}$$
Estimates: 2400, 3200, 3600

B
$$\begin{array}{r} 609 \\ \times\,31 \\ \hline \end{array}$$
Estimates: 180, 1800, 18 000

11. Find the products.

A
$$\begin{array}{r} 78 \\ \times\,59 \\ \hline \end{array}$$

B
$$\begin{array}{r} 288 \\ \times\,543 \\ \hline \end{array}$$

12. Find the quotients and remainders.

A $39\overline{)7272}$

B $265\overline{)38\,742}$

13. Find the number for n.

A $17\overline{)\,n}$ = 326

B $n\overline{)473}$ = 11

C $61\overline{)33\,855}$ = n

D $n\overline{)33\,453}$ = 81

TEST YOURSELF

1. Find the output number in the flow chart.

2. Write two subtraction equations using the numbers in the addition equation $9 + 7 = 16$.

3. Write two division equations using the numbers in the multiplication equation $8 \cdot 4 = 32$.

4. Complete the function table.

Function Rule: $4 \cdot n - 3$

	Input	Output
A	2	
B	5	
C	10	

5. Solve the equation $7 \cdot (39 \div 3) = n$.

6. Solve each equation. Think of the basic principles.
   - A $14 \cdot x = 14$
   - B $84 + 0 = n$
   - C $27 + 89 = 89 + y$
   - D $(12 \cdot 31) \cdot 28 = 12 \cdot (31 \cdot w)$
   - E $8 \cdot (20 + 6) = 8 \cdot r + 8 \cdot 6$

7. Find the sum and difference.

   A $\begin{array}{r} 2356 \\ +1098 \\ \hline \end{array}$ B $\begin{array}{r} 8003 \\ -2896 \\ \hline \end{array}$

8. Solve the equations.
   - A $30 \cdot 600 = n$
   - B $500 \cdot n = 40\,000$
   - C $3500 \div 50 = n$

9. Find the products.

   A $\begin{array}{r} 261 \\ \times 75 \\ \hline \end{array}$ B $\begin{array}{r} 742 \\ \times 386 \\ \hline \end{array}$

10. Find the quotients and remainders.
    - A $61\overline{)7488}$
    - B $304\overline{)77\,777}$

RESEARCH PROJECTS

A The English mathematician John Napier invented a device called **Napier's rods** to multiply whole numbers. Find out how the rods were made and learn how to use them in multiplication. (See *The Giant Golden Book of Mathematics* by Irving Adler; New York: Golden Press, 1960 available from Whitman Goldman, Ltd.)

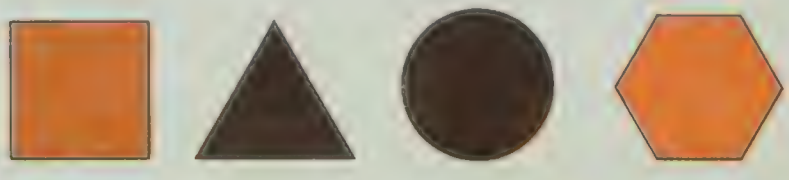
B Learn how to operate a desk calculator. Make up several addition, subtraction, multiplication, and division problems and then solve them yourself. Check your answers using the desk calculator.

C A common method of dividing large numbers in the Middle Ages was the **galley** or **scratch method**. Find out how this method works and then try to use it with some division problems. (See *History of Mathematics* by D. E. Smith; New York: Dover Publications, 1953, Vol. II available from General Publishing Co., Ltd.)

D Construct your own computer. (See *I Can Learn About Calculators and Computers* by Raymond Kenyon; Evanston, Illinois: Harper and Row, 1961, pp. 25–104 available from Fitzhenry & Whiteside, Ltd.)

A Four Color and Shape Attribute Puzzle

Make four copies of each of the geometric shapes shown below. (Sixteen pieces in all). Color the four pieces of each shape in four different colors; for example, red, green, yellow and blue.



The object of the puzzle is to try to place the 16 pieces on a 4 by 4 square in such a way that no two pieces that have the same shape or the same color are in the same row, same column, or in either diagonal of the grid.

MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

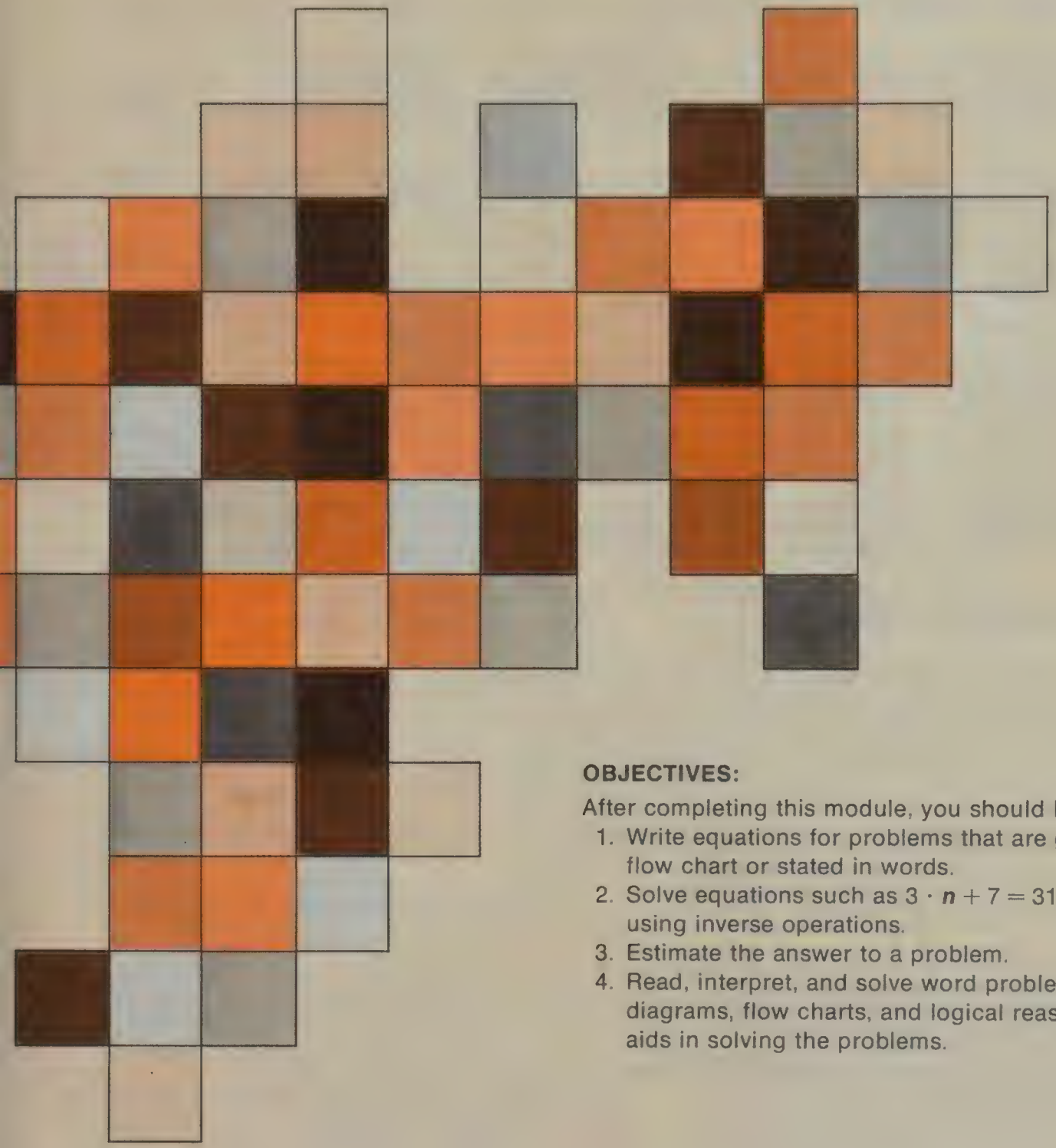
1. 26 **2.** 16 − 9 = 7, 16 − 7 = 9 **3.** 32 ÷ 4 = 8, 32 ÷ 8 = 4

4. A 5; B 17; C 37 **5.** 91 **6.** A 1; B 84; C 27; D 28; E 20

7. A 3454; B 5107 **8.** A 18 000; B 80; C 70

9. A 19 575; B 286 412 **10.** A 122 R 46; B 255, R 257

MODULE 3: *Problem Solving*



OBJECTIVES:

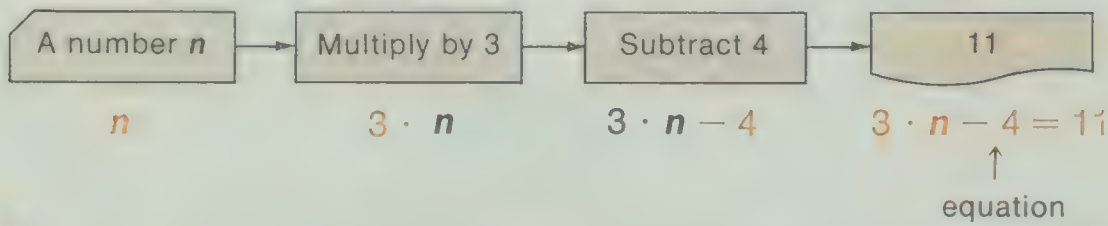
After completing this module, you should be able to:

1. Write equations for problems that are given by a flow chart or stated in words.
2. Solve equations such as $3 \cdot n + 7 = 31$ by using inverse operations.
3. Estimate the answer to a problem.
4. Read, interpret, and solve word problems using diagrams, flow charts, and logical reasoning as aids in solving the problems.

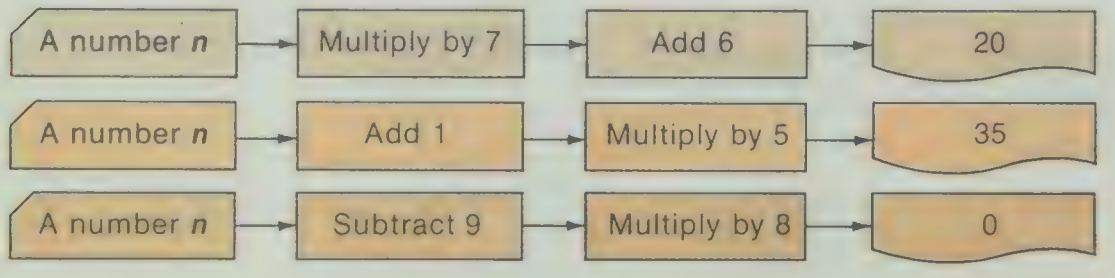
Writing Equations

Investigating the Ideas

The flow chart below suggests an equation.



Can you write an equation for each of the flow charts below?

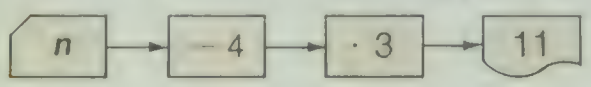


Discussing the Ideas

1. The letters used in equations are called **variables**. A variable is used to represent any number. Any letter may be used as a variable. What is the variable in each of these equations?

   A $y + 7 = 13$ **B** $2 \cdot x + 19 = 37$ **C** $14 - t = 6$

2. The flow chart below suggests an equation.

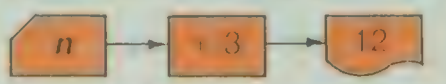


   Which equation do you think it is?

   A $3 \cdot n - 11 = 4$ **B** $3 \cdot n - 4 = 11$ **C** $(n - 4) \cdot 3 = 11$

3. The equation $n + 3 = 12$ is a mathematical way of writing the sentence "If three is added to a certain number, the sum is 12."

   The flow chart for the equation is:

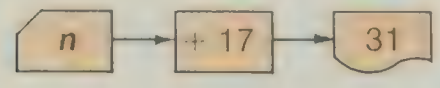


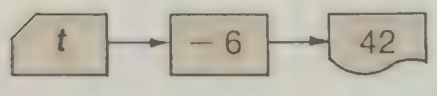
   Write an equation for each of these statements. (A flow chart may help you think about the equation.)

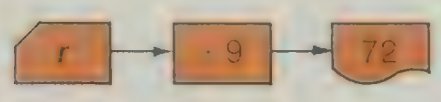
   A When a certain number is multiplied by 4, the product is 28.

   B If a certain number is multiplied by 3 and then 1 is subtracted, the result is 11.

1. Write an equation for each flow chart.

A 

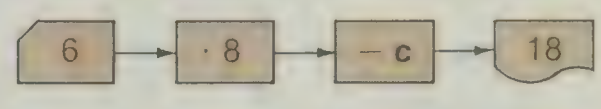
B 

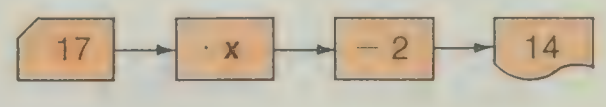
C 

D

E

F

G 

H 

2. Choose a variable for the unknown number in each sentence and then write an equation for each sentence.
   - A When seven is added to a certain number, the sum is 19.
   - B When 38 is subtracted from a certain number, the difference is 29.
   - C When a certain number is multiplied by 8, the product is 96.
   - D When a certain number is multiplied by 2 and then 17 is added to the product, the result is 35.

3. Sometimes parentheses must be used when writing equations.

   EXAMPLE: When the sum of a certain number and 7 is multiplied by 3, the result is 21.

   EQUATION: Let n represent the "certain number." $3 \cdot (n + 7) = 21$

   Write an equation for each of these sentences.
   - A When 4 is subtracted from a certain number and the difference is multiplied by 5, the result is 15.
   - B When a certain number is added to 9 and the sum is multiplied by 6, the result is 120.
   - C When the sum of 19 and 12 is subtracted from a certain number, the result is 11.

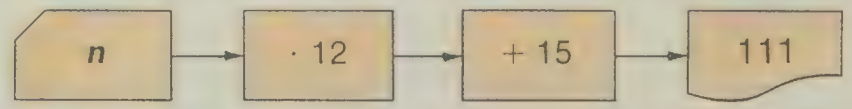
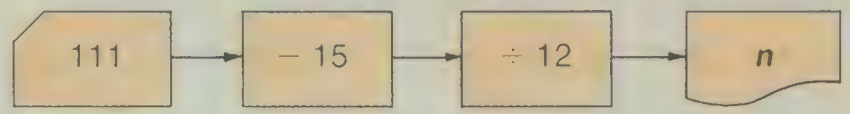
think

"If you had been born in the year of 1980 then you would be 45 years old in the year 45^2 or 2025 A.D." said Jean. "That is true" replied Carlos, "But my grandmother was n years old in the year n^2 and she is still living!" In what year was Carlos' grandmother born?

More practice, page S-8, Set 14

Investigating the Ideas

The flow charts below may help you to understand how to solve equations for certain types of problems.

Problem	A certain number is multiplied by 12, and 15 is added to this product. The result is 111. What was the original number?
Equation	Let n represent the number. $12 \cdot n + 15 = 111$
Flow chart for the equation	A 
Inverse flow chart	B 

Can you find the output number n for flow chart B?

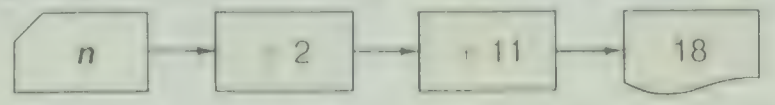
What is the input number n for flow chart A?

Discussing the Ideas

1. How do the instructions for flow chart A compare with those for flow chart B?

2. Explain why flow chart B is called the **inverse flow chart** of A.

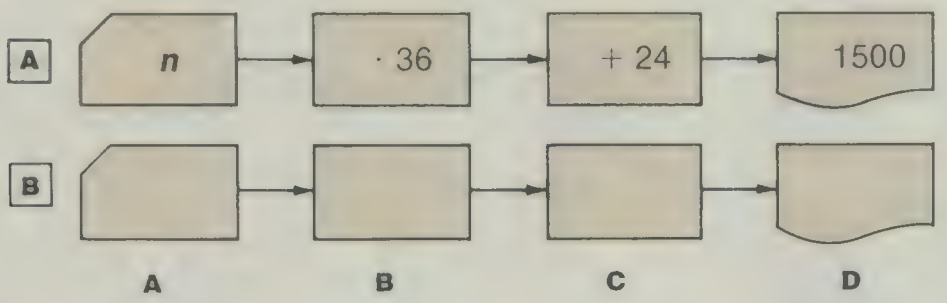
3. The number for the variable in an equation that will form a true statement is called a **solution** for the equation. How can you check to see if the output number for flow chart B is a solution to the equation $12 \cdot n + 15 = 111$?

4. You can think of flow chart B as "undoing" the operations in flow chart A. What would the inverse flow chart be for this one? 

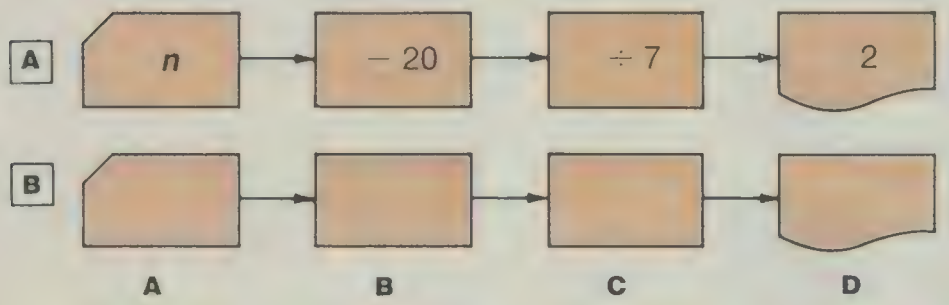
5. What is the solution to the equation $(n \div 2) + 11 = 18$? Explain how to solve the equation?

In Exercises 1, 2, and 3, study flow chart A. Then give the instructions and numbers that go with each part of inverse flow chart B.

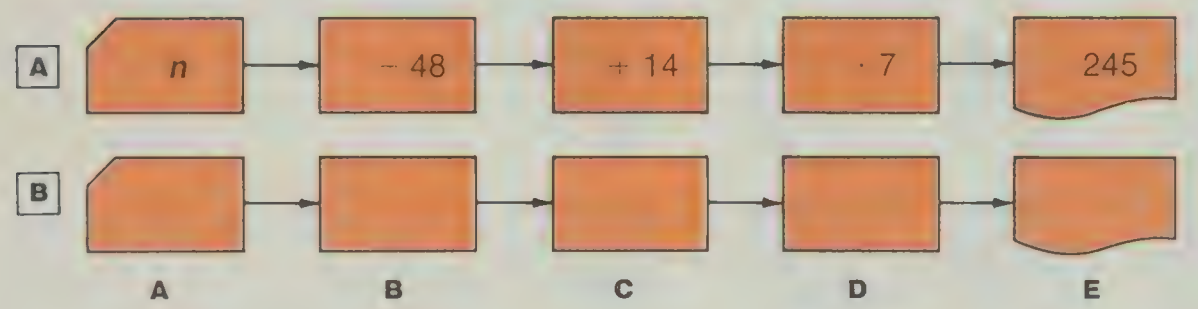
1.



2.



3.



4. Write an equation for flow chart A in Exercises 1, 2, and 3. Then use flow chart B to help you solve the equations.

5. Solve each equation. Use flow charts if necessary.

   A $(n - 7) \div 4 = 14$

   B $n \cdot 9 + 6 = 33$

   C $(n + 17) \cdot 14 - 209 = 71$

   D $\left(\frac{n}{2} - 9\right) \cdot 7 = 14$

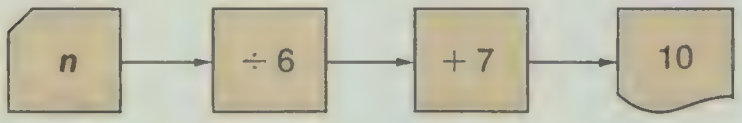
6. **A** Construct a flow chart for the following problem:
   If 427 is added to a certain number n and the sum is multiplied by 12, the result will be 5760.

   B Construct the inverse flow chart for part **A** and then find the output number for this flow chart.

Writing and Solving Equations

1. Write an equation for each flow chart. Then solve the equation. Think about inverse operations and inverse flow charts to help you solve the equations. Part **A** is completed as an example.

A



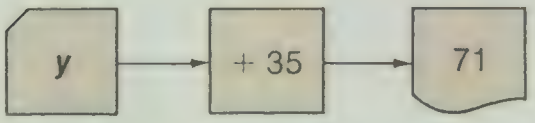
EQUATION: $(n \div 6) + 7 = 10$

SOLUTION: If $(n \div 6) + 7 = 10$,
then $n \div 6 = 10 - 7$.
And if $n \div 6 = 3$,
then $n = 6 \cdot 3 = 18$

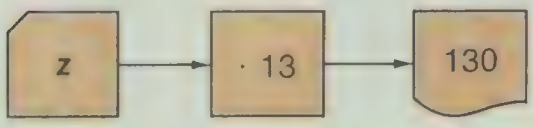
CHECK: $(18 \div 6) + 7 = 10$

B

C



D



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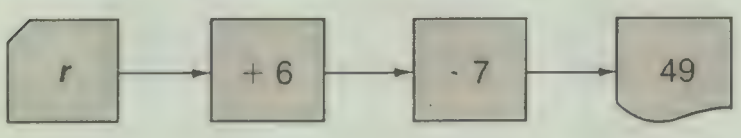

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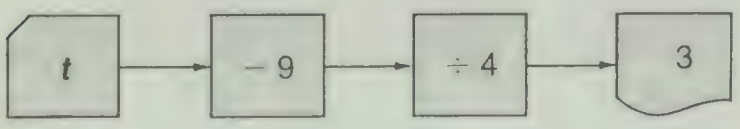


G

2. Solve the equations. Check your answers.

A $y + 35 = 71$

B $13 \cdot z = 91$

C $r - 12 = 99$

D $3 \cdot a - 7 = 11$

E $21 - 7 \cdot b = 0$

F $14 \cdot t + 8 = 120$

G $(x \div 2) + 5 = 9$

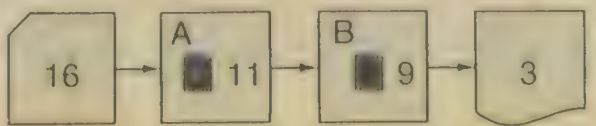
H $(n - 4) \div 6 = 2$

I $(24 \cdot w) \div 9 = 8$

J $135 - 9 \cdot t = 81$

think

The operation signs in the instruction boxes A and B have been covered. What are the two operations?



Solving Word Problems With Equations

Write an equation for each word problem, then solve the equation.
The first exercise is completed as an example.

1. A certain number is multiplied by 8 and then 11 is subtracted from the product. The result is 13. What is the number?

   SOLUTION: Let n represent the number.

   $$8 \cdot n - 11 = 13$$
   $$8 \cdot n = 13 + 11 = 24$$
   $$n = 24 \div 8 = 3$$
   $$n = 3$$

   CHECK: $8 \cdot 3 - 11 = 13$

2. Diane is 153 centimetres (cm) tall. She is 19 cm taller than Elaine. How tall is Elaine?

3. A man averaged 63 kilometres (km) per hour on a trip. If his trip was 441 km, how long did it take him to make the trip?

4. Bill said, "I am thinking of a number. If you multiply it by 10 and then subtract 10, you get 100." What was Bill's number?

5. Karen had a recipe for a small number of cookies but she wanted to make more. First she doubled the amount of flour and then used another extra portion of flour. Altogether she used 7 portions. How many portions of flour did the original recipe call for?

6. Janice had 23 pieces of candy. She ate three pieces and then divided the remaining pieces equally among 4 of her friends. How many pieces of candy did each friend get?

7. If 6 is added to a certain number and the sum is multiplied by 4, the result is 40. What is the number?

8. Larry scored the same number of points in each of 4 games of a card game. In the next game he lost 47 points so that his final score was only 1 point. How many points did he score in each of the first 4 games?

9. Sarah measured the distance around her classroom and found that it was 40 metres. The length of the room was 12 metres. What was the width of the room?

10. David had a certain amount of money. He spent half of it for a notebook and then spent 30 cents for a milk shake. Finally, he had 14 cents left. How much money did he have in the beginning?

Think

A man needs to take a fox, a goose, and a sack of corn across a river. He has a boat in which he cannot take more than one across at a time. He cannot leave the fox and the goose together or the goose and the corn, for one will eat the other. How can he get all three across the river, taking one at a time with him in the boat? Draw a diagram to show your solution.

Investigating the Ideas

Study the problems shown on the chalkboard.

1. A car is traveling at 80 kilometres per hour. It uses a litre of gasoline every 6 kilometres. How long will it take for the car to travel 320 kilometres?

2. Kathy and Andrea together had $3.24. The girls went shopping and Kathy spent $1.15. How much money does Kathy have left?

Can you solve either one or both of these problems?	Give your solutions and be able to explain them.	

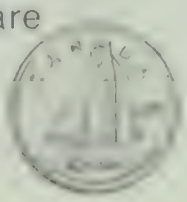
Discussing the Ideas

1. A Which problem contained some numerical information that you did not need in order to solve the problem?
   B What was the information?

2. A Which problem did not contain enough information for you to solve the problem?
   B What other information would you need in order to solve the problem?

3. None of these problems contains enough information to solve the problem. What extra facts are needed?
   A There are 90 students in the seventh grade at Centre Junior High. They plan to use buses on a field trip. How many will they need?
   B Students in the seventh-grade class sold 180 tickets for the class play. Adult tickets were $1.00 and student tickets were 50¢. How much did they earn for the class?

4. In each of these problems, information that is not needed is given. Pick out the unnecessary information.
   A A set of five Canadian coins has a value of 51¢. There is only one penny in the set. What are the other coins?



   B A number is multiplied by 5 and to this product 4 is added. The original number is an odd number less than 10 and the final result is 39. Find the original number.

Try to solve each of the problems. Some may not contain enough information while others contain more than enough.

1. Jill is in the seventh grade. Her brother is 14 years old. Jill added 365 to her age in years, subtracted 52, divided by 13, and ended with a result of 25. How old is Jill?

2. The distance between two cities is 2351 kilometres. The larger city has a population of 129 740. A jet plane has already flown 854 kilometres from the larger city toward the smaller city. How far is the plane from the smaller city?

3. In five days of vacation driving the Smith family drove 425, 513, 524, 411, and 387 kilometres. They used 294 litres of gasoline. If their odometer reading was 23 496 when they left home, what was it after the five days of driving?

4. There are 1512 trees in a large orchard with 36 trees in each row. 432 of the trees are apricot trees. How many rows of trees are there?

5. From a board that was 3 metres long, Dave cut off a piece 1 metre 26 centimetres long. How long was the piece of board that was left?

6. Mrs. Charles saw some small boxes of strawberries priced at 59¢ a box. How much would she have to pay for a crate of strawberries?

7. Carla found that the average length of one of her paces was 55 centimetres. About how many paces would she have to walk to go a distance of one kilometre?

8. A box of a certain kind of cereal contains 380 grams of cereal. How many kilograms of cereal would there be in a carton of cereal?

9. If you save $2.00 per day, how many weeks would you have to save until you would have enough money to buy a bicycle that costs $112?

10. Sue and Jane together had $2.95. Jane spent 98 cents. How much money did Jane have left?

11. There are 463 students in grades 7, 8, and 9 at Jordan Junior High School. There are 31 more students in the eighth grade than in the seventh grade. There are 17 more ninth graders than eighth graders. How many students are there in each grade?

think

Cathy has 10 coins. None of the coins is a half dollar. The value of the coins is 59 cents. Can you determine which coins she must have?

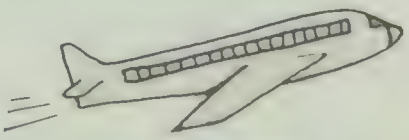
Investigating the Ideas

In many mathematical problems, you can come very close to the correct answer by making a careful estimate.

Can you give an estimate for each problem below?

1. The population of Thunder Bay, is 115 000. The population of Toronto is more than six times the size of Thunder Bay. What is the approximate population of Toronto?

2. A non-stop flight between two large cities took 4 hours and 53 minutes. If the distance between cities is 3974 kilometres, what was the average rate of speed?



3. Maryann bought ten items at the supermarket. Every item cost between 59¢ and $1.00. What is the total cost of the items?

Discussing the Ideas

1. Explain the reasoning you used in estimating the answer for problem 1 in the Investigation.

2. Explain how you made your estimate for problem 2.

3. What was your estimate for problem 3 and how did you make it?

4. Why do you think it is important to make estimates of answers to problems?

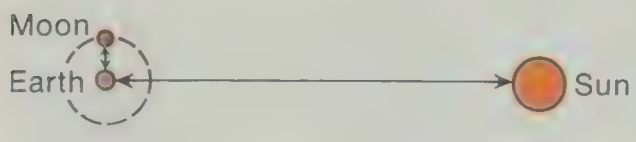
5. A factory worker earns $132.50 per week. His weekly deductions for taxes, insurance, and so forth, amount to $23.67. What do you estimate is his take-home pay for a year?

After each problem, three possible answers are given. None of them are exact answers but one of them will be quite close to the correct result. Use estimation to pick out this answer.

1. A basketball team scored between 45 and 65 points in each of 19 games. About how many points did they score in all of their games? (ANSWERS: 1500; 1100; 700)

2. For every 4 students at a track meet, there were 3 adults. If 124 student tickets were sold, how many adult tickets were sold? (ANSWERS: 40; 90; 150)

3. The distance from the earth to the sun is about 149 000 000 kilometres, and that from the earth to the moon is about 382 000 kilometres. About how many times as far is it from the earth to the sun as from the earth to the moon? (ANSWERS: 4 000 000; 4000; 400)



4. What is the sum of the whole numbers from 1 to 100? (ANSWERS: 500; 5000; 50 000)

5. Cris counted the number of times the letter "e" occurred in each of 10 lines of type. Here is the data.

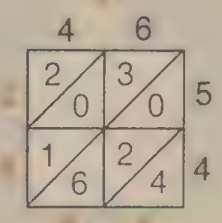
Line	1	2	3	4	5	6	7	8	9	10
Number of e's	5	8	6	8	2	6	5	7	5	7

About how many times, on the average, did "e" occur in each line? (ANSWERS: 4; 6; 8)

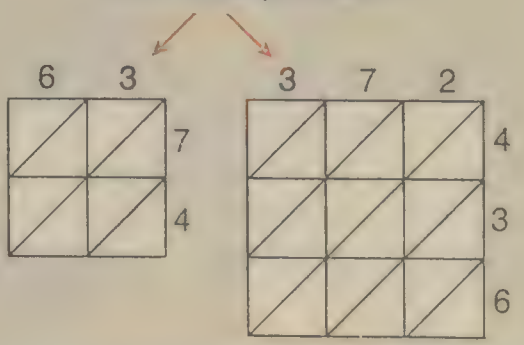
★6. Suppose that a rocket ship can travel from the earth to Mars in 260 days. It takes 45 years and 273 days to reach the planet Pluto. Approximately how many round trips could one rocket ship make between the earth and Mars while a second ship was traveling to Pluto? (ANSWERS: 30; 80; 6000)

think

The diagram shows an old method of multiplying **46** and **54** to get the product 2484



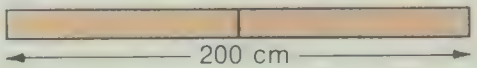
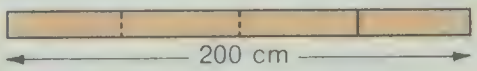
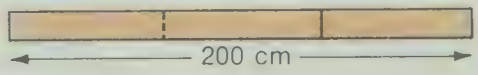
Using this method, find these products.



Using Drawings to Solve Problems

Investigating the Ideas

A drawing or sketch of a problem may often help you solve a problem. Study the problem and then decide which drawing best illustrates the problem.

PROBLEM	DRAWINGS
A board 200 centimetres long is cut into two pieces. One piece is 3 times as long as the other. How long is each piece?	A  B  C 

Can you use the correct drawing to help you solve the problem?

Discussing the Ideas

1. How did you decide which drawing was the correct one?

2. Explain how you used the drawings to solve the problem.

3. A Draw a diagram for this problem:
   The perimeter (distance around) of a rectangle is 40 centimetres (cm). The rectangle is 12 cm long. How wide is the rectangle?
   B What is the distance halfway around the rectangle?
   C If half of the perimeter is 20 cm, how can you use this fact to find the width of the rectangle?
   D Can you find the width in another way?

4. A Tell what facts are shown by the sketch at the right.
   B Make up a problem about the sketch and explain how to solve your problem.

In Exercises 1 through 5, make a drawing to help you analyze each problem. Then solve the problem.

1. A board is cut into two pieces. The shorter piece is 28 centimetres long and the longer piece is twice as long. How long was the board before cutting?

2. The perimeter of a rectangle is 50 metres. If it is 8 metres wide, how long is the rectangle?

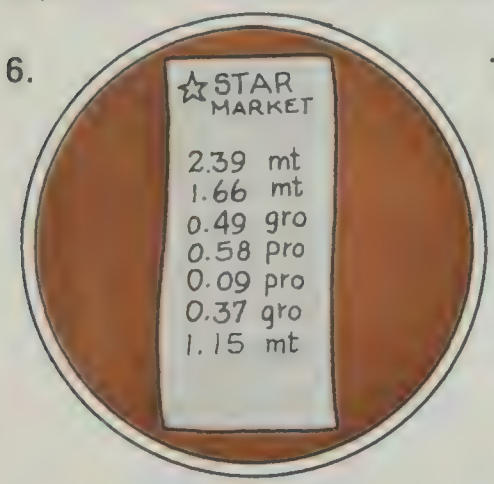
3. A triangle has two sides of equal length. The perimeter of the triangle is 20 cm. The length of one of the two sides of equal length is 8 cm. How long is the third side?

4. There are two rows of cards on a table. One row contains 12 cards. If we moved 5 cards from this row to the second row, the second row would have twice as many cards as the first. How many cards are in the second row?

5. Two cars were 100 kilometres apart. They drove toward each other and met in 2 hours at a point which was 40 kilometres from the starting point of the slower car. How fast was each car travelling?

Each sketch or drawing in Exercises 6 through 11 suggests a problem. Write and solve your own problem for each.

6.



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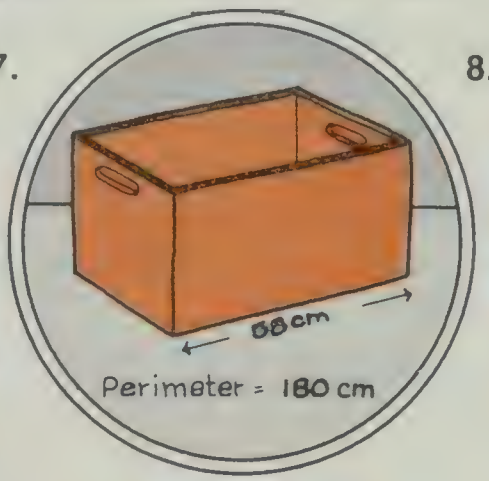

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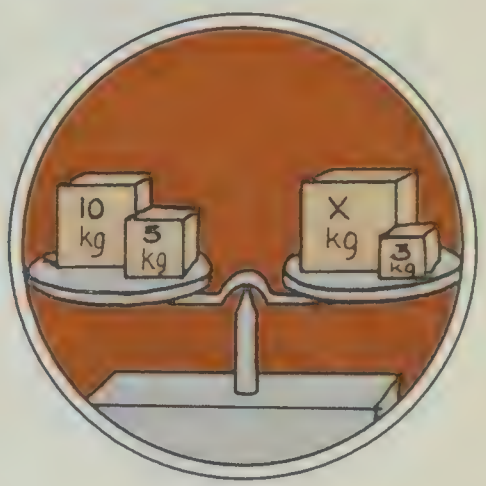

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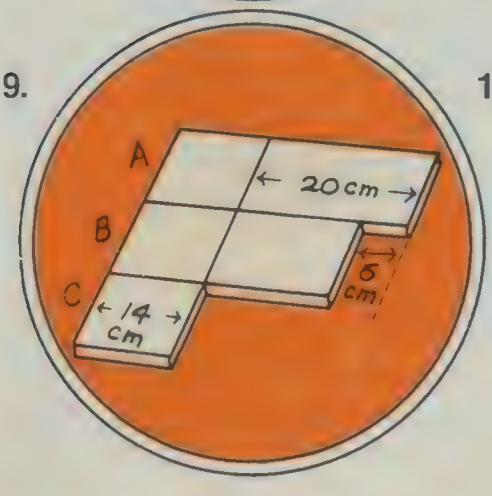

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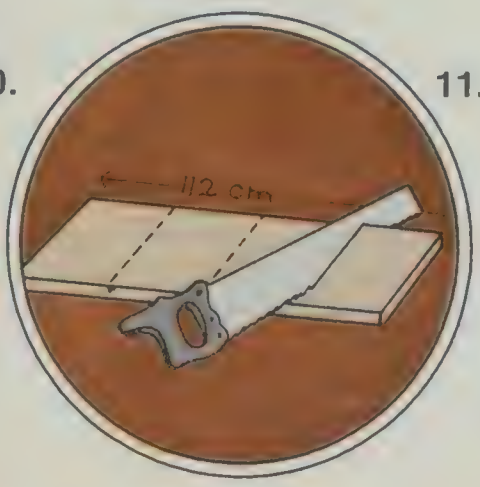

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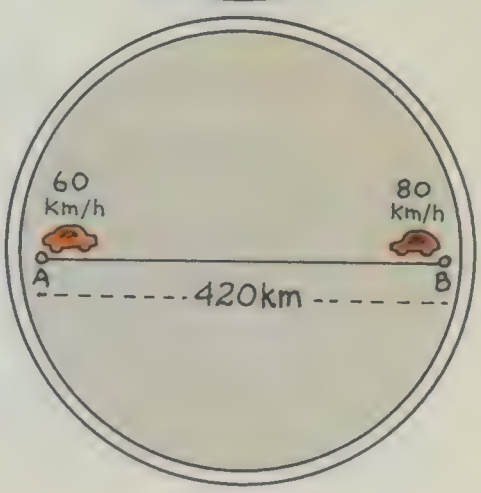
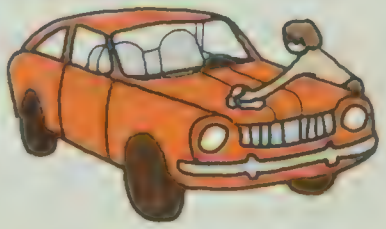


More practice, page S-9, Set 15

Investigating the Ideas

Careful thinking and reasoning are important in problem solving. Read the problem below carefully.

Jim can wax a car in 6 hours. Bill can wax a car in 3 hours. If they work together, how long will it take them to wax one car?



Which of these times is the best estimate for the answer to the problem?

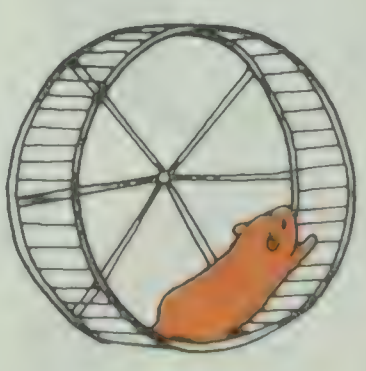
A 9 hours **B** 6 hours **C** $4\frac{1}{2}$ hours **D** 3 hours **E** 2 hours

Discussing the Ideas

1. What is wrong with the reasoning of a student who thought that "9 hours" was the answer to the Investigation problem?

2. Explain the reasoning you used to select your estimate.

3. Unless you carefully read a problem and use logical reasoning, you may easily miss simple story problems.
In each problem below, several answers are given but only one is correct. Find the correct answer and explain your reasoning for each problem.



A Harry had 20 hamsters. All but 7 died. How many were left?
ANSWERS: 27 13 20 7

B How many hours are there in 24 days?
ANSWERS: 12 24 144 576

C A baseball team has played 150 games. It has won 30 more games than it has lost. How many games has it lost?
ANSWERS: 120 60 90 180

In Exercises 1 through 4, choose the best answer for each problem. Think carefully!

1. John had 45 baseball cards. He gave Rick 8 of them. Now John and Rick have the same number of cards. How many do they have together?

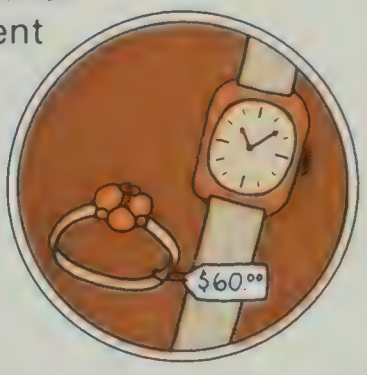
   A 90 **B** 53 **C** 74 **D** 16

2. Mary spent half of her money for a birthday gift and $1.50 for lunch. She has $1.00 left. How much money did she have in the beginning?

   A $5.00 **B** $2.50 **C** $10.00 **D** $1.25

3. Six years ago Sam was 6 years old and Mike was 18. What is the number by which you must multiply Sam's present age in order to get Mike's present age?

   A 4 **B** 3 **C** 2

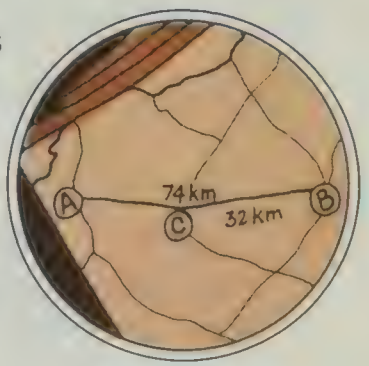


4. A ring cost $60. A watch cost $40 more than the ring. What was the total cost of the watch and ring?

   A $20 **B** $100 **C** $160

In Exercises 5 through 8, solve the problems.

5. It is 74 kilometres from town A to town B. It is 32 kilometres from town B to town C, and town C is on the road between A and B. How far is it from town A to town C?



6. Joan is now 6 years old. Ed is twice as old as Joan. When Joan is 12, how old will Ed be?

7. Beth had $20. She spent $2.00 in one store and half of the remaining money in a second store. How much money did she have left?

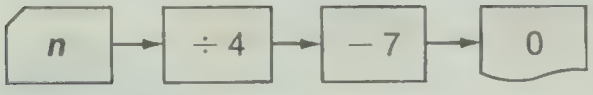
★8. One pipe can fill a tank in 15 minutes. A second pipe can fill the tank in 12 minutes. If the first pipe is used for 10 minutes, then shut off and the tank is then filled using the second pipe, how long will it take in all to fill the tank?

think

A man fenced his garden so that the fence had the form of a square. When he finished, there were 10 fence posts on each side. How many posts did he use?

REVIEWING THE IDEAS

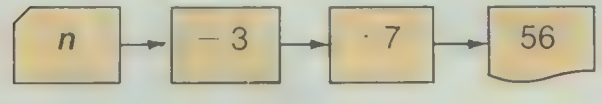
1. Write an equation for the flow chart.



2. Make the inverse flow chart for the one in Exercise 1.

3. Find the output number for the flow chart in Exercise 2.

4. Write and solve an equation for the flow chart.



5. Solve the equations.
   - A $3 \cdot n + 7 = 19$
   - B $(n \div 3) + 1 = 6$
   - C $(n - 12) + 7 = 9$

6. 236 is added to 928, and then the sum is doubled. If this result is divided by 6, what is the quotient?

7. Which of the three answers is correct? The average life of a man is about 2 000 000 000 seconds. The giant turtle lives $\frac{1}{2}$ again as long as man. How many seconds is the average life of the giant turtle?
   - A 1 000 000 000
   - B 1 500 000 000
   - C 3 000 000 000

8. Write and solve your own problem for this picture.

9. Estimate the answers to the following following problems.
   - A A cubic centimetre of gold weighs about 19 grams. About how much does 43 cubic centimetres of gold weigh?
   - B What is the product of 987 and 864?
   - C How much are 315 kilograms of soybeans worth if the market price is $0.89 per kilogram?

10. Draw a diagram that will show the information in each problem. Then solve the problem.
   - A The length of a rectangle is 4 centimetres longer than its width. The perimeter of the rectangle is 56 centimetres. Find the length and width of the rectangle.
   - B A piece of wire 36 metres long is cut into two pieces. One piece is twice as long as the other. How long is each piece?

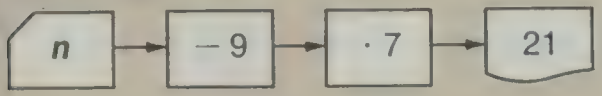
11. A rectangular lot is 125 metres long. If the distance around is 380 metres, how wide is the lot?

12. Janet is 11 years old. Her grandfather is five times as old as she is now. How old will her grandfather be when Janet is 20 years old?

TEST YOURSELF

1. Write the equation suggested by the flow chart.

2. Make the inverse flow chart for this flow chart.



3. What is the number for n in Exercise 2?

4. Solve the equation. Check your answer.

$$6 \cdot n - 15 = 39$$

5. Write an equation for this sentence:

   If 7 is added to a certain number and the sum is multiplied by 4, the result is 48.

6. Solve the equation in Exercise 5.

7. Estimate the answer to this problem:

   A jet airplane travels at 800 km/h. If it flies nonstop for a distance of 1592 km, about how long will the flight take?

8. Draw a diagram that will help you understand the problem. Then solve the problem.

   The perimeter of a triangle is 42 centimetres (cm). Two of the sides have the same length and the third side is 6 cm longer than each of the other two. How long is each side?

9. Which answer is correct?

   A pencil cost 10¢. A pen costs one dollar more than the pencil. How much will they cost together?

   A 90¢ **B** $1.10 **C** $1.20

10. Barry took 28 shots at the basket during a basketball game. He missed 3 times as many baskets as he made. How many baskets did he make?

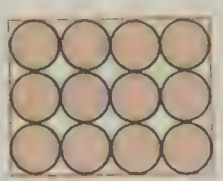
RESEARCH PROJECTS

A Read about some of the famous mathematicians of the past. Write or give a report to your class about some of the interesting things about the lives of the mathematicians. Try to find what their main contributions to mathematics were. You may find that one or more of them may be of interest to you in other fields also.

Fermat	Euler	Archimedes
Gauss	Euclid	Newton

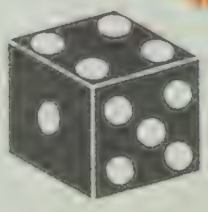
(see *Men of Mathematics* by E. T. Bell; New York: Simon and Schuster, 1962.) available from Musson Book Co.)

B Suppose you had 12 round pies that fit exactly into a shallow rectangular tray when they are arranged as in the diagram below.



Could you design a differently shaped tray that would hold these pies but take up less area than the rectangular one? How much less area would your tray take?

PROBABILITY PRODUCT GAME



This is a game for two or more players. The players take turns tossing a pair of dice. Using the product of the numbers tossed, the player scores the number shown on the chart below. For example, if a 4 and a 2 come up on a toss of the dice, the product is 8. According to the chart, your score 5 points!

PRODUCT	SCORE
4	20
6 or 12	10
1, 9, 16, 25, or 36	15
2, 3, 5, 8, or 10	8
15, 18, 20, 24, 30	5

The first player to score a total of 100 points wins the game.

MATHEMATICAL RECREATIONS

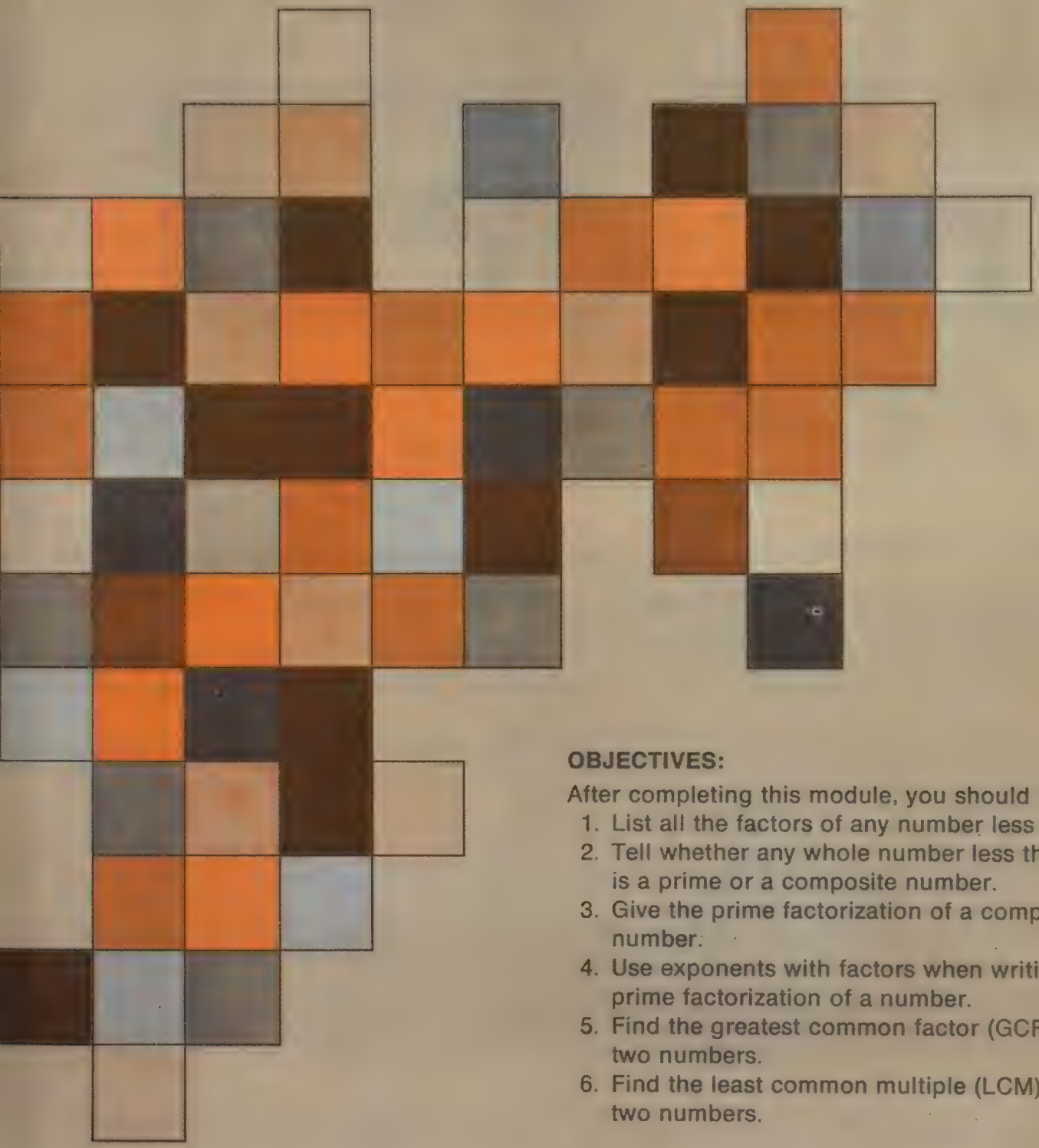
TEST YOURSELF Answers

1. $6 \cdot n + 2 = 44$ **2.**

3. 12
4. 9 **5.** $(n + 7) \cdot 4 = 48$ **6.** 5 **7.** 2 hours **8.** 12 cm, 12 cm, 18 cm
9. c $1.20 **10.** 7 baskets

UNIT A: *The Arithmetic of Whole Numbers*

MODULE 4: *Number Theory*



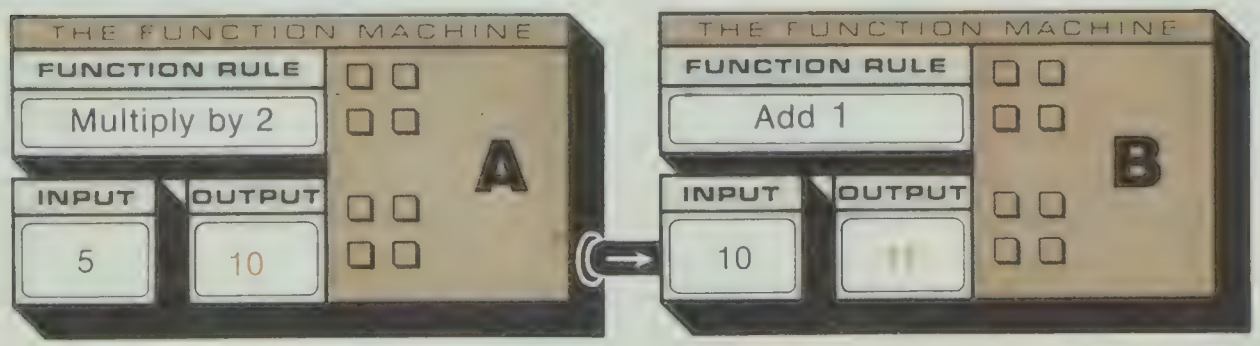
OBJECTIVES:

After completing this module, you should be able to:

1. List all the factors of any number less than 100.
2. Tell whether any whole number less than 100 is a prime or a composite number.
3. Give the prime factorization of a composite number.
4. Use exponents with factors when writing the prime factorization of a number.
5. Find the greatest common factor (GCF) of any two numbers.
6. Find the least common multiple (LCM) of any two numbers.

Investigating the Ideas

The two function machines are connected together so that the output number for A is always the input number for B. The input for A is any whole number from the set of whole numbers {0, 1, 2, 3, 4, 5, . . .}.



?	Choose ten whole numbers as input numbers for function machine A. Can you find the output numbers for both function machines?	Record your answers.

Discussing the Ideas

1. A number that is *2 times a whole number* is an **even number** or a **multiple of 2**. Which function machine produced only even numbers for outputs?

2. A number that is *1 more than an even number* is called an **odd number**. Will the output numbers for function machine B always be an odd number?

3. How can you decide whether a number is even or odd?

4. If each whole number is multiplied by 3, the resulting numbers are **multiples of 3**. Give the missing multiples of 3 for each ▒.

Whole numbers	0	1	2	3	4	5	6	7	8	9	. . .	n	. . .
	×3	×3	×3	×3	×3	×3	×3	×3	×3	×3		×3	
Multiples of 3	0	3	6	▒	▒	▒	▒	21	▒	▒	. . .	$3 \cdot n$	. . .

5. Is 51 a multiple of 3? How can you decide?

6. What are the first four multiples of 5? of 7?

1. For each number listed, tell whether it is an **even** or an **odd** number.

A 17	**C** 79	**E** 0	**G** 1007	**I** 12 004
B 42	**D** 217	**F** 358	**H** 2393	**J** 237 986

2. Write **even** or **odd** to complete each sentence.
   - **A** The sum of any two even numbers is ? .
   - **B** The sum of any two odd numbers is ? .
   - **C** The sum of any even number and any odd number is ? .
   - **D** The product of any two odd numbers is ? .
   - **E** The product of any two even numbers is ? .
   - **F** The product of any even number and any odd number is ? .

3. **A** List the first ten multiples of 3.
   - **B** Choose any two of the multiples of 3. Is their sum a multiple of 3?
   - **C** Find the product of any two multiples of 3. Is the product also a multiple of 3?

4. **A** List the first ten multiples of 5.
   - **B** What is the smallest number, other than 0, that is a multiple of both 3 and 5?

5. The two examples below show how you can use division to decide whether one number is a multiple of another.

EXAMPLE 1: Is 91 a multiple of 7?

$$\begin{array}{r} 13 \\ 7\overline{)91} \\ 7 \\ \hline 21 \\ 0 \end{array}$$

Since the remainder is 0,
$7 \cdot 13 = 91$,
91 **is a multiple of** 7.

EXAMPLE 2: Is 274 a multiple of 14?

$$\begin{array}{r} 19 \\ 14\overline{)274} \\ 14 \\ \hline 134 \\ 126 \\ \hline 8 \end{array}$$

The remainder is not 0.
Therefore, 274 is **not** a multiple of 14.

Use division to answer these questions.

- **A** Is 442 a multiple of 17?
- **B** Is 1008 a multiple of 9?
- **C** Is 551 a multiple of 23?
- **D** Is 165 a multiple of 15?
- **E** Is 1001 a multiple of 143?
- **F** Is 2486 a multiple of 36?

Investigating the Ideas

Since $6 \cdot 3 = 18$, we know that { 6 is a factor of 18. / 3 is a factor of 18.

Since $9 \cdot 2 = 18$, we know that { 9 is a factor of 18. / 2 is a factor of 18.

Since $18 \cdot 1 = 18$, we know that { 18 is a factor of 18. / 1 is a factor of 18.

There are no other pairs of whole numbers whose product is 18.
The set of all factors of 18 is {1, 2, 3, 6, 9, 18}.

Can you find an example of each of the numbers described below?	Record your examples.

A A number with only two factors.

B A number with exactly three factors.

C A number with more than six factors.

Discussing the Ideas

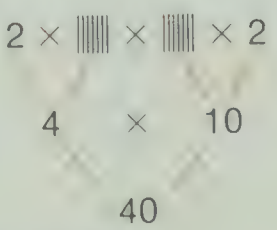
1. Every whole number, except the number 1, has at least two factors. Also, since $0 = 0 \cdot 0 = 1 \cdot 0 = 2 \cdot 0 = \ldots$, every whole number is a factor of zero.

   $1 = 1 \cdot 1$ The only factor of 1 is 1.
   $2 = 1 \cdot 2$ The only factors of 2 are 1 and 2.
   $6 = 1 \cdot 6 = 2 \cdot 3$ The factors of 6 are 1, 2, 3, and 6.

   What are the factors of the following numbers?

   A 7 **B** 8 **C** 9 **D** 10 **E** 11 **F** 13

2. An interesting way to find the factors of a number is to build "factor trees" for numbers. What are the missing factors in the factor tree for 40?

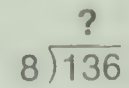


3. What number, other than 1, is a factor of every even number?

4. Explain how the equation below can help you list all the factors of 48.

   $48 = 1 \cdot 48 = 2 \cdot 24 = 3 \cdot 16 = 4 \cdot 12 = 6 \cdot 8$

5. Is 8 a factor of 136? Explain how you can use division to help you decide.



1. Use the equations to help you list all the factors for each number.

   A $25 = 1 \cdot 25 = 5 \cdot 5$

   B $77 = 1 \cdot 77 = 7 \cdot 11$

   C $143 = 1 \cdot 143 = 11 \cdot 13$

   D $27 = 1 \cdot 27 = 3 \cdot 9$

   E $24 = 1 \cdot 24 = 2 \cdot 12 = 3 \cdot 8 = 4 \cdot 6$

   F $100 = 1 \cdot 100 = 2 \cdot 50 = 4 \cdot 25 = 5 \cdot 20 = 10 \cdot 10$

2. Copy the factor trees and supply the names for each ▨.

   A $3 \times 2 \times$ ▨; 3×4; 12

   B $2 \times$ ▨ $\times 3$; 6×3; 18

   C $5 \times$ ▨ $\times$ ▨; 5; 50

   D $2 \times$ ▨ $\times$ ▨; 10, 2; 20

   E ▨ $\times$ ▨ $\times$ ▨; 5×6; 30

   F ▨ $\times$ ▨ $\times$ ▨ $\times$ ▨; $\times$; 24

   G ▨ $\times$ ▨ $\times$ ▨ $\times$ ▨; $\times$; 36

3. List all the factors of these numbers.

   A 12 **B** 15 **C** 40 **D** 18 **E** 26 **F** 6 **G** 30 **H** 16 **I** 27 **J** 50 **K** 29 **L** 64

4. Which number less than 10 have exactly two factors?

5. Use division to help you answer each question.

   A Is 9 a factor of 297?

   B Is 8 a factor of 1000?

   C Is 13 a factor of 211?

   D Is 26 a factor of 130?

   E Is 18 a factor of 556?

   F Is 37 a factor of 999?

think

The three examples below will help you understand the meaning of **abundant**, **deficient**, and **perfect** numbers.

A The factors of 12: {1, 2, 3, 4, 6} $1 + 2 + 3 + 4 + 6 = 16$ ← Sum of the factors
(except 12) Since $16 > 12$, 12 is an **abundant** number.

B The factors of 10: {1, 2, 5} $1 + 2 + 5 = 8$ ← Sum of the factors
(except 10) Since $8 < 10$, 10 is a **deficient** number.

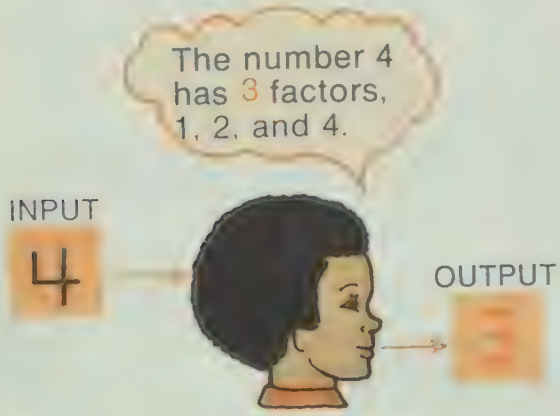
C The factors of 6: {1, 2, 3} $1 + 2 + 3 = 6$ ← Sum of the factors
(except 6) Since $6 = 6$, 6 is a **perfect** number.

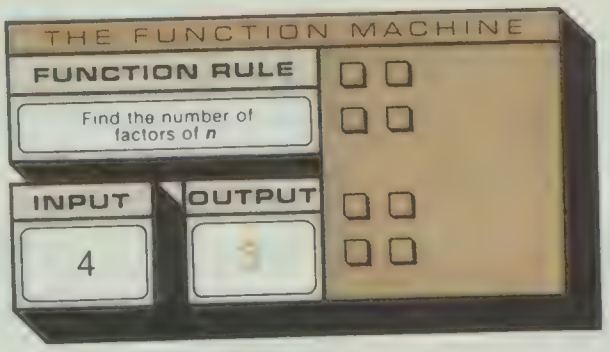
Tell what kind of number each of these are: **abundant**, **deficient**, or **perfect**.

A 18 **B** 26 **C** 28 **D** 30 **E** 16 **F** 32 **G** 45

Investigating the Ideas

William thought of an interesting function rule.
Study his rule and the way he thought about it.





Can you make a function table for the function machine and give the output numbers for each input number from 1 through 20?

Discussing the Ideas

1. A Which number from 1 through 20 has the fewest factors?
   B What is the greatest number of factors that any of the numbers from 1 through 20 has?

2. A whole number which has exactly two factors is called a **prime** number. Which numbers from 1 through 20 are prime numbers?

3. Are any of the even numbers prime numbers? How many?

4. Numbers greater than 1 which have more than two factors are called **composite** numbers.
   A What is the smallest composite number?
   B Is 39 a prime number or a composite number?

5. A Is 1 a prime number, a composite number, or neither? Explain your answer.
   B Is 0 a prime number, a composite number, or neither? Explain your answer.

1. List all the prime numbers less than 100. (If you find all of them, there should be 25 numbers in your list.)

2. Tell how many composite number there are
   A less than 10. **B** less than 100.

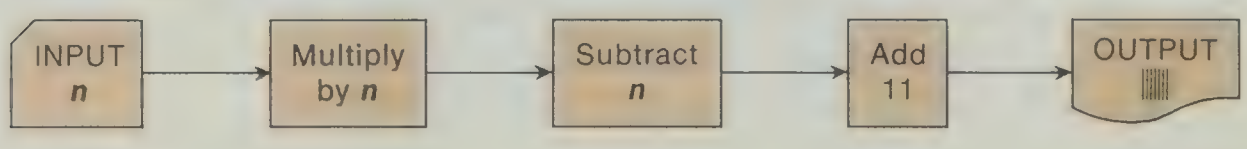
3. How many of the prime numbers are even numbers?

4. There is just one pair of prime numbers whose difference is 1. What is the pair?

5. The smallest pair of prime numbers with a difference of 2 is 3 and 5. Such numbers are called **twin primes**. Find six other pairs of twin primes.

6. **A** Find two prime numbers whose difference is 3.
   B Do you think there is any other such pair?

7. The set of **all** factors of a certain number is {1, 17, 289}. Another number has the set of factors {1, 151}.
   A Which number is prime? **B** What are the two numbers?

8. 

   A Find the output number for the flow chart using the numbers 1 through 10 as input numbers.
   B What kind of numbers does the flow chart produce?
   C Try 11 as an input number for the flow chart. Is the output a prime number?

think

No even number after 2 is a prime number. But every even number larger than 2 seems to be the sum of two primes. No one has ever been able to prove that this is so, but most mathematicians believe it.

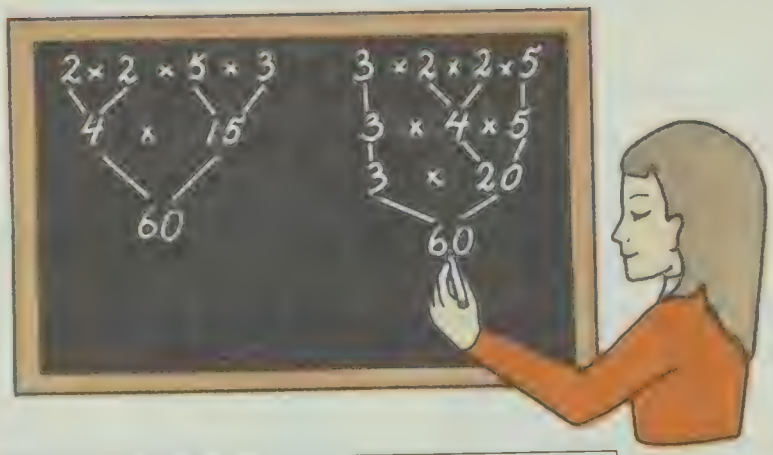
1. Check this for the even numbers up to 50.

2. Express 200 as the sum of two primes.

$4 = 2 + 2$
$6 = 3 + 3$
$8 = 3 + 5$
$10 = 3 + 7$
$12 = 5 + 7$
$14 = 3 + 11$
$16 = 5 + 11$
$18 = 7 + 11$
$20 = 7 + 13$

Investigating the Ideas

In order to find which prime numbers can be multiplied together to give a product of 60, Kathy made some **factor trees**.



How many more factor trees can you build for the number 60?	Do not use 1 as one of the factors.

Discussing the Ideas

1. What do you notice about the "top row" of all of your factor trees for 60?

2. When a composite number is written as a product of primes, we say that we have **factored the number completely** or have given the **prime factorization** of the number. If the order of the factors is disregarded, each composite number can be expressed as a product of primes in one and only one way.

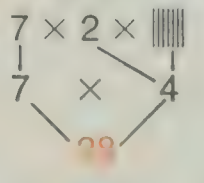
   $12 = 2 \cdot 2 \cdot 3$ — No primes other than 2, 2, and 3 have a product of 12.
   $70 = 2 \cdot 5 \cdot 7$ — No primes other than 2, 5, and 7 have a product of 70.
   $30 = 2 \cdot 3 \cdot 5$ — No primes other than 2, 3, and 5 have a product of 30.

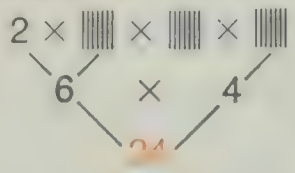
   A What is the prime factorization of 60?
   B What is the prime factorization of 24?

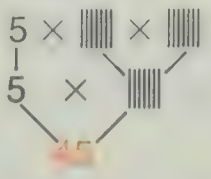
3. The top row of a factor tree for a number gives the prime factorization of the number. What number has been factored completely in each one of these factor trees?

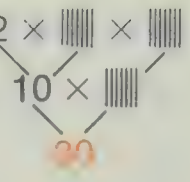
   A $2 \times 3 \times 3 \times 7$ **B** $3 \times 3 \times 3 \times 11$ **C** $7 \times 11 \times 13$

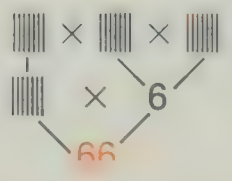
1. Copy and complete the factor trees. The top row of factors is the prime factorization of the bottom number of the tree.

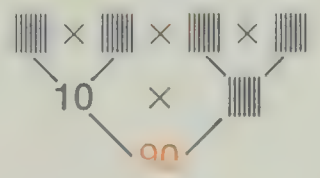
A 

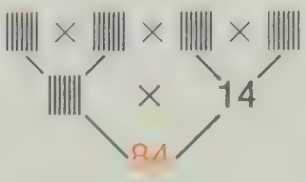
B 

C 

D 

E 

F 

G 

2. Copy and complete the equations so that you have the prime factorization of each given number.

A $10 = 2 \cdot d$

B $12 = 3 \cdot 2 \cdot e$

C $14 = 2 \cdot f$

D $16 = 2 \cdot 2 \cdot 2 \cdot h$

E $18 = 2 \cdot 3 \cdot i$

F $20 = 2 \cdot 2 \cdot j$

G $24 = 2 \cdot 3 \cdot 2 \cdot m$

H $27 = 3 \cdot 3 \cdot n$

I $28 = 2 \cdot 2 \cdot p$

3. Factor each of the following numbers completely.

A 33 C 36 E 40 G 50 I 100 K 231

B 35 D 38 F 42 H 81 J 250 I 126

4. Give the number for each prime factorization shown.

A $2 \cdot 5 \cdot 5 \cdot 2$

B $2 \cdot 3 \cdot 7$

C $2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

D $5 \cdot 7 \cdot 13$

E $2 \cdot 11 \cdot 11 \cdot 17$

F $3 \cdot 3 \cdot 3 \cdot 3 \cdot 5$

G $29 \cdot 31$

H $2 \cdot 3 \cdot 13$

I $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

J $3 \cdot 19 \cdot 23$

K $2 \cdot 37 \cdot 61$

L $2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7$

5. Give the prime factorization of 1024.

think

1. How many factors has 7?
   How many has $2 \cdot 7$?

2. How many factors has 9?
   How many has $2 \cdot 9$?

3. How many factors has 15?
   How many has $2 \cdot 15$?

4. How many factors has 45?
   How many has $2 \cdot 45$?

5. How many factors has 10?
   How many has $2 \cdot 10$?

6. How many factors has 14?
   How many has $2 \cdot 14$?

7. An **odd** number N has 100 factors.
   $\{1, \ldots, N\}$
   How many factors has $2 \cdot N$?

More practice, page 9-10, Set 15

Investigating the Ideas

These factor trees are unusual.

Can you complete them to find out why they are unusual?

Discussing the Ideas

1. What did you find out about the factor trees that made them unusual?

2. When the same factor is repeated several times, you can use exponents just as you did for powers of ten.

Symbol	We think	We say
8^2	$8 \cdot 8$ Two factors of 8	8 squared or 8 to the second power
3^3	$3 \cdot 3 \cdot 3$ Three factors of 3	3 cubed or 3 to the third power
11^5	$11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$ Five factors of 11	11 to the fifth power

How would you represent each number in the Investigation using exponents?

3. The prime factorization of some numbers can be simplified by using exponents.

EXAMPLES: $\blacksquare = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2$

$200 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 = 2^3 \cdot 5^2$

How would you write the prime factorization of these numbers using exponents?

A 27 **B** 100 **C** 90 **D** 64 **E** 225

1. Write each product using exponents.

EXAMPLE: $7 \cdot 7 \cdot 7 = 7^3$

A $4 \cdot 4$ **C** $10 \cdot 10 \cdot 10$ **E** $2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

B $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$ **D** $15 \cdot 15 \cdot 15 \cdot 15 \cdot 15 \cdot 15$ **F** $5 \cdot 4 \cdot 5 \cdot 4 \cdot 5 \cdot 4 \cdot 5$

2. Write each number in expanded form and find the results.

EXAMPLE: $8^5 = 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 = 32\,768$

A 16^2 **C** 2^8 **E** 3^4 **G** 13^3 ★ **I** $4^0 \cdot 3^0 \cdot 2^0$

B 4^4 **D** 9^2 **F** 17^1 **H** $2^3 \cdot 3^2 \cdot 4^1$ ★ **J** $4^3 \cdot 3^2 \cdot 2^1 \cdot 1^0$

3. Give the prime factorization of each number. Use exponents.

A 80 **B** 108 **C** 243 **D** 1000 **E** 144

4. Solve the equations.

A $5^3 = n$ **D** $2^6 = a$ **G** $9^3 = t$ **J** $1^{15} = m$

B $3^3 = h$ **E** $13^2 = r$ **H** $8^4 = b$ **K** $23^1 = w$

C $5^5 = y$ **F** $4^5 = c$ **I** $4^8 = s$ ★ **L** $4^3 = 2^n$

5. Find the products.

A $2^3 \cdot 3^2$ **C** $7 \cdot 3^3$ **E** $10^3 \cdot 10^2$ **G** $7^1 \cdot 11^1 \cdot 13^1$

B $5^2 \cdot 10^2$ **D** $2^4 \cdot 2^3$ **F** $2^2 \cdot 3^2 \cdot 5^2$ **H** $8^2 \cdot 11^2$

★ **6.** For each number below, a factorization is given. Decide whether or not it is a prime factorization. If not, factor the number completely.

A $72 = 2 \cdot 2 \cdot 2 \cdot 9$ **C** $713 = 23 \cdot 31$ **E** $1331 = 11 \cdot 121$ **G** $1001 = 77 \cdot 13$

B $216 = 8 \cdot 3 \cdot 9$ **D** $627 = 11 \cdot 57$ **F** $360 = 2^3 \cdot 45$ **H** $6923 = 161 \cdot 43$

★ **7.** Use the facts $2 \cdot 5 = 10$, $2^2 \cdot 5^2 = 100$, $2^3 \cdot 5^3 = 1000$, and $2^4 \cdot 5^4 = 10\,000$ to find the products quickly.

A $2 \cdot 7 \cdot 5$ **D** $2^3 \cdot 11 \cdot 5^2$ **G** $2^4 \cdot 23 \cdot 5^4$ **J** $2^5 \cdot 7 \cdot 11 \cdot 5^4$

B $2^2 \cdot 13 \cdot 5^2$ **E** $2^2 \cdot 12 \cdot 5^3$ **H** $2^4 \cdot 71 \cdot 5^3$ **K** $2^4 \cdot 3 \cdot 7 \cdot 5^4$

C $2^2 \cdot 45 \cdot 5^2$ **F** $2^3 \cdot 147 \cdot 5^3$ **I** $2^3 \cdot 101 \cdot 5^4$ **L** $2^6 \cdot 3^2 \cdot 5^6$

★ **8.** Use the fact that $7 \cdot 11 \cdot 13 = 1001$ to find the products quickly.

A $7^2 \cdot 11 \cdot 13$ **C** $7 \cdot 11 \cdot 13^2$ **E** $7 \cdot 11 \cdot 13 \cdot 17$ **G** $7 \cdot 11^2 \cdot 13 \cdot 23$

B $7 \cdot 11^2 \cdot 13$ **D** $7^2 \cdot 11^2 \cdot 13$ **F** $7 \cdot 11 \cdot 13 \cdot 53$ **H** $7^2 \cdot 11^2 \cdot 13^2$

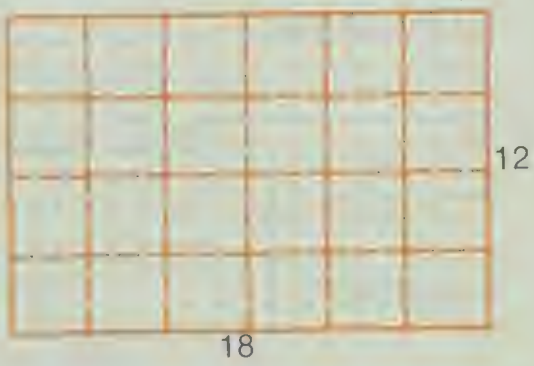
Greatest Common Factor (GCF)

Investigating the Ideas

Draw a rectangle 18 units long and 12 units wide on graph paper.

Can you divide the entire rectangular region into square regions so that the squares are all the same size and as large as they can possibly be?

Discussing the Ideas



1. Tam divided the rectangular region into squares that were 3 units on each side. Did she find the **largest possible** square that could be used?

2. John said, "To find the largest sided square, I thought about the greatest number that is a factor of both 12 and 18."

   What is that number?

Set of factors of 18 $A = \{1, \_, \_, 6, 9, 18\}$

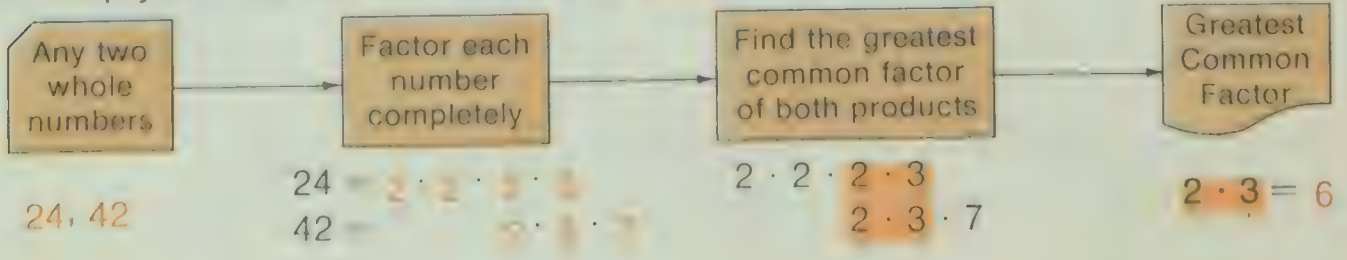
Set of factors of 12 $B = \{1, \_, \_, 4, \_, 12\}$

Set of common factors of 18 and 12 is the **intersection** of sets A and B.

$$A \cap B = \{1, 2, 3, 6\}$$

The **greatest common factor** (GCF) of 18 and 12 is __.

3. Study the flow chart to learn how prime factorizations can be used to find the GCF of two numbers. Use the flow chart to help you find the GCF of 70 and 105


Any two whole numbers
Factor each number completely
Find the greatest common factor of both products
Greatest Common Factor
24, 42
24 = 2 · 2 · 2 · 3
42 = 2 · 3 · 7
2 · 2 · 2 · 3
2 · 3 · 7
2 · 3 = 6

1. **A** List the factors of 12.
   B List the factors of 20.
   C List the common factors of 12 and 20.
   D What number is the GCF of 12 and 20?

2. **A** List the factors of 28.
   B List the factors of 35.
   C List the common factors of 28 and 35.
   D What number is the GCF of 28 and 35?

3. Find the GCF of each pair of numbers. The complete factorization is given and may help you. The answer to Part **A** is 14.

   A $42 = 2 \cdot 3 \cdot 7$
   $70 = 2 \cdot 5 \cdot 7$

   B $70 = 2 \cdot 5 \cdot 7$
   $30 = 2 \cdot 3 \cdot 5$

   C $55 = 5 \cdot 11$
   $15 = 3 \cdot 5$

   D $12 = 2 \cdot 2 \cdot 3$
   $18 = 2 \cdot 3 \cdot 3$

   E $10 = 2 \cdot 5$
   $70 = 2 \cdot 5 \cdot 7$

   F $18 = 2 \cdot 3 \cdot 3$
   $15 = 3 \cdot 5$

   G $42 = 3 \cdot 7 \cdot 2$
   $24 = 3 \cdot 2 \cdot 2 \cdot 2$

   H $24 = 3 \cdot 2^3$
   $6 = 2 \cdot 3$

   I $40 = 5 \cdot 2^3$
   $10 = 5 \cdot 2$

   J $90 = 2 \cdot 3^2 \cdot 5$
   $60 = 2^2 \cdot 3 \cdot 5$

   K $105 = 3 \cdot 5 \cdot 7$
   $225 = 3^2 \cdot 5^2$

   L $385 = 5 \cdot 7 \cdot 11$
   $175 = 5^2 \cdot 7$

4. Find the GCF of each pair of numbers. (Sometimes it is 1.)

   A 10, 25
   B 6, 18
   C 8, 15
   D 24, 36
   E 40, 100
   F 8, 27
   G 8, 18
   H 15, 25
   I 6, 8
   J 12, 16
   K 11, 12
   L 14, 35
   M 100, 60
   N 75, 125
   O 48, 72
   P 85, 68
   Q 45, 36
   R 90, 120
   S 150, 270
   T 450, 360

5. The letter N represents a number between 50 and 60. The GCF of N and 16 is 8. Find the number.

6. Thirteen is not a factor of N. What is the GCF of 13 and N?

7. The GCF of A and B is 5. What is the GCF of $2 \cdot A$ and $2 \cdot B$?

8. Two numbers have a GCF of 12. One number is 5 times as large as the other. Find the two numbers.

Think

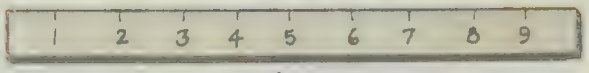
Use the whole numbers from 1 through 20 as input numbers. Then find the output numbers.

How many of the output numbers are prime numbers?

More practice, page S-11, Set 17

Least Common Multiple (LCM)

Investigating the Ideas



You will need some adding machine tape or strips of paper, a centimetre ruler, and a pair of scissors for this Investigation.

What is the shortest length of paper that can be cut so that

A you can cut it into pieces 8 centimetres long?

B you could cut it into pieces 14 centimetres long and have nothing left over in either case?

Discussing the Ideas

1. The number that is the shortest length of a strip of paper that could be cut either into 8 centimetre (cm) pieces or 14 cm pieces is the **least common multiple** (LCM) of 8 and 14. What is the LCM of 8 and 14?

2. Thinking about the multiples of a pair of numbers can help you choose the LCM of the pair.

   EXAMPLE: Multiples of 4 → $A = \{0, 4, 8, 12, 16, 20, 24, 28, 32, 36, \ldots\}$

   Multiples of 6 → $B = \{0, 6, 12, 18, 24, 30, 36, 42, \ldots\}$

   Common nonzero multiples of 4 and 6 → $\{12, 24, 36, \ldots\}$

   The LCM of 4 and 6 is 12.

   After studying the example, try this problem.

   A List the multiples of 3 to 36.

   B List the multiples of 4 to 36.

   C List the common multiples of 3 and 4 to 36.

   D Pick out the LCM of 3 and 4.

3. You may find it easier just to list multiples of the larger number until you find a number that is a multiple of the smaller. EXAMPLE: Find the LCM of 8 and 6.

   Multiples of 8 → {0, 8, 16, 24}.

   The LCM of 8 and 6 is 24.

   Use this method to find the LCM of each pair of numbers.

   A 6, 9 B 4, 14 C 12, 20 D 3, 8 E 5, 7

1. **A** List the multiples of 2 that are less than 25.
   B List the multiples of 3 that are less than 25.
   C List the common multiples of 2 and 3 that are less than 25.
   D What is the LCM of 2 and 3?

2. For each pair of numbers, find the LCM by finding the smallest multiple of the larger number that is also a multiple of the smaller number.

   EXAMPLE: 8, 10; Multiples of 10 are {10, 20, 30, 40}.
   ↑ 40 is also a multiple of 8.

A 3, 5	**D** 6, 12	**G** 5, 6	**J** 10, 11	**M** 4, 6
B 10, 4	**E** 8, 12	**H** 1, 15	**K** 15, 25	**N** 40, 60
C 2, 7	**F** 8, 3	**I** 7, 10	**L** 16, 24	**O** 12, 15

3. If you were working these addition problems, what common denominator would you choose? Think about the LCM of the denominators.

A $\frac{1}{2}+\frac{3}{4}$	**C** $\frac{3}{5}+\frac{1}{2}$	**E** $\frac{7}{12}+\frac{1}{8}$	**G** $\frac{7}{10}+\frac{3}{8}$	**I** $\frac{7}{12}+\frac{1}{16}$
B $\frac{2}{3}+\frac{1}{2}$	**D** $\frac{3}{8}+\frac{5}{6}$	**F** $\frac{4}{3}+\frac{2}{5}$	**H** $\frac{5}{8}+\frac{3}{16}$	**J** $\frac{4}{15}+\frac{3}{20}$

4. To find the LCM of 4, 6, and 10, we can first find the LCM of 6 and 10. It is 30. Now find the LCM of 30 and 4. It is 60. Thus 60 is the LCM of 4, 6, and 10. Find the LCM of the following groups of numbers.

A 2, 3, 5	**C** 2, 4, 6	**E** 4, 6, 12	**G** 3, 4, 8	**I** 10, 20, 30
B 2, 3, 4	**D** 2, 4, 8	**F** 2, 3, 6	**H** 5, 10, 15	**J** 8, 12, 15

5. You can use prime factorizations to find the LCM of two numbers. Study the example, then use this method to find the LCM of each of these pairs of numbers.

   EXAMPLE: $12 = 2 \cdot 2 \cdot 3$
   $20 = 2 \cdot 2 \cdot 5$
   The LCM of 12 and 20 is $2 \cdot 2 \cdot 3 \cdot 5 = 60$.

   A 10, 15 **B** 6, 20 **C** 12, 14 **D** 9, 24 **E** 18, 12

6. Find the LCM of each number pair.
   A 2^2 and 3^2 **C** 2^3 and $2 \cdot 3^2$ **E** $2^2 \cdot 3$ and $3^2 \cdot 5$
   B 3^3 and 5^2 **D** $2^2 \cdot 5$ and $3 \cdot 5^2$ **F** $3^2 \cdot 5^3 \cdot 7$ and $3^3 \cdot 5 \cdot 11$

7. Two cars are driving around a 2-km track. They start off together. Car A makes one lap every 80 seconds. Car B makes one lap every 60 seconds.
   A How long will it take the faster car to gain a lap on the slower one?
   B How long will it take the faster car to gain a lead of 10 laps?

More practice, page S-11, Set 18

REVIEWING THE IDEAS

1. Give a definition for
   A an even number. **B** an odd number.

2. List all the factors of
   A 14 **B** 60 **C** 210

3. Show that 48 is a factor of 5808.

4. **A** A prime number has exactly ? factors.
   B A ? number has more than two factors.
   C Is 1 a prime number?
   D Is 0 a composite number?

5. Tell whether or not each number is prime.

A 2	**E** 25	**I** 81
B 3	**F** 35	**J** 83
C 13	**G** 51	★ **K** 121
D 18	**H** 63	★ **L** 441

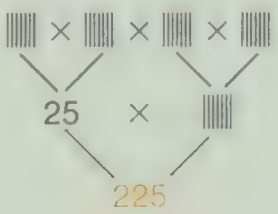
6. Complete the factor trees.

A

B

C

D



7. Replace each factorization by a prime factorization.
   A $72 = 8 \cdot 9$ **C** $100 = 4 \cdot 25$
   B $144 = 16 \cdot 9$ **D** $675 = 5 \cdot 135$

8. Give the prime factorization of 84.

9. Write each product using exponents.
   A $2 \cdot 2 \cdot 2 \cdot 2$ **B** $23 \cdot 23$
   C $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ **D** $6 \cdot 6 \cdot 6 \cdot 4 \cdot 4$
   E $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

10. Find the prime factorization of each number. Use exponents to express the factorization.

A 27	**C** 48	**E** 400
B 60	**D** 120	**F** 72

11. If set A contains the factors of 135 and set B contains the factors of 175, list all the numbers in
    A set A. **B** set B. **C** set $A \cap B$.

12. Find the greatest common factor (GCF) of these pairs of numbers.

A 8, 16	**C** 12, 29	**E** 60, 18
B 8, 15	**D** 14, 49	**F** 26, 182

13. Use the prime factorizations to find the GCF of each pair of numbers.
    A $15 = 3 \cdot 5$ $25 = 5 \cdot 5$
    B $44 = 2^2 \cdot 11$ $52 = 2^2 \cdot 13$
    C $77 = 7 \cdot 11$ $143 = 11 \cdot 13$
    D $81 = 3^4$ $72 = 2^3 \cdot 3^2$

14. **A** List the first 5 multiples of 6.
    B List the first 5 multiples of 8.
    C What is the least common multiple of 6 and 8?

15. Find the least common multiple (LCM) of 6 and
    A 3 **B** 9 **C** 8 **D** 4 **E** 5

16. Find the LCM of each group of numbers.
    A 4, 8, 12 **C** 4, 6, 9
    B 3, 6, 10 **D** 10, 12, 20

TEST YOURSELF

1. List all the factors of 40.

2. Is 23 a factor of **431** or **437**?

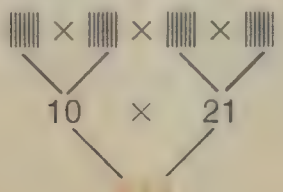
3. Complete the sentence:

   If a number has exactly two different factors, then it is a ? number.

4. Which numbers in the set are prime number?

   {4, 5, 21, 29, 39, 47, 87}

5. Complete the factor tree.



6. If the prime factorization of n is $2 \cdot 5 \cdot 11$, what number is n?

7. Give the prime factorization of 54.

8. Write each product using exponents.
   - A $3 \cdot 3 \cdot 3 \cdot 3$
   - B $5 \cdot 5 \cdot 7 \cdot 7 \cdot 13 \cdot 13 \cdot 13$
   - C $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

9. Which number is larger? 2^5 or 5^2?

10. What is the greatest common factor (GCF) of each pair of number?
    - A 10, 25
    - B 42, 77

11. What is the least common multiple (LCM) of each pair of numbers?
    - A 8, 12
    - B 20, 24

RESEARCH PROJECTS

A The **Seive of Eratosthenes** is a method that can be used to find prime numbers. Look up this method in a mathematics reference book and use it to find all the prime numbers less than 200. (See *A Panorama of Numbers* by Robert J. Wisner; Glenview, Illinois: Scott, Foresman and Company, 1970 available from Gage Educational Publishing, Ltd.)

B How can you tell when a number is exactly divisible by another number? Find out about some **divisibility rules**. In particular, find out how to tell when a number is divisible by 3, 4, 7, 9, 11, or 13. (See *Enrichment Mathematics for the Grades, Twenty-seventh Yearbook*; Washington, D.C.: National Council of Teachers of Mathematics, 1963.)

C Find out more about **perfect numbers**. How many perfect numbers less than 10 000 are known? Is there a formula for perfect numbers? (See *The Enjoyment of Mathematics* by Hans Rademacher and Otto Toelitz; Princeton, New Jersey: Princeton University Press, 1957.)

D The **Euclidean Algorithm** is a method of finding the greatest common factor of two numbers. It is particularly useful if both numbers are large. Find out how to use this method and use it to find the GCF of some pairs of large numbers such as 806 and 1333. (See *Fundamental Concepts of Elementary Mathematics* by Clarence H. Heinke; Belmont, California: Dickenson Publishing Company, 1970, pp.164–165.)

CUMULATIVE REVIEW

1. Set $A = \{0, 1, 3, 5, 7\}$
   Set $B = \{0, 2, 5, 8, 11\}$
   - A List the elements in $A \cup B$.
   - B List the elements in $A \cap B$.

2. In the numeral **18 473 905**, which digit is in the
   - A hundreds' place?
   - B ten thousands' place?
   - C one millions' place?

3. Write the usual numeral for
   $9 \cdot 1000 + 3 \cdot 100 + 5 \cdot 10 + 1$

4. 10^3 means 10 used as a factor ? times.

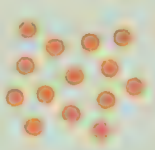
5. Give the usual numeral for 10^5.

6. Express 8 000 000 in scientific notation.

7. Round 6 746 283 to the nearest
   - A hundred.
   - B ten thousand.

8. Give the correct symbol ($<$, $=$, or $>$) for each ▒.
   - A 2467 ▒ 2475
   - B 18 429 ▒ 18 249

9. Write a numeral telling how many dots there are using
   - A base-ten numerals.
   - B base-eight numerals.
   - C base-five numerals.

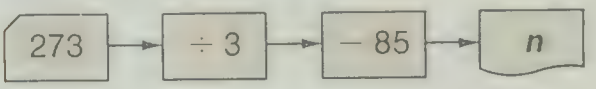


10. Find the sum and difference.
    - A $5_{(8)} + 6_{(8)} =$ ▒
    - B $14_{(8)} - 5_{(8)} =$ ▒

11. Write the Roman numeral for 126.

12. Write a decimal numeral for LXXIV.

13. Find the output number for the flow chart.



14. Write two subtraction equations related to the addition equation $14 + 8 = n$.

15. Write two division equations related to the multiplication equation $9 \cdot n = 27$.

16. A Complete the function table.

Function Rule: $5 \cdot n + 2$

n	$f(n)$
2	12
1	▒
▒	22
0	▒
▒	52

B Give the function rule.

Function Rule: ▒

n	$f(n)$
2	5
3	7
0	1
5	11
7	15

17. Match the equation with the basic principle.
    - A $17 \cdot 99 = 99 \cdot 17$
    - B $6 \cdot (20 + 3) = 6 \cdot 20 + 6 \cdot 3$
    - C $(59 \cdot 2) \cdot 5 = 59 \cdot (2 \cdot 5)$
    - D $106 \cdot 1 = 106$
    - E $38 = 38 + 0$

    1. Commutative principle (+)
    2. Commutative principle (×)
    3. Associative principle (+)
    4. Associative principle (×)
    5. Distributive principle
    6. Zero principle
    7. One principle

18. Solve the equations.

A $(20 - 5) \div 3 = r$

B $3 \cdot 9 + 8 = s$

C $(10 \cdot 2 - 6) \div 2 = t$

19. Find the sum and difference.

A
$$\begin{array}{r} 475 \\ 963 \\ +\ 49 \\ \hline \end{array}$$

B
$$\begin{array}{r} 1075 \\ -\ 499 \\ \hline \end{array}$$

20. Find the products and quotients.

A $37 \cdot 10$

B $900 \cdot 80$

C $350 \div 7$

D $4800 \div 60$

21. What is the best estimate for 829×69?

A 48 000

B 55 000

C 60 000

22. Find the products.

A
$$\begin{array}{r} 79 \\ \times 23 \\ \hline \end{array}$$

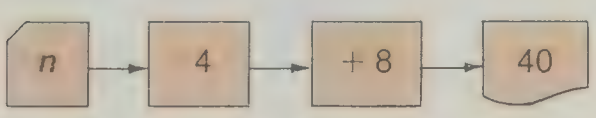
B
$$\begin{array}{r} 6438 \\ \times\ 305 \\ \hline \end{array}$$

23. Find the quotients and remainders.

A $32\overline{)729}$

B $495\overline{)19\,408}$

24. Write the equation suggested by this flow chart.



25. Make the inverse flow chart for Exercise 24.

26. What is *n* in Exercise 24?

27. Write and solve an equation for this statement.

When a certain number is multiplied by 8 and then 17 is subtracted from the product, the result is 39.

28. Estimate the answer to this problem.

A satellite travels at about 28000 km/h. About how far will it be in 48 hours?

29. Solve the problem. (A diagram may help you.)

The perimeter of a rectangle is 460 cm. The width is 100 cm. What is the length of the rectangle?

30. List the first five multiples of 4.

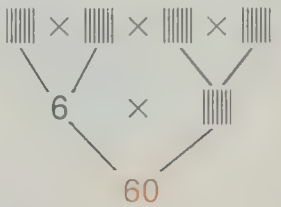
31. List all the factors of 24.

32. Is 17 a factor of **221** or **225**?

33. Which numbers in the set are prime?

$\{2, 9, 13, 27, 49, 59\}$

34. Complete the factor tree.



35. Write $2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$ using exponents.

36. Give the prime factorization of 45.

37. Find the product for $2^3 \cdot 3^2 \cdot 10^4$.

38. Find the GCF of these pairs of numbers.

A 15, 27

B 24, 40

39. Find the LCM of these groups of numbers.

A 9, 15

B 8, 13

C 3, 10, 12

THE REPEATING DIFFERENCE

Here is an interesting number investigation for you.

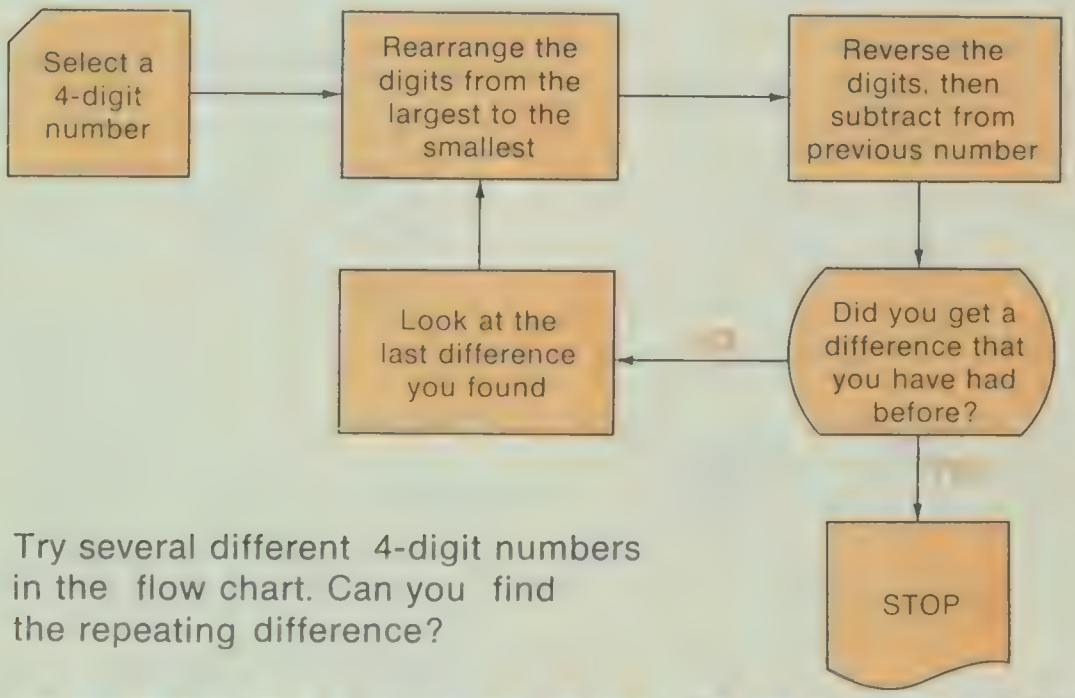
Choose any 4-digit number so that not all four digits are the same.

Rearrange the digits of the number from the largest to the smallest. Then reverse the order of the digits of the number and subtract from the previous number.

Rearrange the digits of the difference from the largest to the smallest. Then reverse the digits and subtract as before.

If you repeat this process several times, you will eventually get a difference which upon rearrangement and subtraction will give the same difference again and again.

The flow chart below may help you follow the steps for this investigation.


Select a 4-digit number
Rearrange the digits from the largest to the smallest
Reverse the digits, then subtract from previous number
Did you get a difference that you have had before?
Look at the last difference you found
STOP

Try several different 4-digit numbers in the flow chart. Can you find the repeating difference?

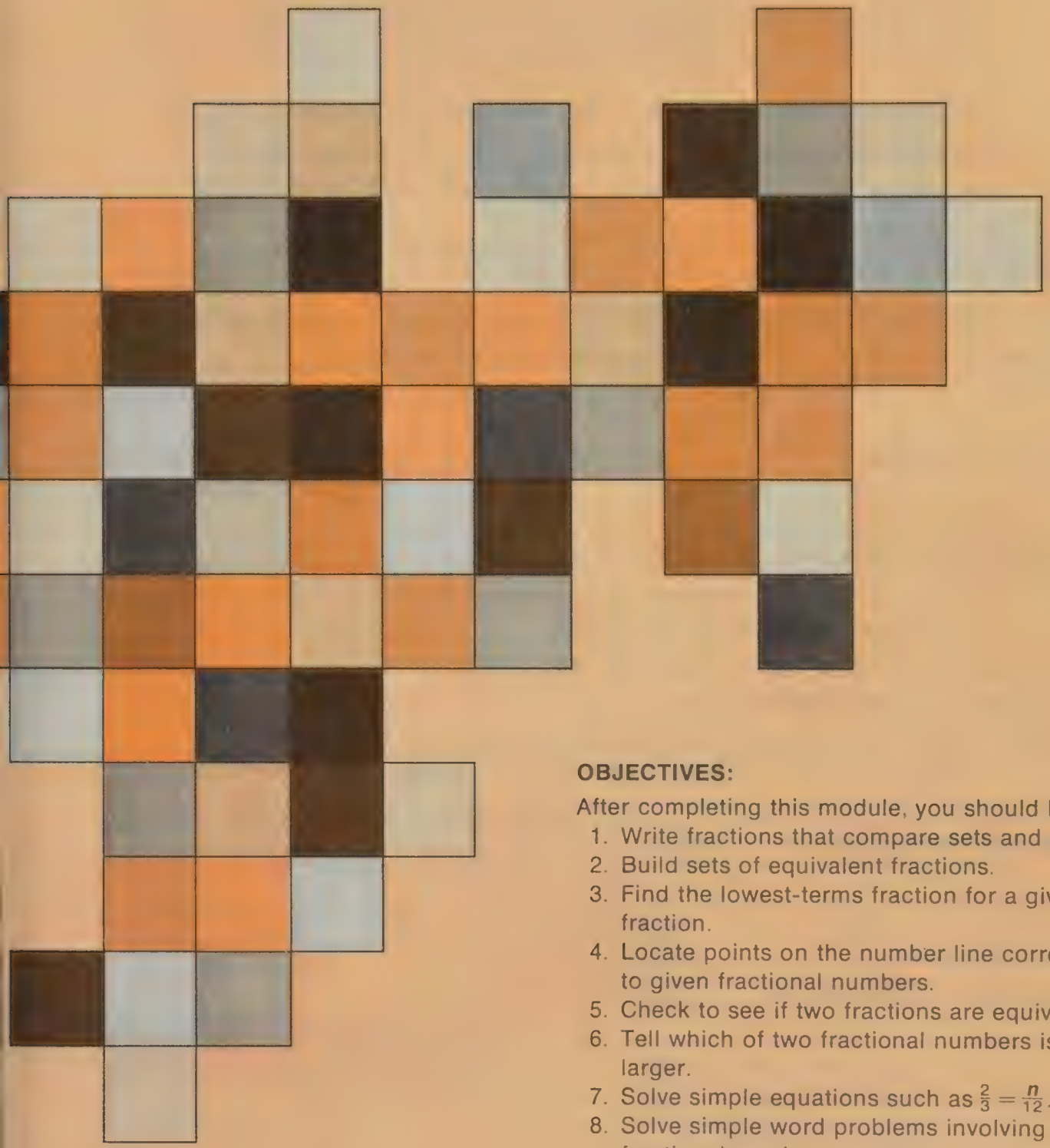
MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

1. 1, 2, 4, 5, 8, 10, 20, 40 **2.** 437 **3.** prime **4.** 5, 29, 47
5. $2 \cdot 5 \cdot 3 \cdot 7$ **6.** 110 **7.** $2 \cdot 3^3$ **8.** A 3^4; B $5^2 \cdot 7^2 \cdot 13^3$; C 10^6
9. 2^5 **10.** A 5; B 7 **11.** A 24; B 120

UNIT B: *The Arithmetic of Fractional Numbers*

MODULE 1: *Fractions and Fractional Numbers*



OBJECTIVES:

After completing this module, you should be able to:

1. Write fractions that compare sets and regions.
2. Build sets of equivalent fractions.
3. Find the lowest-terms fraction for a given fraction.
4. Locate points on the number line corresponding to given fractional numbers.
5. Check to see if two fractions are equivalent.
6. Tell which of two fractional numbers is the larger.
7. Solve simple equations such as $\frac{2}{3} = \frac{n}{12}$.
8. Solve simple word problems involving fractional numbers.

Investigating the Ideas

Teenagers Enjoy Space Age Rocketry

In the newspaper headline, 2 of the 30 letters are T's.
The **fraction** $\frac{2}{30}$ compares the number of times the letter T appears to all the letters in the headline.
We say that $\frac{2}{30}$ of the letters are T's.

Can you write a fraction that compares the number of times each of the other letters appear to all of the letters in the headline?

Discussing the Ideas

1. The numeral above the bar of a fraction represents a number which is the **numerator** of the fraction. The numeral below the bar represents a number which is the **denominator** of the fraction.

$\frac{2}{30}$ Numerator
Denominator

   A What is the denominator of each fraction that you found in the Investigation?
   B What are the numerators of the fractions that you found?

2. What fraction would compare the number of times a vowel (a, e, i, o, u) appears to all the letters in the headline?

3. A What part of the circular region is shaded?
   B What part is unshaded?

4. The fraction $\frac{3}{4}$ compares the length of $\overline{AB}$ to the unit $\overline{CD}$. The fraction $\frac{7}{4}$ is called an **improper fraction** because the numerator is greater than or equal to the denominator. The fraction $\frac{4}{7}$ is called a **proper fraction**.

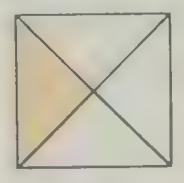
AB: 7 parts
CD: 4 parts

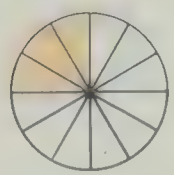
Tell whether each fraction below is a proper or an improper fraction.

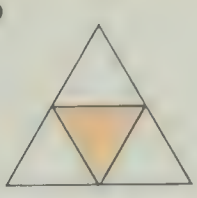
A $\frac{3}{8}$ B $\frac{9}{4}$ C $\frac{5}{5}$ D $\frac{1}{2}$ E $\frac{2}{3}$ F $\frac{3}{2}$ G $\frac{3}{3}$ H $\frac{5}{1}$

1. For each figure, write a fraction that indicates the part of the region that is shaded.

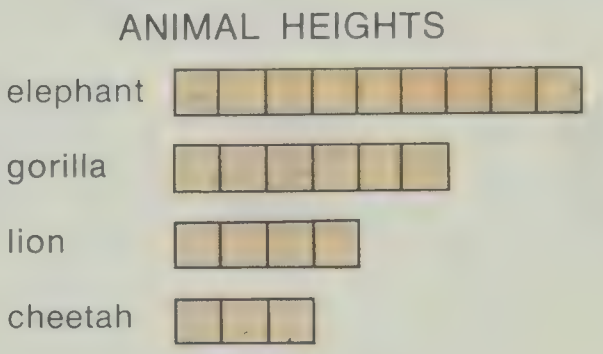
A

B 

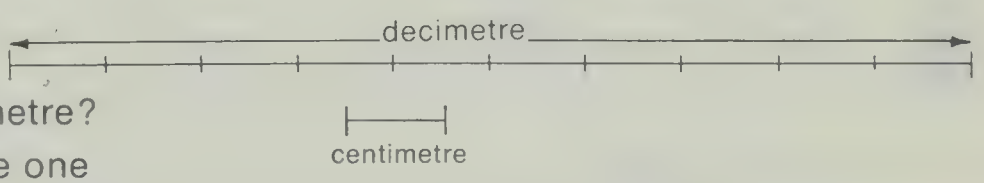
C 

D 

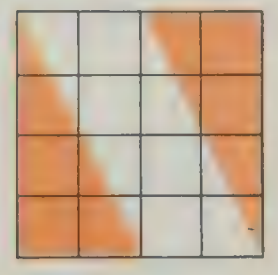
2. The bar graph compares the heights of four animals. Write a fraction that compares the heights in each part.
   - A lion to elephant
   - B cheetah to gorilla
   - C lion to cheetah
   - D elephant to gorilla
   - E cheetah to elephant

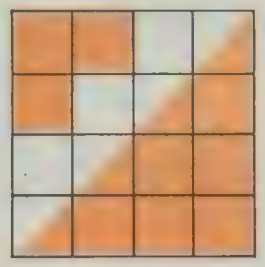


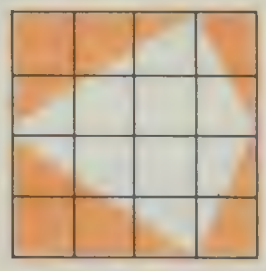
3. A A centimetre is what part of a decimetre?
   B Ten decimetres make one metre. What part of a metre is a centimetre?



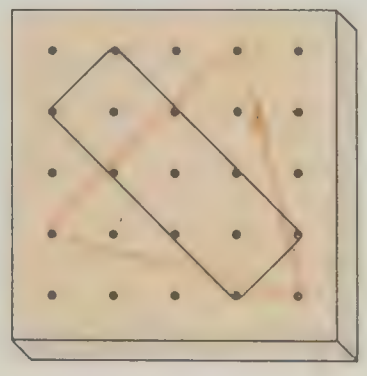
4. Write a fraction that gives the part of each region that is shaded.

A 

B 

C 

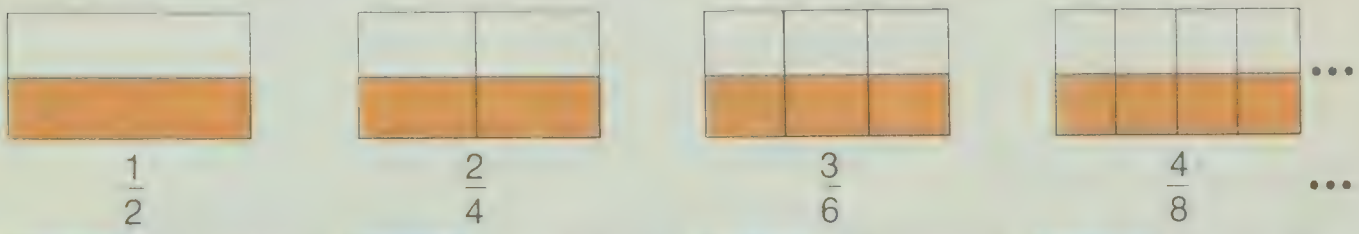
5. What part of the geoboard nails are
   - A inside the triangle?
   - B inside the rectangle?
   - C inside both figures?
   - D outside of the rectangle?
   - E outside of the triangle?
   - F outside both figures?



Investigating the Ideas

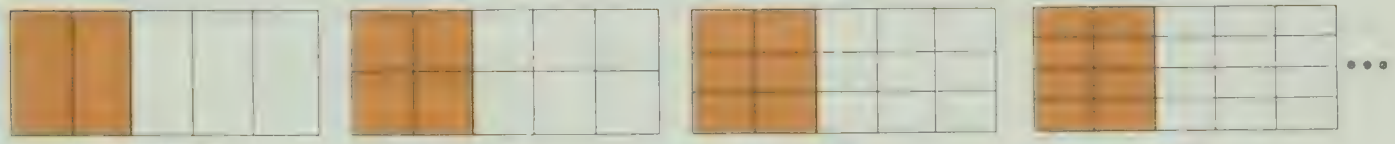
Two different fractions may be suggested by the same subset of a set or by the same part of a region. The fractions in **A** and **B** name the shaded part of each region. Give four more fractions for each sequence.

A



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$ ...

B



$\frac{2}{5}$ $\frac{4}{10}$ $\frac{6}{15}$ $\frac{8}{20}$ ...

Can you draw and shade some regions for the fraction $\frac{3}{4}$?	Record the fractions for each of your regions.

Discussing the Ideas

1. Two different fractions which show the same part of a region or the same part of a set are called **equivalent fractions**. Use the figures in **A** and **B** above to answer these questions.

   A Is $\frac{1}{2}$ equivalent to $\frac{2}{4}$? **B** Is $\frac{2}{5}$ equivalent to $\frac{6}{15}$? **C** Is $\frac{1}{2}$ equivalent to $\frac{2}{5}$?

2. The diagram below may help you find other fractions for **B** above. What are the fractions for **A**, **B**, **C**, and **D**?

$\frac{2}{5}$ $\frac{2 \cdot 1}{5 \cdot 1}$ $\frac{2 \cdot 2}{5 \cdot 2}$ $\frac{2 \cdot 3}{5 \cdot 3}$ $\frac{2 \cdot 4}{5 \cdot 4}$ $\frac{2 \cdot 5}{5 \cdot 5}$ $\frac{2 \cdot 6}{5 \cdot 6}$

$\frac{4}{10}$ **A** **B** **C** **D**

3. If n represents any whole number except zero, what can you conclude about the pair of fractions $\frac{7}{8}$ and $\frac{7 \cdot n}{8 \cdot n}$?

1. Each pair of figures suggests a pair of equivalent fractions. Write the pair of fractions.

A

B

C

D

2. Write the missing fractions for each row.

$\frac{5}{6}$ $\frac{5 \cdot 1}{6 \cdot 1}$ $\frac{5 \cdot 2}{6 \cdot 2}$ $\frac{5 \cdot 3}{6 \cdot 3}$ $\frac{5 \cdot 4}{6 \cdot 4}$ $\frac{5 \cdot 5}{6 \cdot 5}$ $\frac{5 \cdot 6}{6 \cdot 6}$

↓ ↓ ↓ ↓ ↓ ↓

$\frac{5}{6}$ A B C D E

$\frac{4}{3}$ $\frac{4 \cdot 1}{3 \cdot 1}$ $\frac{4 \cdot 2}{3 \cdot 2}$ $\frac{4 \cdot 3}{3 \cdot 3}$ $\frac{4 \cdot 4}{3 \cdot 4}$ $\frac{4 \cdot 5}{3 \cdot 5}$ $\frac{4 \cdot 6}{3 \cdot 6}$

↓ ↓ ↓ ↓ ↓ ↓

F G H I J K

3. Write the next three fractions in each **set of equivalent fractions**.

A $\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\right\}$

B $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\right\}$

C $\left\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21}, \frac{12}{28}, \ldots\right\}$

D $\left\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \ldots\right\}$

E $\left\{\frac{3}{8}, \frac{6}{16}, \frac{9}{24}, \ldots\right\}$

F $\left\{\frac{1}{9}, \frac{2}{18}, \frac{3}{27}, \ldots\right\}$

G $\left\{\frac{1}{5}, \frac{2}{10}, \ldots\right\}$

H $\left\{\frac{2}{7}, \frac{4}{14}, \ldots\right\}$

I $\left\{\frac{7}{10}, \ldots\right\}$

J $\left\{\frac{3}{2}, \ldots\right\}$

K $\left\{\frac{4}{9}, \ldots\right\}$

L $\left\{\frac{5}{5}, \ldots\right\}$

4. One of the fractions in each set is not equivalent to any of the other fractions in that set. Which fraction is it?

A $\left\{\frac{3}{4}, \frac{6}{8}, \frac{8}{12}, \frac{12}{16}, \frac{15}{20}\right\}$

B $\left\{\frac{1}{3}, \frac{2}{3}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}\right\}$

C $\left\{\frac{3}{8}, \frac{6}{16}, \frac{9}{24}, \frac{12}{36}, \frac{15}{40}\right\}$

D $\left\{\frac{2}{3}, \frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}\right\}$

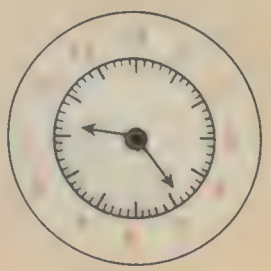
E $\left\{\frac{5}{4}, \frac{10}{6}, \frac{15}{12}, \frac{20}{16}, \frac{25}{20}\right\}$

F $\left\{\frac{6}{24}, \frac{5}{20}, \frac{2}{8}, \frac{4}{18}, \frac{3}{12}\right\}$

think

What part of the clock face is shaded?

Time: 9:24



More practice, page S-12, Set 19

Investigating the Ideas

Read each statement below.

1. Mary bought $\frac{50}{100}$ litre of milk.
2. $\frac{20}{30}$ of the students walk to school.
3. A dime is $\frac{10}{100}$ of a dollar.
4. The team won $\frac{14}{16}$ of its games.

Can you replace each fraction in the statements above with a "simpler" equivalent fraction?

Discussing the Ideas

1. For each set of equivalent fractions there is one fraction that is the "simplest" fraction in the set. This fraction is the **lowest-terms** fraction. What is the lowest-terms fraction for the set of equivalent fractions that contains $\frac{50}{100}$?

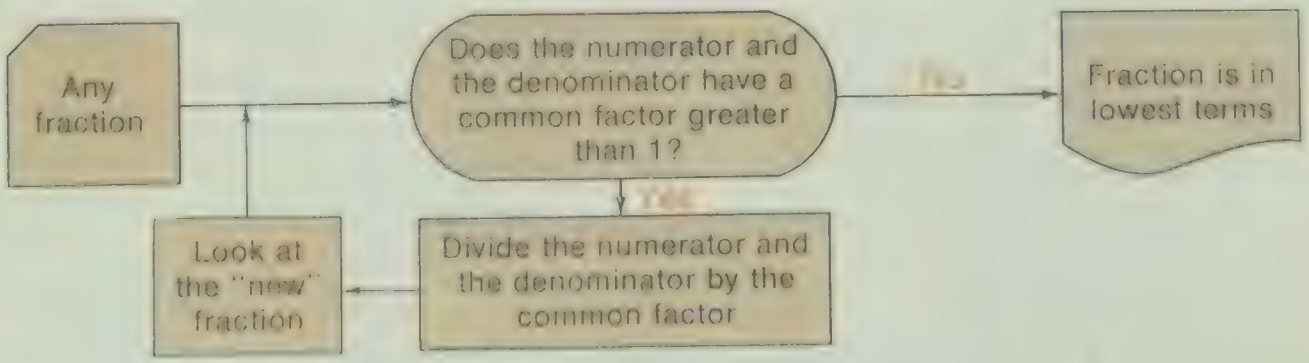
2. A fraction is in lowest terms if the greatest common factor (GCF) of both the numerator and the denominator is 1.

   A Which of the fractions $\frac{12}{15}$ or $\frac{9}{16}$ is in lowest terms?

   B What is the lowest-terms fraction for each fraction in the chart?

Fraction	Factors	GCF	Original fraction
$\frac{12}{15}$	{1, 2, 3, 4, 6, 12}	3	$\frac{4 \cdot 3}{5 \cdot 3}$
	{1, 3, 5, 15}		
$\frac{9}{16}$	{1, 3, 9}	1	$\frac{9 \cdot 1}{16 \cdot 1}$
	{1, 2, 4, 8, 16}		

3. Study the flow chart. Then explain how to use it in order to find the lowest-terms fraction for each fraction below.


Any fraction
Does the numerator and the denominator have a common factor greater than 1?
Yes
Fraction is in lowest terms
Look at the "new" fraction
Divide the numerator and the denominator by the common factor

A $\frac{12}{15}$ B $\frac{20}{24}$ C $\frac{10}{4}$ D $\frac{24}{36}$ E $\frac{7}{7}$ F $\frac{6}{29}$ G $\frac{18}{12}$

1. Give the lowest-terms fraction for each fraction.

A	$\frac{8}{10}$	**C**	$\frac{10}{16}$	**E**	$\frac{18}{18}$	**G**	$\frac{35}{50}$	**I**	$\frac{18}{81}$	**K**	$\frac{77}{98}$	**M**	$\frac{45}{120}$
B	$\frac{12}{14}$	**D**	$\frac{9}{15}$	**F**	$\frac{36}{40}$	**H**	$\frac{12}{20}$	**J**	$\frac{60}{45}$	**L**	$\frac{105}{175}$	**N**	$\frac{72}{180}$

2. Tell whether or not each fraction is in lowest terms. If not, give the lowest-terms fraction.

A	$\frac{10}{12}$	**C**	$\frac{8}{9}$	**E**	$\frac{13}{39}$	**G**	$\frac{11}{44}$	**I**	$\frac{0}{30}$	**K**	$\frac{70}{66}$	**M**	$\frac{12}{121}$
B	$\frac{15}{24}$	**D**	$\frac{14}{15}$	**F**	$\frac{25}{36}$	**H**	$\frac{21}{28}$	**J**	$\frac{4}{15}$	**L**	$\frac{28}{331}$	**N**	$\frac{38}{95}$

3. Find the lowest-terms fraction for each set.

A $\left\{\frac{4}{6}, \frac{6}{9}, \frac{10}{15}, \frac{20}{30}\right\}$

B $\left\{\frac{15}{60}, \frac{16}{64}, \frac{40}{160}, \frac{8}{32}\right\}$

C $\left\{\frac{90}{100}, \frac{900}{1000}, \frac{9000}{10\,000}, \frac{90\,000}{100\,000}\right\}$

D $\left\{\frac{65}{52}, \frac{25}{20}, \frac{100}{80}, \frac{15}{12}\right\}$

E $\left\{\frac{36}{27}, \frac{68}{51}, \frac{140}{105}\right\}$

F $\left\{\frac{10}{35}, \frac{24}{84}, \frac{42}{147}\right\}$

4. If the prime factorizations of the numerator and the denominator are known, then you can easily find the lowest-terms fraction. Study the example and then give the lowest-terms fraction for each part.

EXAMPLE: $\frac{30}{42} = \frac{2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 7} = \frac{5}{7}$

A	$\frac{2 \cdot 3}{2 \cdot 5}$	**D**	$\frac{19 \cdot 23}{19 \cdot 31}$	**G**	$\frac{7 \cdot 11 \cdot 19}{11 \cdot 13 \cdot 19}$	**J**	$\frac{3 \cdot 5 \cdot 5 \cdot 7}{3 \cdot 5 \cdot 7 \cdot 13}$
B	$\frac{2 \cdot 3}{3 \cdot 5}$	**E**	$\frac{2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 7}$	**H**	$\frac{3 \cdot 3 \cdot 7}{3 \cdot 5 \cdot 7}$	**K**	$\frac{2 \cdot 7 \cdot 11 \cdot 11}{7 \cdot 11 \cdot 13}$
C	$\frac{7 \cdot 11}{11 \cdot 13}$	**F**	$\frac{5 \cdot 7 \cdot 11}{2 \cdot 5 \cdot 7}$	**I**	$\frac{11 \cdot 11 \cdot 13}{11 \cdot 13 \cdot 31}$	**L**	$\frac{3 \cdot 3 \cdot 19 \cdot 31}{7 \cdot 3 \cdot 19}$

★5. Give the missing numerator or denominator in order to form fractions equivalent to the given lowest-terms fraction.

A $\frac{3}{4} \rightarrow \left\{\frac{\square}{20}, \frac{9}{\square}, \frac{\square}{40}, \frac{15}{\square}\right\}$

B $\frac{2}{7} \rightarrow \left\{\frac{\square}{21}, \frac{14}{\square}, \frac{6}{\square}, \frac{\square}{70}\right\}$

C $\frac{8}{9} \rightarrow \left\{\frac{24}{\square}, \frac{\square}{45}, \frac{72}{\square}, \frac{\square}{72}\right\}$

D $\frac{5}{6} \rightarrow \left\{\frac{\square}{18}, \frac{\square}{72}, \frac{40}{\square}, \frac{75}{\square}\right\}$

E $\frac{7}{11} \rightarrow \left\{\frac{\square}{22}, \frac{77}{\square}, \frac{\square}{77}, \frac{91}{\square}\right\}$

think

Discounting wind resistance and starting from rest, an object will fall approximately as indicated in the table below.

$490 \cdot 1^2$ cm in 1 sec	$490 \cdot \square^2$ cm in $\square$ sec
$490 \cdot 2^2$ cm in 2 sec	$490 \cdot \square^2$ cm in $\square$ sec

If a skydiver jumps from 2960 metres, about how many seconds should he wait in order to pull his ripcord at about 1000 metres?

More practice, page S-12, Set 20

Investigating the Ideas

Fractions are symbols for **fractional numbers**. For each set of equivalent fractions, we think of exactly one fractional number. Each fractional number can be paired with exactly one point on the number line.

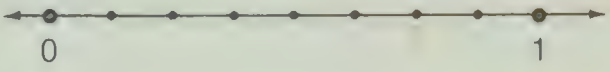
A set of equivalent fractions	One fractional number	One point on the number line
$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \ldots\}$	two thirds	

Can you give a fraction that names the fractional number for the point above the arrow on each number line below?	Record your answers.	

A

B

C

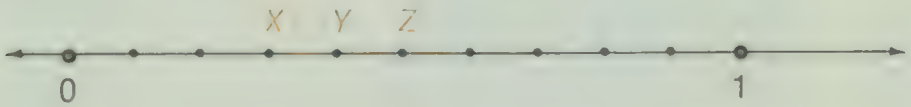


D

Discussing the Ideas

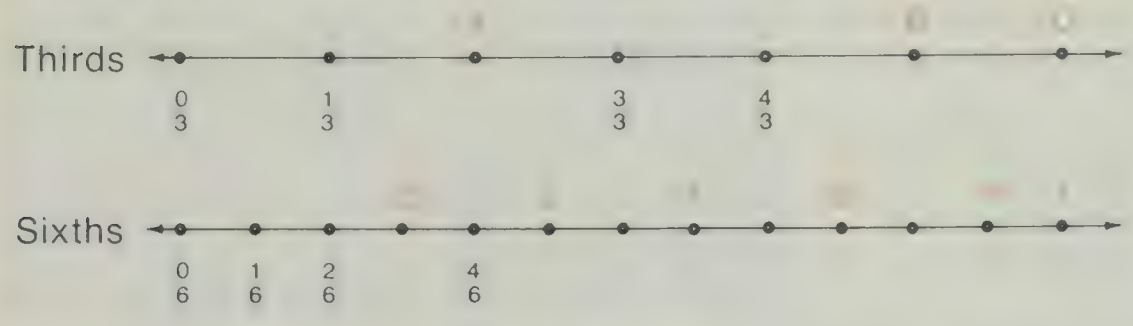
1. For the point on number line A, Steve wrote $\frac{2}{6}$. Margaret wrote $\frac{1}{3}$. Which student is correct?

2. Any fraction in a set of equivalent fractions can be used as a name for the fractional number associated with that set. Usually, the lowest-terms fraction is used as the name for the fractional number. Give the lowest-terms fraction for points *X*, *Y*, and *Z* on the number line.



3. The arrows designate points halfway between points shown on the number line. What number goes with each lettered point?

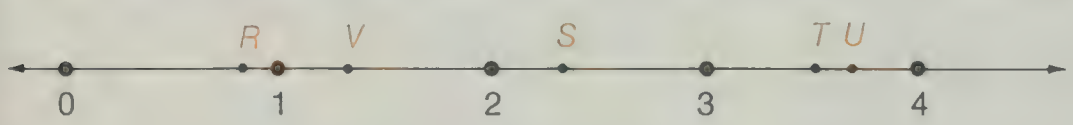
1. Use the denominator suggested at the left of each number line to give the fraction for the point indicated by each letter.



2. The number line is **scaled in fourths** because each segment between two whole-number points is divided into **four** congruent segments. Give the fractional number for each lettered point.

3. Give the fractional number for each lettered point.

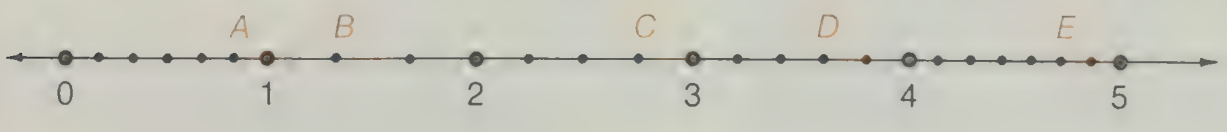
4. One of the five numbers $\frac{7}{3}, \frac{4}{5}, \frac{9}{7}, \frac{7}{2}, \frac{11}{3}$, is matched with each point on the number line. Match the numbers with the lettered points.



5. Draw a number line and mark the points for 0, 1, 2, 3, 4, and 5. Then mark the approximate location of the point for each number below.

A $\frac{3}{2}$ **B** $\frac{12}{5}$ **C** $\frac{10}{3}$ **D** $\frac{25}{6}$ **E** $\frac{9}{2}$ **F** $\frac{1}{10}$ **G** $\frac{11}{10}$ **H** $\frac{21}{10}$

6. Give the fractional numbers that go with the points *A*, *B*, *C*, *D*, and *E*.

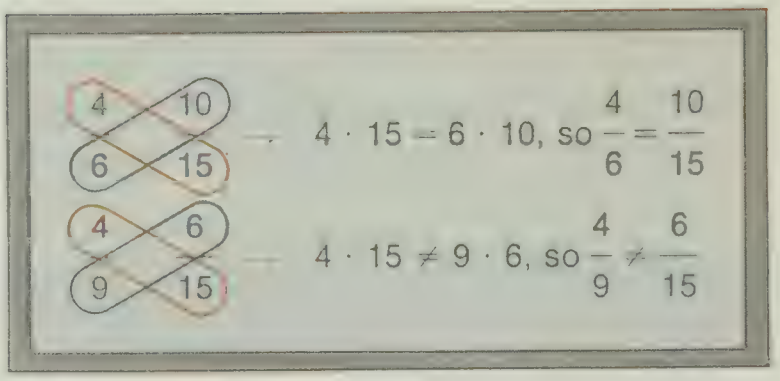


Equality of Fractional Numbers

Investigating the Ideas

Pete showed his classmates a method they could use to tell whether or not two fractions were equivalent.

Study his method in the examples shown on the chalkboard.



Can you use this method to decide which pairs of fractions are equivalent?

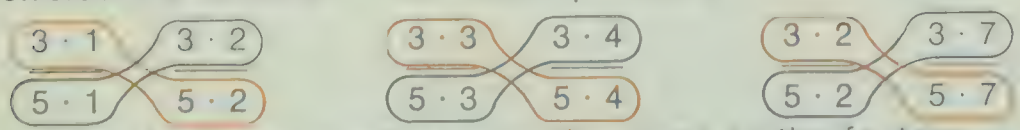
A $\frac{3}{8}, \frac{6}{16}$ B $\frac{4}{6}, \frac{6}{9}$ C $\frac{4}{7}, \frac{6}{10}$ D $\frac{5}{3}, \frac{15}{9}$

Discussing the Ideas

1. The class wondered whether Pete's method would always work. The teacher suggested that they look at some sets of equivalent fractions. She listed the set of fractions equivalent to $\frac{3}{5}$.

$$\left\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{15}{25}, \frac{18}{30}, \frac{21}{35}, \ldots\right\}$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$\left\{\frac{3\cdot1}{5\cdot1}, \frac{3\cdot2}{5\cdot2}, \frac{3\cdot3}{5\cdot3}, \frac{3\cdot4}{5\cdot4}, \frac{3\cdot5}{5\cdot5}, \frac{3\cdot6}{5\cdot6}, \frac{3\cdot7}{5\cdot7}, \ldots\right\}$$

Then she chose several pairs of fractions from the set and circled the factors in the "cross products."



Are the factors circled in color the same as the factors circled in black for each pair of fractions?

2. How would you use the "cross products" method to decide which pairs of fractions are equivalent?

A $\frac{3}{4}, \frac{9}{12}$ B $\frac{6}{18}, \frac{4}{9}$ C $\frac{2}{2}, \frac{10}{10}$ D $\frac{3}{1}, \frac{12}{4}$ E $\frac{5}{6}, \frac{3}{4}$

3. The "cross products" method can be used to solve equations involving fractional numbers.
Study the example. Then explain how you would solve these equations.

A $\frac{3}{4} = \frac{n}{20}$ B $\frac{n}{2} = \frac{9}{6}$ C $\frac{4}{n} = \frac{12}{15}$

EQUATION: $\frac{2}{3} = \frac{n}{12}$

SOLUTION: $3 \cdot n = 2 \cdot 12$
$3 \cdot n = 24$
$n = 8$

1. Which pairs of fractions are equivalent?

A $\frac{5}{6}, \frac{9}{10}$ **B** $\frac{8}{6}, \frac{12}{9}$ **C** $\frac{3}{10}, \frac{9}{100}$ **D** $\frac{4}{20}, \frac{5}{25}$ **E** $\frac{8}{12}, \frac{5}{25}$ **F** $\frac{1}{8}, \frac{3}{20}$ **G** $\frac{5}{3}, \frac{20}{12}$ **H** $\frac{0}{7}, \frac{0}{8}$ **I** $\frac{2}{1}, \frac{18}{9}$ **J** $\frac{1}{3}, \frac{33}{100}$

2. Copy each exercise and insert the correct symbol ($=$ or $\neq$) for each ▨.

A $\frac{6}{9} ▨ \frac{4}{6}$ **B** $\frac{4}{10} ▨ \frac{10}{25}$ **C** $\frac{20}{24} ▨ \frac{5}{6}$ **D** $\frac{8}{9} ▨ \frac{7}{8}$ **E** $\frac{10}{12} ▨ \frac{12}{14}$ **F** $\frac{8}{21} ▨ \frac{4}{10}$ **G** $\frac{7}{6} ▨ \frac{21}{18}$ **H** $\frac{5}{1} ▨ \frac{40}{8}$ **I** $\frac{3}{3} ▨ \frac{11}{11}$ **J** $\frac{90}{100} ▨ \frac{9}{10}$ **K** $\frac{11}{15} ▨ \frac{99}{135}$ **L** $\frac{23}{31} ▨ \frac{713}{961}$

3. Answer true (T) or false (F).

A $\frac{1}{3} = \frac{3}{9}$ **B** $\frac{2}{5} = \frac{5}{2}$ **C** $\frac{3}{7} = \frac{7}{13}$ **D** $\frac{7}{8} = \frac{56}{64}$ **E** $\frac{2}{15} = \frac{12}{90}$ **F** $\frac{5}{6} = \frac{45}{48}$ **G** $\frac{5}{9} = \frac{26}{45}$ **H** $\frac{11}{12} = \frac{55}{60}$ **I** $\frac{14}{16} = \frac{35}{80}$ **J** $\frac{63}{18} = \frac{49}{15}$ ★**K** $\frac{7}{13} = \frac{91}{169}$ ★**L** $\frac{11}{17} = \frac{187}{289}$

4. Solve the equations.

A $\frac{2}{3} = \frac{n}{15}$ **B** $\frac{n}{4} = \frac{7}{2}$ **C** $\frac{3}{n} = \frac{15}{40}$ **D** $\frac{16}{40} = \frac{n}{5}$ **E** $\frac{6}{5} = \frac{24}{n}$ **F** $\frac{8}{12} = \frac{6}{n}$ **G** $\frac{10}{12} = \frac{n}{18}$ **H** $\frac{5}{8} = \frac{n}{8}$ **I** $\frac{n}{10} = \frac{40}{100}$ ★**J** $\frac{4}{n} = \frac{n}{9}$

★5. Choose any four whole numbers greater than 0. Form two fractions, using these four numbers in the following way: Select two of your numbers. Their product is the numerator of the first fraction. The product of the remaining pair of numbers is the denominator of the second fraction. Now make another selection of two of your original numbers. Their product is the denominator of the first fraction. The product of the remaining pair is the numerator of the second fraction.

A Are the two fractions equivalent?

B Can you choose the numbers so that the two fractions will not be equivalent?

think

Try to arrange the nine letters in the nine squares so that each letter is moved one square to the right or left or one square above or below its original position.

If this can be done, show an arrangement of the letters. If you feel it cannot be done, tell why it is not possible.

A	B	C
D	E	F
G	H	I

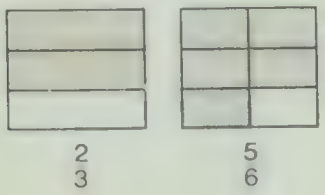
More practice, page S-13, Set 21

Comparing Fractional Numbers

Investigating the Ideas

You can use different methods to compare two fractional numbers.

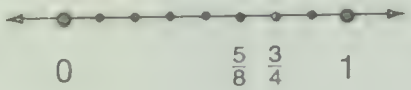
A Compare parts of regions



$\frac{2}{3}$ $\frac{5}{6}$

$\frac{2}{3}$ is ? than $\frac{5}{6}$.

B Use points on the number line



$\frac{3}{4}$ is ? than $\frac{5}{8}$.

C Use common denominators

$\frac{7}{8} = \frac{7 \cdot 6}{8 \cdot 6} = \frac{42}{48}$

$\frac{5}{6} = \frac{5 \cdot 8}{6 \cdot 8} = \frac{40}{48}$

$\frac{7}{8}$ is ? than $\frac{5}{6}$.

Which is the correct word for each blank, **greater** or **less**?

Can you choose any two fractional numbers and compare them using each of the methods?

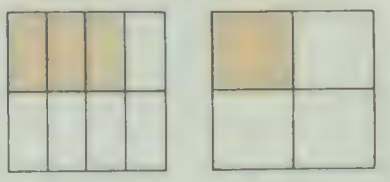
Discussing the Ideas

1. Which method above do you think is most practical to use? Why?

2. You can use the symbol $>$ for "greater than" and $<$ for "less than" for fractional numbers just as you did for the whole numbers.
   Tell which symbol goes in each ▒.

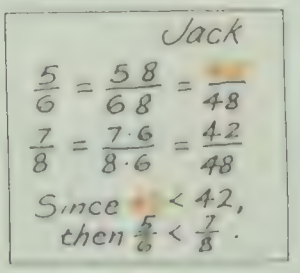
   A $\frac{1}{2}$ ▒ $\frac{3}{8}$ **B** $\frac{3}{5}$ ▒ $\frac{2}{3}$ **C** $\frac{1}{6}$ ▒ $\frac{5}{4}$ **D** $\frac{9}{10}$ ▒ $\frac{1}{8}$

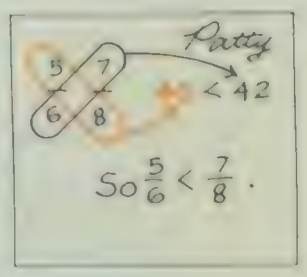
3. **A** Give a fractional number for the shaded part of each region.
   B Which fractional number is greater?



4. **A** Which number is greater, the number for X or the number for Y?
   B Write an inequality statement about the numbers for X and Y.

5. Jack used the common denominator method to compare $\frac{5}{6}$ and $\frac{7}{8}$.
   Patty used a shortcut.
   Use either method to decide which fractional number in each pair is the larger.





A $\frac{4}{5}, \frac{7}{8}$ **B** $\frac{5}{9}, \frac{7}{12}$ **C** $\frac{4}{5}, \frac{7}{10}$ **D** $\frac{12}{10}, \frac{7}{6}$ **E** $\frac{12}{5}, \frac{5}{2}$ **F** $\frac{1}{4}, \frac{1}{6}$

1. Give the numbers for *a* and *b*. Then give the correct symbol ($<$ or $>$) for each ▒.

A $\frac{1}{4} = \frac{a}{12}$, $\frac{1}{3} = \frac{b}{12}$ → $\frac{1}{4}$ ▒ $\frac{1}{3}$

B $\frac{5}{3} = \frac{a}{6}$, $\frac{5}{2} = \frac{b}{6}$ → $\frac{5}{3}$ ▒ $\frac{5}{2}$

C $\frac{3}{4} = \frac{a}{36}$, $\frac{7}{9} = \frac{b}{36}$ → $\frac{3}{4}$ ▒ $\frac{7}{9}$

D $\frac{2}{5} = \frac{a}{35}$, $\frac{3}{7} = \frac{b}{35}$ → $\frac{2}{5}$ ▒ $\frac{3}{7}$

E $\frac{5}{8} = \frac{a}{48}$, $\frac{7}{12} = \frac{b}{48}$ → $\frac{5}{8}$ ▒ $\frac{7}{12}$

F $\frac{7}{6} = \frac{a}{30}$, $\frac{6}{5} = \frac{b}{30}$ → $\frac{7}{6}$ ▒ $\frac{6}{5}$

G $\frac{3}{10} = \frac{a}{100}$, $\frac{33}{100} = \frac{b}{100}$ → $\frac{3}{10}$ ▒ $\frac{33}{100}$

H $\frac{1}{3} = \frac{a}{90}$, $\frac{7}{30} = \frac{b}{90}$ → $\frac{1}{3}$ ▒ $\frac{7}{30}$

2. Copy each problem and use any method you wish to find the correct symbol ($<$ or $>$) for each ▒.

A $\frac{1}{3}$ ▒ $\frac{1}{2}$ **B** $\frac{1}{4}$ ▒ $\frac{3}{16}$ **C** $\frac{1}{4}$ ▒ $\frac{5}{16}$

D $\frac{3}{8}$ ▒ $\frac{2}{5}$ **E** $\frac{1}{9}$ ▒ $\frac{1}{10}$ **F** $\frac{2}{3}$ ▒ $\frac{5}{7}$

G $\frac{7}{4}$ ▒ $\frac{8}{5}$ **H** $\frac{3}{7}$ ▒ $\frac{4}{9}$ **I** $\frac{9}{10}$ ▒ $\frac{10}{11}$

J $\frac{2}{5}$ ▒ $\frac{3}{8}$ **K** $\frac{5}{12}$ ▒ $\frac{3}{8}$ **L** $\frac{5}{16}$ ▒ $\frac{1}{3}$

M $\frac{7}{8}$ ▒ $\frac{11}{12}$ **N** $\frac{19}{25}$ ▒ $\frac{77}{100}$ **O** $\frac{9}{11}$ ▒ $\frac{35}{43}$

3. Which number in each pair is the larger?

A $\frac{1}{4}, \frac{1}{3}$ **B** $\frac{3}{5}, \frac{3}{8}$ **C** $\frac{3}{4}, \frac{4}{5}$ **D** $\frac{1}{2}, \frac{3}{5}$ **E** $\frac{1}{6}, \frac{6}{5}$ **F** $\frac{1}{3}, \frac{1}{2}$ **G** $\frac{3}{4}, \frac{5}{8}$ **H** $\frac{2}{3}, \frac{3}{5}$ **I** $\frac{3}{4}, \frac{7}{8}$ **J** $\frac{8}{7}, \frac{7}{6}$ **K** $\frac{1}{10}, \frac{1}{9}$ **L** $\frac{5}{12}, \frac{6}{11}$ **M** $\frac{4}{19}, \frac{3}{18}$ **N** $\frac{7}{50}, \frac{3}{25}$

think

A set of three consecutive even numbers → {24, 26, 28}

A set of four consecutive odd numbers → {7, 9, 11, 13}

1. Find a set of three consecutive **even** numbers whose sum is 216.
2. Find a set of four consecutive **odd** numbers whose sum is 216.
3. Find a set of three consecutive **whole** numbers whose sum is 216.

▒ + ▒ + ▒ = 216

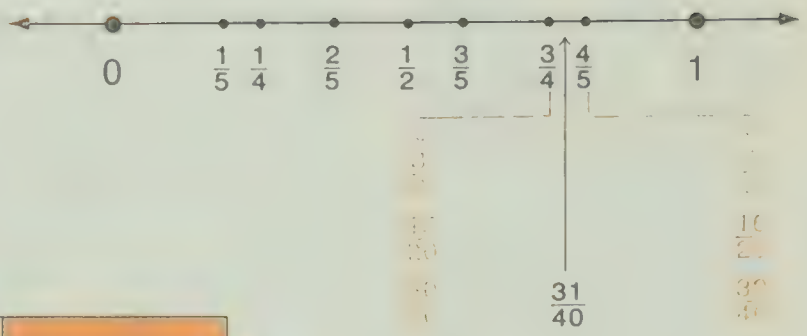
More practice, page S-13, Set 22

Order of Fractional Numbers

Investigating the Ideas

Is there a fractional number for the point that is halfway between the points for $\frac{3}{4}$ and $\frac{4}{5}$?

Study the diagram to see why $\frac{31}{40}$ is halfway between $\frac{3}{4}$ and $\frac{4}{5}$.



Can you use this method to find a fractional number that is halfway between $\frac{1}{2}$ and $\frac{3}{5}$?

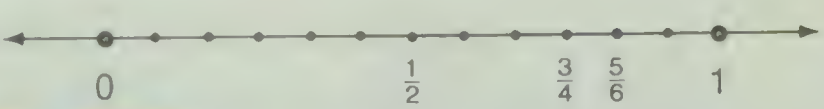
Discussing the Ideas

1. The number $\frac{31}{40}$ is halfway between $\frac{3}{4}$ and $\frac{4}{5}$. Is there a fractional number halfway between $\frac{3}{4}$ and $\frac{31}{40}$? Can you name a fraction for it?

2. Can you give the name of the fractional number that is halfway between $\frac{57}{100}$ and $\frac{58}{100}$ Explain how you found it.

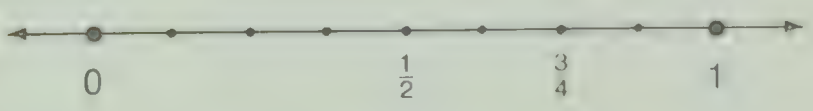
3. Do you think there is always another fractional number between any two different fractional numbers?

4. From the number line



you can see that $\frac{1}{2} < \frac{3}{4}$ and $\frac{3}{4} < \frac{5}{6}$. You can shorten this statement by writing $\frac{1}{2} < \frac{3}{4} < \frac{5}{6}$.

5. A What is the name for the fractional number for the point halfway between the points for $\frac{1}{2}$ and $\frac{3}{4}$?



B Give a fraction for the ____: $\frac{1}{2} < \frac{\quad}{\quad} < \frac{3}{4}$.

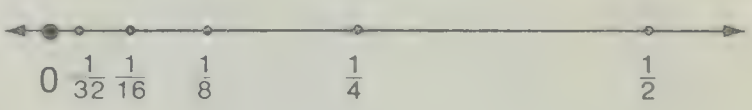
1. What fractional number is halfway between $\frac{2}{5}$ and $\frac{3}{5}$?

2. **A** Find the fractional number for the point on the number line that is just halfway between the points for $\frac{1}{3}$ and $\frac{1}{5}$.
   B Is $\frac{1}{4}$ larger or smaller than this number?
   C Is the point for $\frac{1}{4}$ closer to the point for $\frac{1}{3}$ or the point for $\frac{1}{5}$?
   D Find the fractional number for the point on the number line that is halfway between $\frac{1}{4}$ and $\frac{1}{5}$.

3. Write a shorter statement that has the same meaning as the given statement.
   EXAMPLE: For $\frac{1}{4} < \frac{1}{3}$ and $\frac{1}{3} < \frac{1}{2}$, write $\frac{1}{4} < \frac{1}{3} < \frac{1}{2}$
   A $\frac{1}{2} < \frac{3}{4}$ and $\frac{3}{4} < \frac{7}{8}$
   B $\frac{1}{5} < \frac{3}{10}$ and $\frac{3}{10} < \frac{3}{8}$
   C $\frac{5}{4} < \frac{5}{3}$ and $\frac{5}{3} < \frac{5}{2}$
   D $\frac{1}{7} < \frac{1}{6}$ and $\frac{1}{6} < \frac{1}{5}$
   E $\frac{1}{3} > \frac{1}{6}$ and $\frac{1}{6} > \frac{1}{12}$
   F $\frac{2}{5} < \frac{3}{4}$ and $\frac{7}{8} > \frac{3}{4}$

4. Give the missing numerator for each ▥.
   A $\frac{3}{8} < \frac{▥}{16} < \frac{4}{8}$
   B $\frac{1}{2} < \frac{▥}{8} < \frac{3}{4}$
   C $\frac{6}{10} < \frac{▥}{20} < \frac{7}{10}$
   D $\frac{5}{7} < \frac{▥}{14} < \frac{6}{7}$
   E $\frac{4}{9} < \frac{▥}{18} < \frac{5}{9}$
   F $\frac{2}{3} < \frac{▥}{24} < \frac{3}{4}$
   G $\frac{0}{4} < \frac{▥}{8} < \frac{1}{4}$
   H $\frac{7}{8} < \frac{▥}{48} < \frac{11}{12}$

5. **A** What is the smallest number in the set $\left\{\frac{1}{32}, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}\right\}$?
   B Give the names for two fractional numbers which are less than $\frac{1}{32}$.



6. What is the smallest number in the set $\left\{\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}\right\}$?

7. What is the largest number in the set $\left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}\right\}$?

8. **A** Which number is larger, $\frac{7}{8}$ or $\frac{6}{7}$?
   B Which number is smaller, $\frac{4}{5}$ or $\frac{5}{6}$?

9. If the numbers $\frac{3}{2}, \frac{3}{4}, \frac{7}{8}, \frac{7}{6}, \frac{15}{16}, \frac{5}{8}, \frac{2}{3}, \frac{19}{16}$ were listed in order of their closeness to 1, what would the order be?

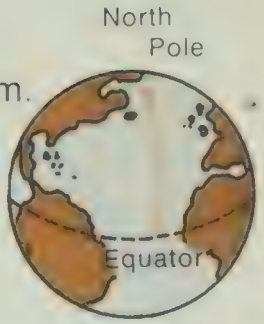
10. Is there a "smallest" fractional number that is still larger than zero?

think

Give the lowest-terms fraction for each point *A* and *B* on the number line.

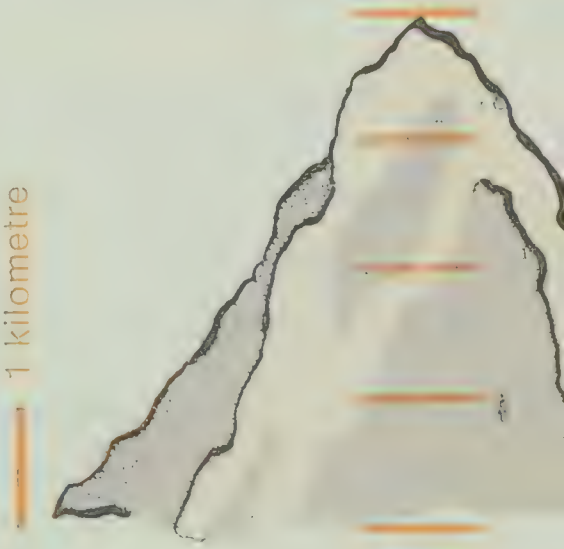
THE METRIC SYSTEM

1. The **metre** (m) is the standard unit of length in the metric system. Originally, the metre was a length such that the distance from the North Pole to the Equator measured along a certain meridian on Earth was 10 000 000 metres. What fraction compares one metre to the distance from the North Pole to the Equator?

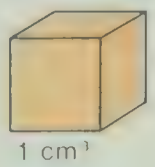


2. A metre is divided into 100 parts of equal length called **centimetres** (cm). What fraction compares the length of a centimetre with the length of a metre.

3. A **kilometre** (km) is 1000 metres. The drawing compares the height of Mt. Robson in British Columbia with a distance of one kilometre. What fraction compares the length of a kilometre with the height of Mt. Robson?



4 The **litre** (ℓ) is the metric unit of capacity. A litre is 1000 cubic centimetres (cm^3). What fraction compares a cubic centimetre to the litre?



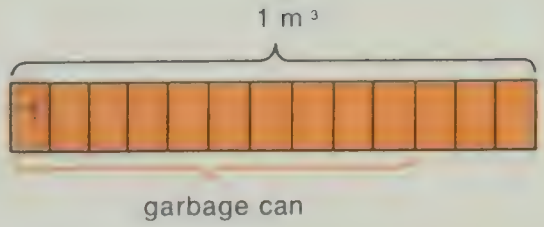
5. The standard unit of weight in the metric system is a **kilogram** (kg). There are 1000 grams (g) in one kilogram. What fraction compares one gram with one kilogram?

6. The **square centimetre** (cm^2) is a metric unit of area. A **square metre** (m^2) is also a metric area. What fraction would you write to compare the size of the square centimetre to that of a square metre?

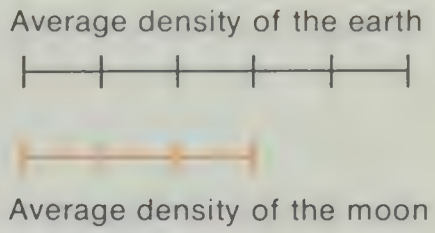
FRACTIONAL NUMBER PROBLEMS

1. A decimetre is $\frac{1}{10}$ of a metre. What lowest-terms fraction compares 4 decimetres with one metre?

2. The shaded region shows what fractional part a large garbage can is of 1 cubic metre (m^3). What is this fraction?

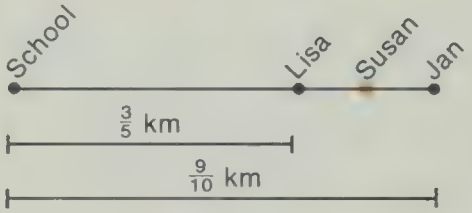


3. The average mass of the earth per cubic metre is greater than that of the moon. We say that the density of the moon is less than the density of the earth. The graph compares these densities. Express the average density of the moon as a fraction of the average density of the earth.

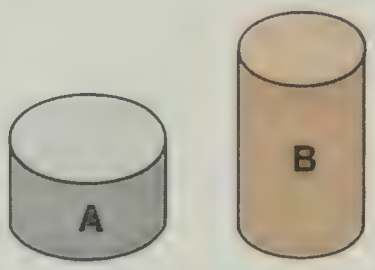


4. A cubic centimetre of gold is heavier than a cubic centimetre of iron. The graph compares the densities of these metals. Express the density of iron as a fraction of the density of gold.

5. Jan's house is $\frac{9}{10}$ km east of school. Lisa lives on the same street $\frac{3}{5}$ km east of school. Susan lives halfway between them. How far does Susan live from school?



6. Container A holds $\frac{3}{5}$ litre.
Container B holds $\frac{2}{3}$ litre.
Which container holds more?



7. A T-bone steak weighed $\frac{3}{10}$ kilogram. A piece of sirloin steak weighed $\frac{33}{100}$ kilogram. Which steak weighed less?

8. A **hectare** is a metric unit of area and is 10 000 square metres. The graph compares the size of a hectare with the size of a building lot. What fraction compares them?

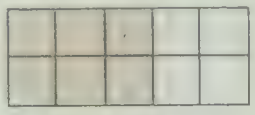
REVIEWING THE IDEAS

1. Each figure suggests a pair of equivalent fractions. Give these fractions.

A

B

C



2. Give three fractions equivalent to the fraction given.

A $\frac{4}{5}$ B $\frac{1}{6}$ C $\frac{5}{8}$ D $\frac{3}{10}$ E $\frac{5}{16}$

3. Give the GCF of the numerator and denominator of each fraction.

A $\frac{12}{15}$ C $\frac{9}{27}$ E $\frac{26}{39}$ G $\frac{25}{30}$

B $\frac{8}{12}$ D $\frac{8}{9}$ F $\frac{14}{21}$ H $\frac{80}{90}$

4. Give the lowest-terms fraction for each fraction.

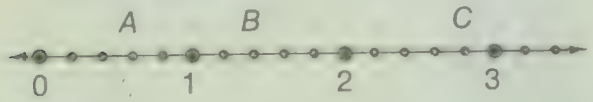
A $\frac{16}{20}$ E $\frac{20}{24}$ I $\frac{12}{9}$

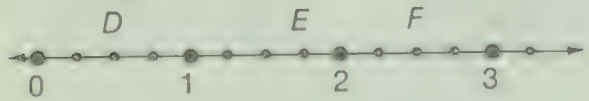
B $\frac{10}{12}$ F $\frac{16}{28}$ J $\frac{48}{100}$

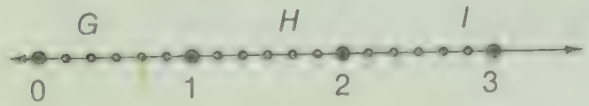
C $\frac{24}{36}$ G $\frac{2 \cdot 5}{7 \cdot 5}$ K $\frac{9}{30}$

D $\frac{16}{40}$ H $\frac{2}{2} \cdot \frac{2}{3} \cdot \frac{3}{5}$ L $\frac{60}{72}$

5. Give the lowest-terms fraction for each letter.







6. Solve the equations.

A $\frac{4}{5} = \frac{n}{20}$ D $\frac{3}{16} = \frac{n}{64}$

B $\frac{3}{4} = \frac{n}{16}$ E $\frac{9}{10} = \frac{n}{100}$

C $\frac{1}{2} = \frac{n}{30}$ F $\frac{2}{3} = \frac{n}{27}$

7. Copy and give the correct symbol ($<$, $=$, or $>$) for each ▨.

A $\frac{5}{8}$ ▨ $\frac{10}{12}$ D $\frac{3}{7}$ ▨ $\frac{4}{9}$

B $\frac{4}{3}$ ▨ $\frac{9}{6}$ E $\frac{5}{6}$ ▨ $\frac{4}{5}$

C $\frac{3}{2}$ ▨ $\frac{9}{6}$ F $\frac{5}{9}$ ▨ $\frac{4}{7}$

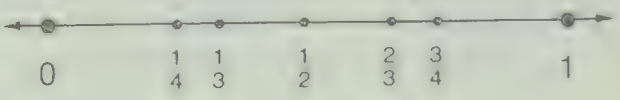
8. Copy and correctly fill in each screen.

A The statement, $\frac{5}{8} < \frac{2}{3} < \frac{5}{6}$, means $\frac{5}{8} <$ ▨ and ▨ $< \frac{5}{6}$.

B $\frac{3}{4} <$ ▨ $< \frac{5}{6}$

C $\frac{1}{2} <$ ▨ $< \frac{5}{8}$

9. Give the fractional number for the point that is halfway between the points for $\frac{2}{3}$ and $\frac{3}{4}$.



10. Give a whole number for n: $\frac{1}{6} < \frac{n}{60} < \frac{1}{5}$.

11. Mr. Knowlton sells a litre of cider for 48 cents. Mr. Shoup sells a litre of cider for 56 cents. What is the fraction that compares Mr. Knowlton's price to Mr. Shoup's price?

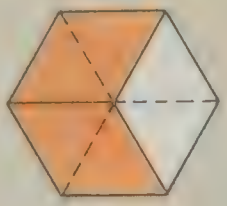
12. The graph compares the density of gasoline to the density of water.

What fraction compares the density of gasoline to the density of water?

TEST YOURSELF

1. What part of the set is composed of stars?

2. Name a pair of equivalent fractions suggested by the figure



3. Write three more fractions in this set of equivalent fractions.
$\left\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \ldots\right\}$

4. Give the lowest-terms fraction for each fraction.
A $\frac{10}{12}$ B $\frac{15}{21}$ C $\frac{42}{36}$

5. Give the fractional numbers for *X*, *Y*, and *Z*.

6. A Is $\frac{9}{24}$ equivalent to $\frac{6}{16}$?
B Is $\frac{8}{12}$ equivalent to $\frac{12}{16}$?

7. Find the number for *n*. $\frac{5}{3} = \frac{n}{12}$

8. Which number is larger, $\frac{5}{6}$ or $\frac{7}{10}$?

9. What number is halfway between $\frac{1}{4}$ and $\frac{1}{2}$?

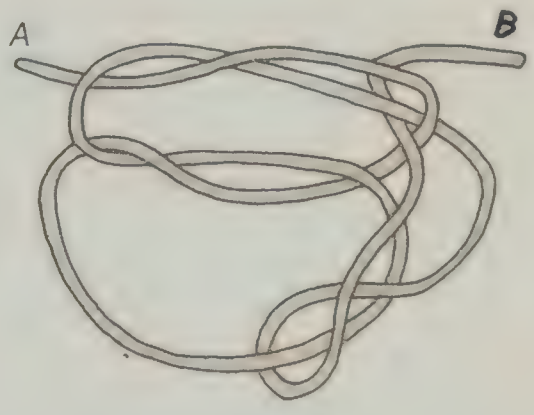
10. Give the missing numerator. $\frac{3}{5} < \frac{\text{IIII}}{80} < \frac{5}{8}$

RESEARCH PROJECTS

A Read about how the Egyptians used "unit fractions." Then explain their method to your class by giving example of your own. (See *Twenty-seventh Yearbook*, National Council of Teachers of Mathematics; Washington, D.C., 1963, pp. 221–226. *Science Awakening* by B. L. Van der Waerden; New York: John Wiley and Sons Canada, Ltd. 1963, pp. 19–23, 48–51.)

B Try writing some fractions using a base other than ten. For example, in base four the fraction $\frac{5}{8}$ would be written as $\frac{11}{13_{(4)}}$. In base two, the fraction $\frac{7}{10}$ would be written $\frac{111}{1010_{(2)}}$. List some sets of equivalent fractions using other bases. Make a chart that compares fractions in other bases with fractions in base ten.

C How are knots and mathematics related? Is the figure below a knot or a false knot? Find out about problems like this one.



(*See Topology: The Rubber-Sheet Geometry* by Donovan Johnson and William H. Glenn; St. Louis: Webster Publishing Co., 1960 available from McGraw-Hill Ryerson, Ltd.)

the pentagonal circuit

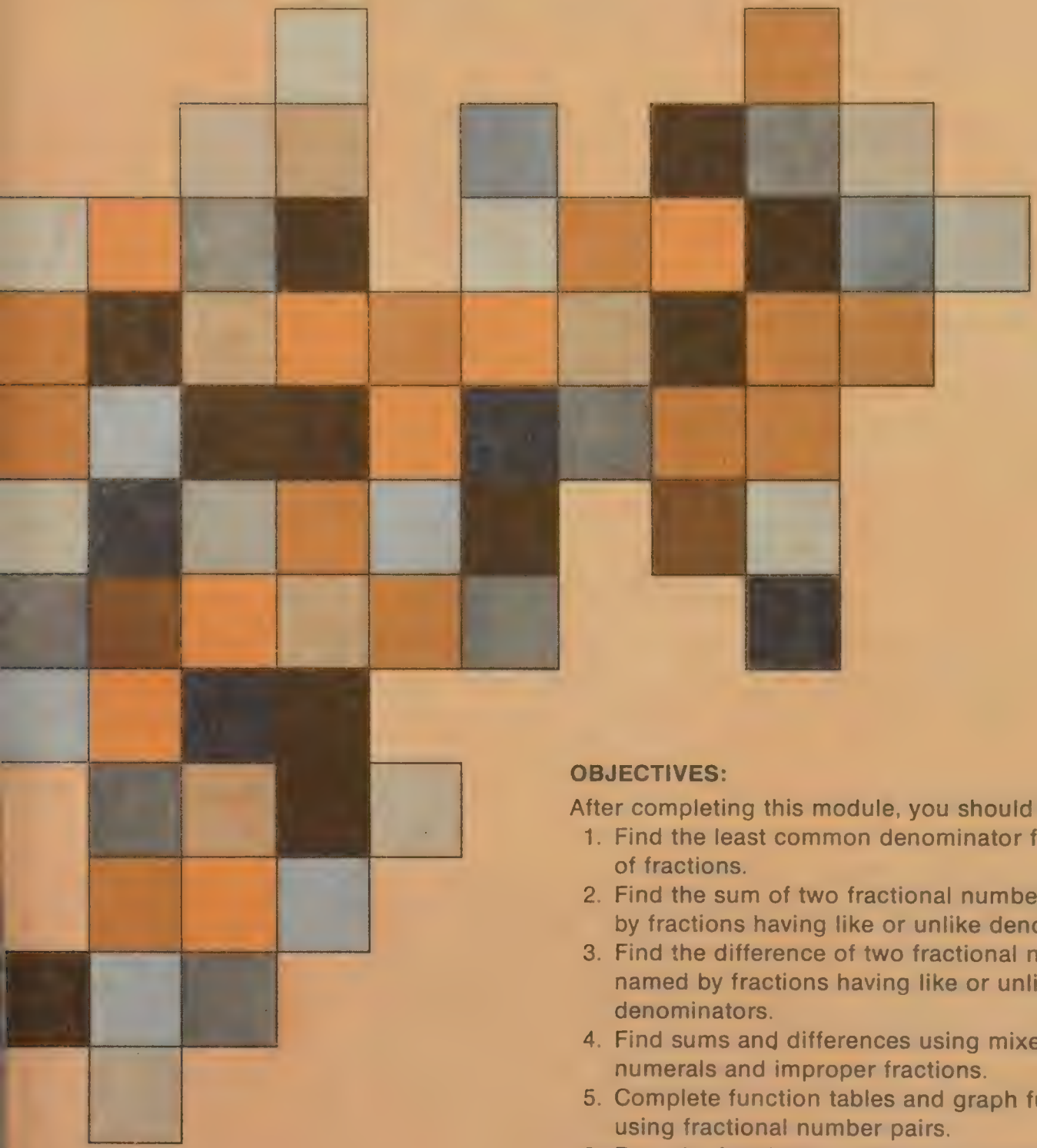
The 20 points on the diagram below represent 20 towns. The segments connecting the points represent roads.

One player selects five towns that can be reached in consecutive order such as I, S, T, P, O and draws a path connecting them. The second player must start at the last town (in this case, O) and draw a path which passes through each of the remaining towns exactly once and then return to the starting point. If the second player cannot find a path, he loses the game. Tracing paper can be placed over the diagram and the paths drawn on the tracing paper. Or, counters can be placed on the diagram to mark the routes taken.

TEST YOURSELF Answers

1. $\frac{4}{9}$ 2. $\frac{2}{3}, \frac{4}{6}$ 3. $\frac{8}{20}, \frac{10}{25}, \frac{12}{30}$ 4. A $\frac{5}{6}$; B $\frac{5}{7}$; C $\frac{7}{8}$ 5. X: $\frac{1}{6}$; Y: $\frac{1}{3}$; Z: $\frac{5}{6}$
6. A yes; B no 7. $n = 20$ 8. $\frac{5}{6}$ 9. $\frac{3}{8}$ 10. 49

MODULE 2: *Adding and Subtracting Fractional Numbers*



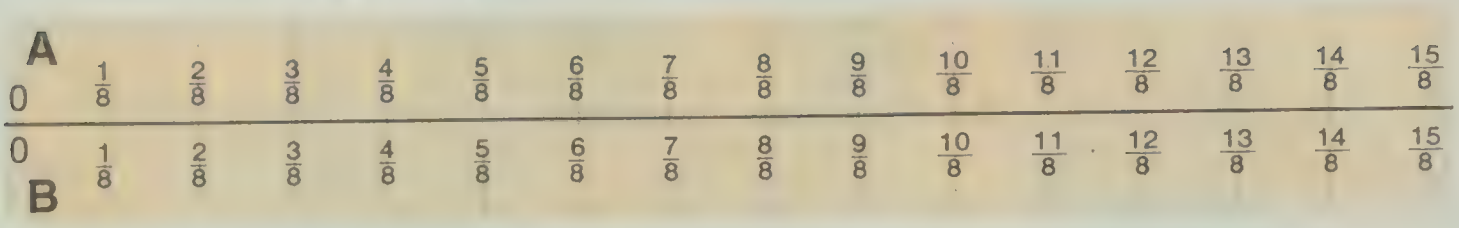
OBJECTIVES:

After completing this module, you should be able to:

1. Find the least common denominator for a pair of fractions.
2. Find the sum of two fractional numbers named by fractions having like or unlike denominators.
3. Find the difference of two fractional numbers named by fractions having like or unlike denominators.
4. Find sums and differences using mixed numerals and improper fractions.
5. Complete function tables and graph functions using fractional number pairs.
6. Round a fractional number to the nearest whole number.
7. Solve word problems involving addition or subtraction of fractional numbers.

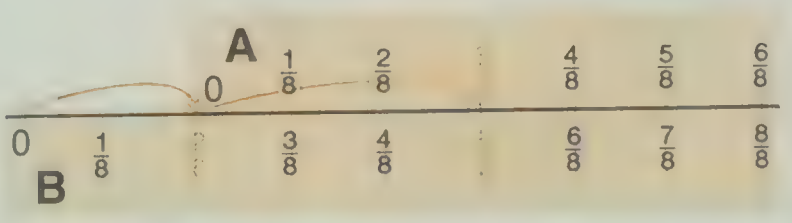
Investigating the Ideas

Make a number line scaled in eighths on both sides of the line on a strip of graph paper. Cut along the line so as to divide the strip into parts **A** and **B**.



You can use the two parts, **A** and **B**, as an addition slide rule. For example, with the strips in the position shown at the right, you can see that

$\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$.



Can you use your slide rule to find at least 10 more sums?	Write an equation for each sum that you find.	

Discussing the Ideas

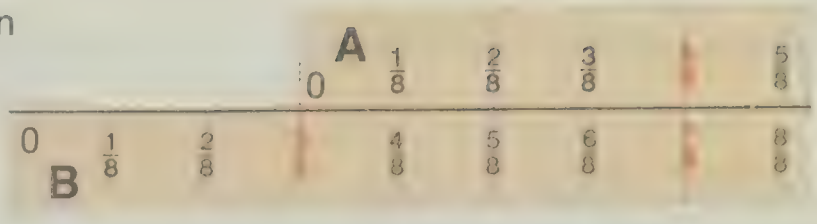
1. **A** How can you use your slide rule to find $\frac{3}{8} + \frac{7}{8}$?
   B What is the sum as a lowest-terms fraction?

2. **A** What fractions would you use on the slide rule to find $\frac{1}{2} + \frac{3}{4}$?
   B What is the lowest-terms fraction for this sum?

3. How can you use the slide rule to find the difference $\frac{5}{8} - \frac{3}{8}$?

4. Study the examples. Then give the missing sum or difference.
   A To find $\frac{1}{2} + \frac{3}{8}$, we think $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10} \ldots\right\}$
   and write $\frac{4}{8} + \frac{3}{8} = \frac{\text{IIII}}{\text{IIII}}$.
   B To find $\frac{11}{8} - \frac{3}{4}$, we think $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\right\}$
   and write $\frac{11}{8} - \frac{6}{8} = \frac{\text{IIII}}{\text{IIII}}$.

1. Give an addition and a subtraction equation for the slide rule. Use the fractions shown in color.



2. Find each sum and difference.

A $\frac{5}{8}+\frac{1}{8}$ C $\frac{8}{8}+\frac{8}{8}$ E $\frac{3}{4}+\frac{7}{4}$ G $\frac{3}{4}+\frac{5}{4}$ I $\frac{5}{16}+\frac{8}{16}$ K $\frac{16}{15}-\frac{4}{15}$

B $\frac{9}{8}-\frac{3}{8}$ D $\frac{9}{8}-\frac{8}{8}$ F $\frac{5}{2}-\frac{3}{2}$ H $\frac{9}{4}+\frac{3}{4}$ J $\frac{4}{12}+\frac{10}{12}$ L $\frac{7}{24}+\frac{9}{24}$

3. Give the numerators x and y.

A $\frac{3}{4}+\frac{1}{2}=\frac{3}{4}+\frac{x}{4}=\frac{y}{4}$ $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots\right\}$

B $\frac{13}{8}-\frac{5}{4}=\frac{13}{8}-\frac{x}{8}=\frac{y}{8}$ $\left\{\frac{5}{4}, \frac{10}{8}, \frac{15}{12}, \ldots\right\}$

C $\frac{2}{3}+\frac{5}{12}=\frac{x}{12}+\frac{5}{12}=\frac{y}{12}$ $\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\right\}$

4. Give the numerators r and s.

A $\frac{7}{8}+\frac{1}{2}=\frac{7}{8}+\frac{r}{8}=\frac{s}{8}$

B $\frac{11}{4}-\frac{3}{2}=\frac{11}{4}-\frac{r}{4}=\frac{s}{4}$

C $\frac{1}{8}+\frac{9}{4}=\frac{1}{8}+\frac{r}{8}=\frac{s}{8}$

D $\frac{9}{2}-\frac{7}{8}=\frac{r}{8}-\frac{7}{8}=\frac{s}{8}$

E $2+\frac{5}{4}=\frac{r}{4}+\frac{5}{4}=\frac{s}{4}$

F $\frac{3}{4}+\frac{9}{16}=\frac{r}{16}+\frac{9}{16}=\frac{s}{16}$

G $\frac{3}{5}-\frac{3}{10}=\frac{r}{10}-\frac{3}{10}=\frac{s}{10}$

H $3-\frac{5}{6}=\frac{r}{6}-\frac{5}{6}=\frac{s}{6}$

I $\frac{11}{3}+\frac{5}{6}=\frac{r}{6}+\frac{5}{6}=\frac{s}{6}$

5. Find the sums and differences.

A $\frac{3}{8}+\frac{3}{2}$ C $\frac{1}{2}+\frac{1}{8}$ E $\frac{9}{4}+\frac{3}{8}$ G $\frac{13}{2}-\frac{9}{8}$ I $\frac{5}{8}-\frac{3}{16}$ K $5+\frac{3}{2}$

B $\frac{11}{8}-\frac{1}{4}$ D $\frac{3}{2}-\frac{3}{4}$ F $\frac{5}{8}-\frac{1}{4}$ H $\frac{3}{4}+\frac{5}{16}$ J $4-\frac{7}{4}$ L $\frac{15}{2}+4$

★6. If a, b, and c are whole numbers and $b \neq 0$, then $\frac{a}{b}+\frac{c}{b}=$ ▨.

think

The ancient Greek mathematicians gave the name **triangular numbers** to the following set: {1, 3, 6, 10, . . .}

The **square numbers** are the numbers in this set: {1, 4, 9, 16, . . .}

1. Give the next five triangular numbers.
2. Give the next five square numbers.
3. Pick any triangular number. Add it to the next one. Do this several times. What do you notice?

Choosing the Least Common Denominator (LCD)

Investigating the Ideas

You can find the sum of two fractional numbers by choosing fractions that have the same denominator.

Consider the sum $\frac{1}{2} + \frac{2}{3}$.

$\frac{1}{2} \rightarrow \left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \frac{9}{18}, \frac{10}{20}, \frac{11}{22}, \frac{12}{24}, \ldots\right\}$

$\frac{2}{3} \rightarrow \left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}, \frac{16}{24}, \frac{18}{27}, \ldots\right\}$

$\frac{3}{6} + \frac{4}{6} =$ ▒▒▒ $\frac{6}{12} + \frac{8}{12} =$ ▒▒▒ $\frac{9}{18} + \frac{12}{18} =$ ▒▒▒ $\frac{12}{24} + \frac{16}{24} =$ ▒▒▒

Can you give each of the missing fractional numbers for the screens above and show that they are equivalent to each other?	Write at least three more addition equations for the sum of $\frac{1}{2}$ and $\frac{2}{3}$.

Discussing the Ideas

1. Although you may use many different pairs of fractions to find a sum, your work will be easier if you choose pairs with the smallest possible common denominator. This denominator is called the **least common denominator** (LCD) for the two fractions given. In the example above, 6 was the LCD for $\frac{1}{2}$ and $\frac{2}{3}$. Notice that 6 is the least common multiple (LCM) of the denominators 2 and 3. What is the LCD for $\frac{5}{8}$ and $\frac{1}{6}$?

2. **A** List the multiples of 8 up to 80.
   B List the multiples of 14 up to 140.
   C What is the LCM of 8 and 14.
   D What is the LCD for $\frac{3}{8}$ and $\frac{5}{14}$?

3. You can find the LCD for two fractions by thinking about the prime factorization of their denominators.

   Consider the sum $\frac{9}{30} + \frac{13}{42}$.

   Prime factorization of 30 = $2 \cdot 3 \cdot 5$
   Prime factorization of 42 = $2 \cdot 3 \cdot 7$
   LCM of 30 and 42: $2 \cdot 3 \cdot 5 \cdot 7 = 210$; LCD for $\frac{9}{30}$ and $\frac{13}{42}$ is 210

   A Find the sum $\frac{9}{30} + \frac{13}{42}$.
   B Explain how to use this method to find the LCD for $\frac{5}{18} + \frac{7}{24}$.

1. **A** Give two common multiples of 16 and 40.
   B Give the LCM of these two numbers.
   C What is the LCD for $\frac{1}{16}$ and $\frac{1}{40}$?
   D Find the sum $\frac{5}{16} + \frac{7}{40}$.

2. Find the LCD for the pair of fractions by finding the LCM of the denominators.

A $\frac{2}{3}, \frac{7}{9}$	**C** $\frac{2}{15}, \frac{3}{10}$	**E** $\frac{1}{4}, \frac{1}{9}$	**G** $\frac{1}{3}, \frac{4}{5}$	**I** $\frac{7}{12}, \frac{9}{16}$	**K** $\frac{5}{9}, \frac{1}{12}$
B $\frac{1}{6}, \frac{1}{8}$	**D** $\frac{5}{8}, \frac{1}{12}$	**F** $\frac{2}{7}, \frac{5}{8}$	**H** $\frac{3}{10}, \frac{7}{100}$	**J** $\frac{1}{2}, \frac{3}{11}$	**L** $\frac{11}{16}, \frac{5}{6}$

3. **A** Find the LCM of 27 and 45.
   B What is the LCD for $\frac{7}{27}$ and $\frac{11}{45}$?
   C Find $\frac{7}{27} + \frac{11}{45}$.
   D Find $\frac{7}{27} - \frac{11}{45}$.

4. In simple addition problems, practice finding the LCD mentally.

PROBLEM:	THINK:	WRITE:
$\frac{3}{4} + \frac{1}{6}$	LCD is 12	$\frac{3}{4} =$ $+\frac{1}{6} =$

Find the sum or difference. Give your answer in lowest terms.

A $\frac{2}{3} + \frac{3}{4}$	**B** $\frac{3}{4} + \frac{3}{8}$	**C** $\frac{11}{12} + \frac{1}{4}$	**D** $\frac{7}{8} - \frac{5}{6}$	**E** $\frac{11}{20} - \frac{3}{8}$
F $\frac{15}{16} - \frac{3}{4}$	**G** $\frac{5}{6} - \frac{1}{3}$	**H** $\frac{4}{5} + \frac{1}{6}$	**I** $\frac{3}{2} - \frac{3}{4}$	**J** $\frac{9}{10} - \frac{1}{3}$
K $\frac{77}{100} - \frac{7}{10}$	**L** $\frac{3}{5} + \frac{1}{2}$	★**M** $\frac{12}{20} + \frac{14}{16}$	★**N** $\frac{18}{27} - \frac{32}{64}$	★**O** $\frac{18}{28} + \frac{4}{56}$

5. For each problem, rename two of the three fractional numbers so that the fractions have the same denominators. Find the sum of the three numbers.

A $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$	**C** $\frac{1}{2}, \frac{3}{4}, \frac{5}{16}$	**E** $\frac{7}{2}, \frac{6}{5}, \frac{3}{10}$	**G** $\frac{3}{5}, \frac{1}{6}, \frac{7}{30}$	**I** $\frac{4}{7}, \frac{7}{10}, \frac{11}{70}$
B $\frac{1}{4}, \frac{3}{8}, \frac{5}{16}$	**D** $\frac{3}{2}, \frac{2}{3}, \frac{5}{6}$	**F** $\frac{1}{3}, \frac{4}{5}, \frac{7}{15}$	**H** $\frac{5}{8}, \frac{6}{7}, \frac{3}{56}$	**J** $\frac{5}{4}, \frac{11}{25}, \frac{71}{100}$

6. Find the sums.

A $\frac{1}{35} + \frac{2}{55}$	**D** $\frac{7}{24} + \frac{4}{9}$	**G** $\frac{1}{3} + \frac{1}{4} + \frac{1}{6}$	★**J** $\frac{7}{35} + \frac{3}{18} + \frac{7}{105}$
B $\frac{2}{15} + \frac{2}{21}$	**E** $\frac{3}{22} + \frac{5}{26}$	**H** $\frac{2}{5} + \frac{1}{6} + \frac{7}{30}$	★**K** $\frac{7}{9} + \frac{5}{24} + \frac{11}{36}$
C $\frac{5}{14} + \frac{3}{22}$	**F** $\frac{8}{105} + \frac{5}{63}$	★**I** $\frac{5}{30} + \frac{7}{42} + \frac{9}{54}$	★**L** $\frac{3}{8} + \frac{5}{12} + \frac{7}{10}$

More practice, page S-14, Set 23

Investigating the Ideas

A fraction whose numerator is greater than or equal to its denominator is called an **improper fraction.** Sometimes a **mixed numeral** is written for an improper fraction.

Improper fraction → $\frac{9}{5} = \frac{5}{5} + \frac{4}{5} = 1 + \frac{4}{5} = 1\frac{4}{5}$ ← **Mixed numeral**

Study the sequences of improper fractions and mixed numerals below.

improper fractions: $\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \ldots$

Mixed numerals: $1, 1\frac{1}{2}, 1\frac{2}{5}, 1\frac{5}{12}, 1\frac{12}{29}, \ldots$

Can you find the next three improper fractions and mixed numerals for each sequence?

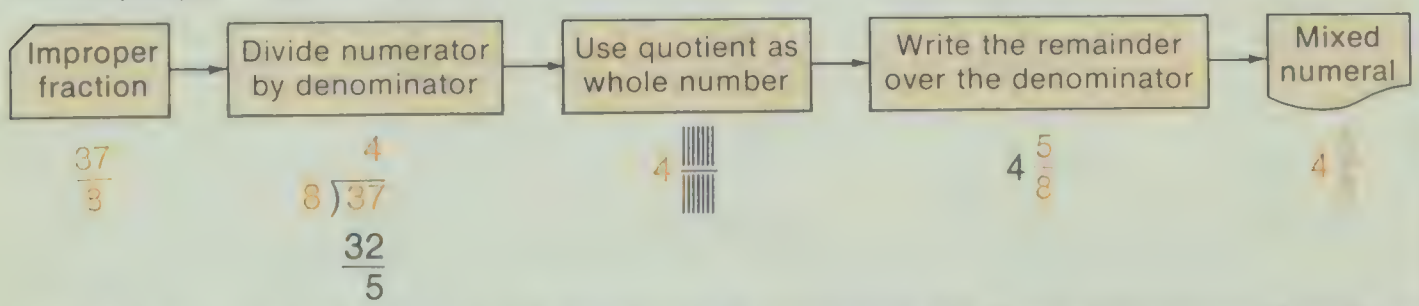
Discussing the Ideas

1. Explain the pattern for the sequences in the Investigation.

2. Give whole numbers for ***a*** and ***b*** and a mixed numeral for ***c***.

   A $\frac{17}{12} = \frac{12}{12} + \frac{a}{12} = 1 + \frac{b}{12} = c$ **B** $\frac{22}{3} = \frac{21}{3} + \frac{a}{3} = 7 + \frac{b}{3} = c$

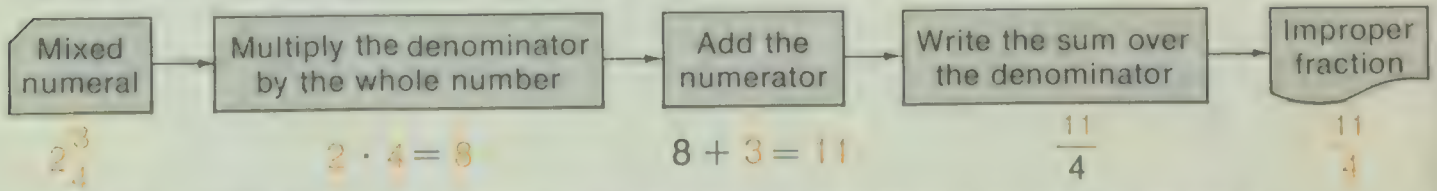
3. The flow chart shows a shortcut for finding mixed numerals for improper fractions.



   Use the flow chart to help you find mixed numerals for these improper fractions.

   A $\frac{12}{5}$ **B** $\frac{11}{2}$ **C** $\frac{45}{8}$ **D** $\frac{100}{3}$ **E** $\frac{38}{9}$ **F** $\frac{17}{17}$ **G** $\frac{63}{15}$

4. This flow chart shows a shortcut for finding an improper fraction for a mixed numeral.



   Use the flow chart to find improper fractions for these mixed numerals.

   A $1\frac{2}{3}$ **B** $3\frac{1}{5}$ **C** $3\frac{4}{5}$ **D** $7\frac{3}{8}$ **E** $9\frac{9}{10}$ **F** $5\frac{3}{7}$ **G** $10\frac{7}{8}$

1. Write a mixed numeral for each improper fraction.

A $\frac{7}{3}$	D $\frac{25}{4}$	G $\frac{39}{8}$	J $\frac{31}{16}$	M $\frac{49}{10}$	P $\frac{610}{80}$
B $\frac{13}{5}$	E $\frac{47}{12}$	H $\frac{66}{7}$	K $\frac{111}{20}$	N $\frac{223}{100}$	Q $\frac{75}{24}$
C $\frac{27}{10}$	F $\frac{27}{2}$	I $\frac{74}{9}$	L $\frac{51}{13}$	O $\frac{5287}{1000}$	R $\frac{90}{60}$

2. Write an improper fraction for each mixed numeral.

A $7\frac{1}{8}$	D $8\frac{2}{3}$	G $6\frac{3}{4}$	J $37\frac{1}{2}$	M $16\frac{2}{3}$	P $30\frac{1}{4}$
B $2\frac{5}{6}$	E $33\frac{1}{3}$	H $10\frac{9}{10}$	K $18\frac{3}{7}$	N $11\frac{3}{7}$	Q $62\frac{1}{2}$
C $3\frac{4}{5}$	F $10\frac{5}{6}$	I $9\frac{7}{8}$	L $3\frac{1}{3}$	O $19\frac{3}{4}$	R $83\frac{1}{3}$

3. Give the missing numerators of the fractions.

A $3 = \frac{\square}{1} = \frac{\square}{2} = \frac{\square}{3}$ C $10 = \frac{\square}{4} = \frac{\square}{8} = \frac{\square}{10}$ E $9 = \frac{\square}{7} = \frac{\square}{8} = \frac{\square}{9}$

B $7 = \frac{\square}{3} = \frac{\square}{5} = \frac{\square}{7}$ D $13 = \frac{\square}{1} = \frac{\square}{2} = \frac{\square}{3}$ F $0 = \frac{\square}{5} = \frac{\square}{11} = \frac{\square}{20}$

4. Give the numerators ***a*** and ***b***.

A $4\frac{1}{8} = 3 + \left(\frac{a}{8} + \frac{1}{8}\right) = 3\frac{b}{8}$ C $3\frac{2}{3} = 2 + \left(\frac{a}{3} + \frac{2}{3}\right) = 2\frac{b}{3}$

B $6\frac{1}{4} = 5 + \left(\frac{a}{4} + \frac{1}{4}\right) = 5\frac{b}{4}$ D $7\frac{4}{5} = 6 + \left(\frac{a}{5} + \frac{4}{5}\right) = 6\frac{b}{5}$

5. Solve the equations. Give your answers in lowest terms.

A $4 + n = 3\frac{5}{4}$ C $12\frac{12}{9} = 13 + n$ E $30\frac{7}{4} = 31 + n$

B $10 + n = 9\frac{11}{8}$ D $23 + n = 22\frac{18}{16}$ F $72\frac{19}{16} = 73 + n$

6. Write each mixed numeral in such a way that the fractional part is not an improper fraction and is also in lowest terms.

A $13\frac{10}{8}$	C $26\frac{9}{6}$	E $10\frac{18}{15}$	G $32\frac{15}{12}$
B $9\frac{10}{4}$	D $1\frac{4}{3}$	F $3\frac{24}{10}$	H $11\frac{28}{24}$

think

The number ***n*** in Problem A is the same as in Problem B. The quotient in Problem A is the same as in Problem B. Find ***n*** and the missing quotient.

A $15\overline{)n}$, R13

B $18\overline{)n}$, R1

More practice, page S-14, Set 24

Discussing the Ideas

1. If $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ represent any fractional numbers, the basic principles in the table below hold for addition of fractional numbers.

Commutative principle	Associative principle	Zero principle

   A Which principle states that the sum will be the same even though the **order** of the addends is changed?

   B Which principle enables you to change the **grouping** of the addends without changing the sum?

2. Since addition is both commutative and associative, we may omit parentheses and arrange the addends in any way that we choose.

$$\frac{1}{3}+\frac{3}{4}+\frac{5}{6}=\frac{5}{6}+\frac{1}{3}+\frac{3}{4}$$

   What are some other ways that you could arrange the addends $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$?

3. A Which principle enables you to find the sum of $\frac{23}{42}+0$ mentally?

   B What is the sum?

4. Explain how you can use the basic principles to help you find these sums.

   A $\frac{1}{8}+\left(\frac{7}{8}+\frac{1}{4}\right)$ B $\left(\frac{1}{3}+0\right)+\frac{1}{3}$ C $\left(2+\frac{1}{4}\right)+\left(6+\frac{3}{4}\right)$

5. Explain the steps used to find this sum.

$$\begin{aligned} 12\tfrac{1}{3} &= 12\tfrac{4}{12} \\ +19\tfrac{3}{4} &= 19\tfrac{9}{12} \\ \hline 31\tfrac{13}{12} &= 32\tfrac{1}{12} \end{aligned}$$

6. Explain the steps used to find this difference.

$$\begin{aligned} 23\tfrac{1}{2} &= 23\tfrac{3}{6} = 22\tfrac{9}{6} \\ -12\tfrac{2}{3} &= 12\tfrac{4}{6} = 12\tfrac{4}{6} \\ \hline & \qquad 10\tfrac{5}{6} \end{aligned}$$

1. Use the commutative, associative, and zero principles to find the sums.

A $\frac{1}{6}+\left(\frac{5}{6}+\frac{1}{4}\right)$ **C** $\frac{3}{10}+\left(\frac{7}{10}+\frac{1}{3}\right)$ **E** $\frac{5}{6}+\frac{9}{10}+\frac{1}{6}$ **G** $\frac{6}{13}+\frac{0}{11}+\frac{2}{13}$

B $\left(\frac{3}{8}+\frac{3}{4}\right)+\frac{1}{4}$ **D** $\left(\frac{1}{2}+0\right)+\frac{1}{2}$ **F** $\frac{7}{11}+\frac{5}{9}+\frac{4}{11}$ **H** $\frac{3}{7}+\frac{2}{7}+\frac{0}{3}$

2. Find the sums.

A $\frac{3}{4}+\frac{2}{3}$ **B** $\frac{9}{10}+\frac{3}{5}$ **C** $\frac{5}{12}+\frac{5}{8}$ **D** $\frac{1}{8}+\frac{8}{9}$ **E** $\frac{7}{15}+\frac{3}{5}$ **F** $\frac{8}{9}+\frac{5}{12}$

G $23\frac{9}{10}+32\frac{1}{5}$ **H** $11\frac{7}{10}+16\frac{3}{4}$ **I** $83\frac{1}{3}+16\frac{2}{3}$ **J** $52\frac{1}{2}+29\frac{7}{10}$ **K** $41\frac{1}{8}+33\frac{5}{6}$ **L** $19\frac{5}{9}+20\frac{5}{6}$

3. Find the differences:

A $7\frac{7}{8}-3\frac{3}{4}$ **B** $16\frac{1}{4}-12\frac{3}{8}$ **C** $35\frac{1}{6}-28\frac{1}{4}$ **D** $8\frac{1}{5}-7\frac{9}{10}$ **E** $24\frac{2}{3}-11\frac{1}{2}$ **F** $40-27\frac{5}{8}$

G $11\frac{1}{10}-5\frac{7}{10}$ **H** $14\frac{1}{2}-8\frac{2}{3}$ **I** $19\frac{3}{8}-13\frac{3}{4}$ **J** $66\frac{2}{3}-33\frac{3}{4}$ **K** $53\frac{3}{16}-42\frac{1}{2}$ **L** $31\frac{1}{8}-17\frac{1}{2}$

4. The price of a certain stock was listed at $68\frac{1}{4}$. Three days later, it was listed at $65\frac{1}{2}$. How much did the price drop?

5. Roberta lives $2\frac{1}{2}$ blocks from Joan. She ran $1\frac{3}{4}$ blocks and walked the rest of the way to Joan's house. How far did she walk?

think

Find the sums for Exercises 1 through 4. Use these results to find the sum for Exercise 5.

1. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}$

2. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}$

3. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}$

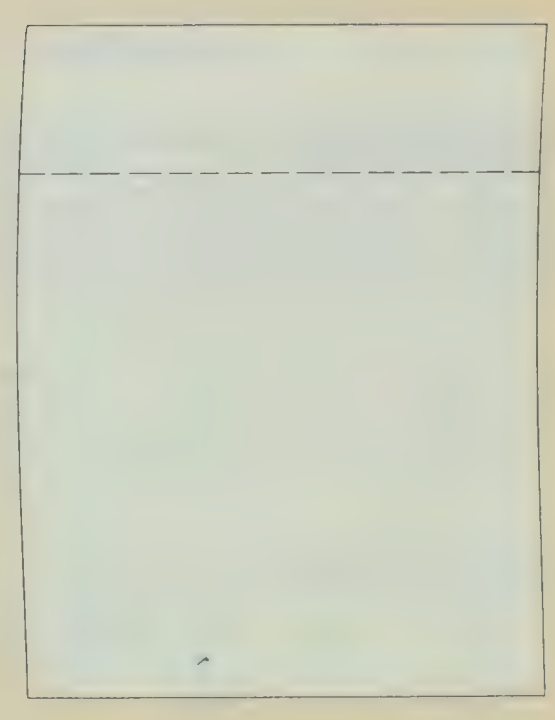
4. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}$

5. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{99\cdot 100}$

More practice, page S-15, Set 25

PAPER PROBLEMS

The problems below refer to the size of a sheet of paper that is ordinarily used for writing and typing.

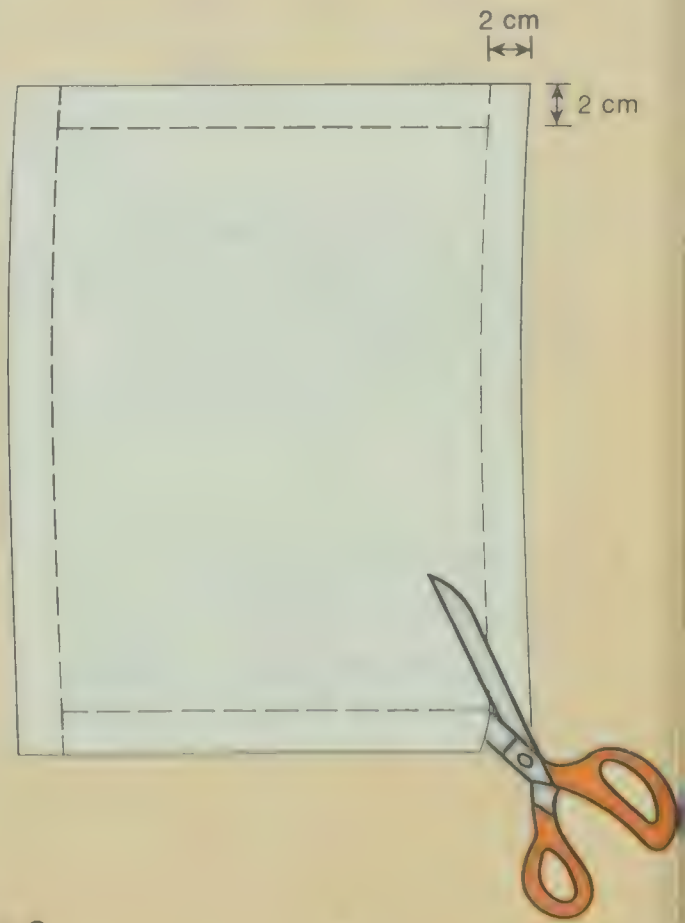


1. What is the perimeter (distance around) of a sheet of typing paper?

2. Measure one of the diagonals of the paper to the nearest $\frac{1}{10}$ centimetre. What is its length?

3. If you cut a square from the paper whose sides are the same length as the shorter side of the sheet of paper, how long and how wide is the rectangular piece of paper that is left?

4. Suppose that you cut a strip of paper two centimetres wide from each of the four sides of a piece of typing paper and then placed the four strips end-to-end lengthwise. What would be the total length of the four strips?



5. What would be the perimeter of the rectangle that is left after the 2-centimetre strips were cut from the sheet?

6. Repeat Exercise 4 using the rectangle that was left after the first set of 2-centimetre strips have been removed? What is the perimeter of the new smaller rectangle that is left after the second cutting?

7. Is the last rectangle more or less than half the size of the original sheet of paper?

1500-METRE OLYMPIC RECORDS

For years it seemed nearly impossible that man would be able to run the Olympic distance of 1500 metres in less than four minutes. But in 12 Olympic meets many runners have run the distance in less than four minutes. The table gives the men who have won this event over the past seventy six years.

NAME	YEAR	TIME	
		min	sec
Edwin Flack (Br.)	1896	4	$33\frac{1}{5}$
Charles Bennett (Br.)	1900	4	6
James Lightbody (U.S.)	1904	4	$5\frac{2}{5}$
James Lightbody (U.S.)	1906	4	12
Mel Sheppard (U.S.)	1908	4	$3\frac{2}{5}$
Arnold Jackson (Br.)	1912	3	$56\frac{8}{10}$
Albert Hill (Br.)	1920	4	$1\frac{4}{5}$
Paavo Nurmi (Fin.)	1924	3	$53\frac{6}{10}$
Harry Larva (Fin.)	1928	3	$53\frac{1}{5}$
Luigi Beccali (Italy)	1932	3	$51\frac{2}{10}$
Jack Lovelock (N.Z.)	1936	3	$47\frac{8}{10}$
Henri Eriksson (Swed.)	1948	3	$49\frac{8}{10}$
Joseph Barthel (Lux.)	1952	3	$45\frac{2}{10}$
Ron Delany (Ire.)	1956	3	$41\frac{2}{10}$
Herb Elliot (Australia)	1960	3	$35\frac{6}{10}$
Peter Snell (N.Z.)	1964	3	$38\frac{1}{10}$
Kipchoge Keino (Kenya)	1968	3	$34\frac{9}{10}$
Pekka Vasala (Fin.)	1972	3	$36\frac{3}{10}$

1. According to the table, who was the first to run the 1500 in less than 4 minutes? How much less than 4 minutes was his time?

2. How many seconds less did Ron Delany take to run the event than did Jack Lovelock?

3. Nurmi and Larva were two famous Finnish runners. How much difference was there in their Olympic times?

4. How many seconds difference is there in James Lightbody's record time and Pekka Vasala's time?

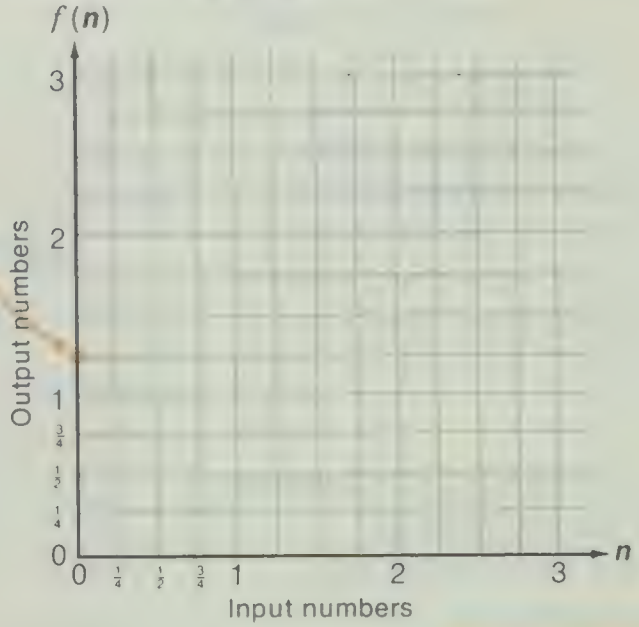
5. Which person in the table was the first to run the 1500 in almost a minute less than Edwin Flack?

6. In 1967, Jim Ryun set a record time for the 1500 metre race. His time for this race was 3 minutes 33 $\frac{1}{10}$ seconds. How much better is this time than the record Olympic time for the event?

Investigating the Ideas

Study the function machine rule to see how the number pairs for the function can be used as co-ordinates on the graph.

Function Rule $n + 1\frac{1}{4}$		
n	$f(n)$	Number pairs
0	$1\frac{1}{4}$	$(0, 1\frac{1}{4})$
$\frac{1}{4}$	$1\frac{1}{2}$	$(\frac{1}{4}, 1\frac{1}{2})$
$\frac{1}{2}$	$1\frac{3}{4}$	$(\frac{1}{2}, 1\frac{3}{4})$
$\frac{3}{4}$	2	$(\frac{3}{4}, 2)$
1	$2\frac{1}{4}$	$(1, 2\frac{1}{4})$
$1\frac{1}{4}$	$2\frac{1}{2}$	$(1\frac{1}{4}, 2\frac{1}{2})$
$1\frac{1}{2}$	$2\frac{3}{4}$	$(1\frac{1}{2}, 2\frac{3}{4})$



Can you mark the points on the grid for the co-ordinates?

Discussing the Ideas

1. Do the points for the co-ordinates for the function above follow a pattern? Describe the pattern.

2. **A** Complete the function table.
   B Use the number pairs of this function as co-ordinates and mark the points for them on a grid.
   C Do the points for this function follow a pattern?

Function Rule $3 - n$	
n	$f(n)$
0	3
$\frac{1}{4}$	$2\frac{3}{4}$
$\frac{1}{2}$	
$\frac{3}{4}$	
1	
$1\frac{1}{2}$	
2	

3. **A** Try the numbers $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$ as input numbers for the function rule in Exercise 2.
   B Estimate the position of the points for the co-ordinates in part **A** and mark them on your graph. Do these points fall in line with your other points?

In Exercises 1, 2, and 3, graph the number pairs for each function. Use a grid similar to the one in the Investigation.

1.

Function Rule: $n + \frac{1}{2}$	
n	$f(n)$
0	$\frac{1}{2}$
$\frac{1}{2}$	1
1	$1\frac{1}{2}$
$1\frac{1}{2}$	2
2	$2\frac{1}{2}$
$2\frac{1}{2}$	3

2.

Function Rule: $n - \frac{3}{4}$	
n	$f(n)$
2	$1\frac{3}{4}$
$1\frac{3}{4}$	$1\frac{1}{2}$
$1\frac{1}{2}$	$1\frac{1}{4}$
$1\frac{1}{4}$	1
1	$\frac{3}{4}$
$\frac{3}{4}$	$\frac{1}{2}$

3.

Function Rule: $2 + n$	
n	$f(n)$
0	2
$\frac{1}{2}$	$2\frac{1}{2}$
1	3
$1\frac{1}{2}$	$3\frac{1}{2}$
$1\frac{3}{4}$	$3\frac{3}{4}$
2	4

In Exercises 4, 5, and 6, complete each function table. Then graph the number pairs of the function.

4.

Function Rule: $n + \frac{3}{4}$		
	n	$f(n)$
A	$\frac{1}{4}$	
B	$\frac{1}{2}$	
C	$\frac{3}{4}$	
D	1	
E	$1\frac{1}{4}$	
F	$1\frac{1}{2}$	
G	$1\frac{3}{4}$	

5.

Function Rule: $n - 1\frac{1}{2}$		
	n	$f(n)$
A	3	
B	$2\frac{3}{4}$	
C	$2\frac{1}{2}$	
D	$2\frac{1}{4}$	
E	2	
F	$1\frac{3}{4}$	
G	$1\frac{1}{2}$	

6.

Function Rule: $n + n$		
	n	$f(n)$
A	0	
B	$\frac{1}{4}$	
C	$\frac{1}{2}$	
D	$\frac{3}{4}$	
E	1	
F	$1\frac{1}{4}$	
G	$1\frac{1}{2}$	

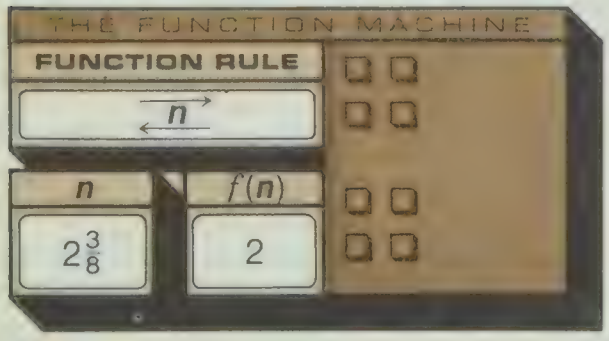
7. Invent a function rule. Make a table for your function and then graph the number pairs of the function.

think

A microbe colony with plenty of food doubles its population every three hours. At noon the colony had a billion microbes. When was the population 500 million?

Investigating the Ideas

The input and output numbers in tables A and B describe a special function rule. The symbol $\overrightarrow{n}$ is used for this function rule.



A

$\overrightarrow{n}$	
n	$f(n)$
2	2
$2\frac{3}{8}$	2
$2\frac{1}{2}$	3
$2\frac{3}{4}$	3
$3\frac{1}{8}$	3
$3\frac{5}{8}$	4

B

$\overrightarrow{n}$	
n	$f(n)$
$\frac{5}{6}$	
$\frac{7}{4}$	
$\frac{1}{8}$	
$\frac{15}{4}$	
$8\frac{7}{12}$	
$16\frac{1}{2}$	

Can you discover the rule for this special function?

Complete function table B to show that you know the rule.

Discussing the Ideas

1. Do you think that the output numbers for the function rule $\overrightarrow{n}$ will always be a whole number?

2. Mike said, "The function rule rounds each input number to the nearest whole number." Do you agree with him?

3. What does the function rule do about numbers halfway between two whole numbers such as $2\frac{1}{2}$ and $3\frac{1}{2}$?

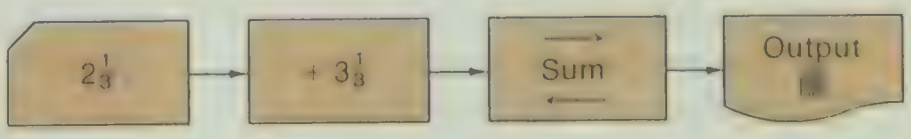
4. Give the missing whole numbers.

A $\overrightarrow{\frac{5}{2}} = ?$ B $\overrightarrow{6\frac{1}{2}} = ?$ C $\overrightarrow{\frac{14}{4}} = ?$ D $\overrightarrow{\frac{33}{2}} = ?$ E $\overrightarrow{\frac{1}{2}} = ?$

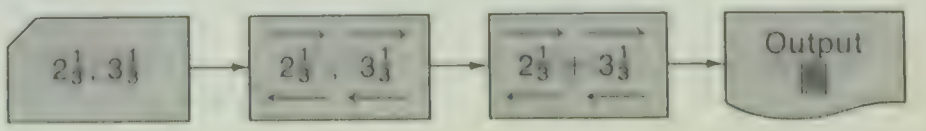
5. A Find the output numbers for flow charts A and B.

   B Explain why the output numbers for the two flow charts are different.

A



B



For Exercises 1, 2, and 3, give the output number $f(n)$ for each input number n.

1.

Function Rule: $\overrightarrow{n}$

	n	$f(n)$
A	1	
B	$\frac{4}{3}$	
C	$\frac{5}{3}$	
D	2	
E	$\frac{7}{3}$	
F	$\frac{11}{3}$	

2.

Function Rule: $\overrightarrow{n}$

	n	$f(n)$
A	$\frac{9}{10}$	
B	$\frac{17}{16}$	
C	$\frac{20}{6}$	
D	$\frac{44}{9}$	
E	$\frac{100}{3}$	
F	$\frac{1}{6}$	

3.

Function Rule: $\overrightarrow{n}$

	n	$f(n)$
A	$18\frac{5}{8}$	
B	$9\frac{4}{11}$	
C	$\frac{127}{241}$	
D	$\frac{289}{144}$	
E	$1099\frac{9}{10}$	
F	$\frac{11}{24}$	

In Exercises 4 through 7, solve the equations.

4. $\overleftarrow{1\frac{3}{8}} + \overleftarrow{2\frac{1}{4}} = n$

5. $\overleftarrow{1\frac{3}{8} + 2\frac{1}{4}} = n$

6. $\overleftarrow{8\frac{3}{4}} - \overleftarrow{3\frac{3}{8}} = n$

7. $\overleftarrow{8\frac{3}{4} - 3\frac{3}{8}} = n$

8. Tell which symbol ($=$ or $<$) is covered by each ▒.

A 3 ▒ $\overleftarrow{3\frac{1}{4}}$ ▒ 4

B 4 ▒ $\overleftarrow{\frac{23}{5}}$ ▒ 5

C 6 ▒ $\overleftarrow{\frac{41}{6}}$ ▒ 7

9. You can find many numbers for N in the equation.

$$\overleftarrow{2\tfrac{2}{3}} \cdot \overleftarrow{N} = 6$$

A What is the smallest number for N?

B The number for N must be less than ▒.

C Is there a largest number for N?

10. Draw a graph of the function $\overleftarrow{n}$. Use fractional numbers from 0 through 5 as input numbers.

think

Discover the rule for the function shown in the table.

Function Rule: ▒

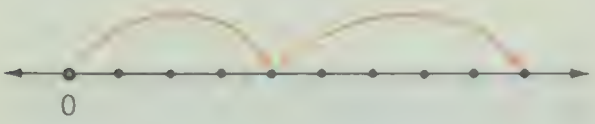
n	$f(n)$
$\frac{3}{4}$	1
$2\frac{1}{7}$	10
$2\frac{6}{7}$	11
$5\frac{7}{8}$	110
$\frac{46}{3}$	1111
$\frac{31}{2}$	10000
$13\frac{2}{5}$	1101

REVIEWING THE IDEAS

1. Tell what addition fact is shown by the number line, given that it is scaled in

   A units **C** ninths

   B halves **D** twenty-fourths



2. Give the number for n.

   A $\frac{3}{5} + n = 1$ **E** $4\frac{1}{3} - n = 3\frac{2}{3}$

   B $\frac{5}{7} + n = 1\frac{3}{7}$ **F** $n - \frac{3}{4} = \frac{5}{4}$

   C $1\frac{1}{3} + n = 2$ **G** $n - \frac{11}{7} = \frac{12}{7}$

   D $\frac{7}{9} - n = \frac{4}{9}$ **H** $\frac{4}{3} \cdot n = \frac{28}{3}$

3. **A** What is the least common multiple (LCM) of 4 and 6?

   B What is the least common denominator (LCD) for $\frac{3}{4}$ and $\frac{5}{6}$?

4. Find the sums. Give your answers in lowest-terms fractions.

   A $\frac{1}{6} + \frac{1}{10}$ **C** $\frac{2}{15} + \frac{2}{21}$

   B $\frac{9}{16} + \frac{1}{4}$ **D** $\frac{3}{8} + \frac{1}{6}$

5. Give an improper fraction for each mixed numeral.

   A $10\frac{1}{3}$ **C** $2\frac{7}{8}$ **E** $6\frac{3}{4}$

   B $7\frac{3}{10}$ **D** $3\frac{2}{3}$ **F** $5\frac{5}{6}$

6. Give a mixed numeral for each improper fraction.

   A $\frac{12}{5}$ **C** $\frac{33}{7}$ **E** $\frac{127}{14}$

   B $\frac{17}{3}$ **D** $\frac{37}{10}$ **F** $\frac{441}{100}$

7. Use the associative and commutative principles to obtain the sums easily.

   A $2\frac{1}{2} + \frac{3}{5} + 1\frac{1}{2}$ **C** $\frac{7}{13} + \frac{5}{11} + \frac{6}{13}$

   B $\frac{7}{9} + 3\frac{2}{5} + \frac{2}{9}$ **D** $2\frac{3}{5} + 3\frac{4}{7} + 5\frac{2}{5}$

8. Find the sums.

   A $7\frac{1}{6} + 4\frac{2}{3}$ **B** $28\frac{3}{4} + 29\frac{7}{8}$

9. Find the differences.

   A $16\frac{2}{3} - 9\frac{1}{4}$ **B** $42\frac{1}{4} - 27\frac{3}{8}$

10. Complete the function rule table.

	Function Rule $n + \frac{3}{4}$	
	n	$n + \frac{3}{4}$
	$\frac{1}{4}$	1
A	$\frac{1}{2}$	
B	$\frac{3}{4}$	
C	1	
D	$\frac{2}{3}$	

11. If $\overrightarrow{\underleftarrow{n}}$ means "round n to the nearest whole number," what are each of the following?

    A $\overrightarrow{\underleftarrow{1\frac{2}{3}}}$ **B** $\overrightarrow{\underleftarrow{7\frac{2}{5}}}$ **C** $\overrightarrow{\underleftarrow{16\frac{1}{2}}}$

12. A strip of paper is $21\frac{1}{10}$ cm long. If you cut off a piece $9\frac{7}{10}$ cm long how long is the remaining piece?

TEST YOURSELF

1. Give the sum as a lowest-terms fraction.
   $\frac{3}{8} + \frac{7}{8}$

2. Give the difference as a lowest-terms fraction.
   $\frac{13}{4} - \frac{7}{4}$

3. What is the least common denominator (LCD) for $\frac{1}{4}$ and $\frac{5}{6}$?

4. Find the sum. $\frac{1}{4} + \frac{5}{6}$

5. Find the difference. $\frac{4}{3} - \frac{7}{8}$

6. Give a mixed numeral for each improper fraction.
   A $\frac{15}{4}$ B $\frac{25}{10}$

7. Give an improper fraction for each mixed numeral.
   A $3\frac{1}{5}$ B $8\frac{5}{6}$

8. Find the sums.
   A $12\frac{1}{3} + 29\frac{3}{4}$ B $53\frac{7}{8} + 19\frac{3}{4}$

9. Find the differences.
   A $31\frac{1}{2} - 12\frac{1}{5}$ B $100 - 76\frac{9}{10}$

10. Complete the function table.

	Function Rule $n + \frac{2}{3}$	
	$\frac{1}{3}$	1
A	$\frac{1}{6}$	
B	$\frac{1}{2}$	

11. Round each fractional number to the nearest whole number.
    A $1\frac{7}{8}$ B $5\frac{3}{16}$ C $3\frac{1}{2}$

RESEARCH PROJECTS

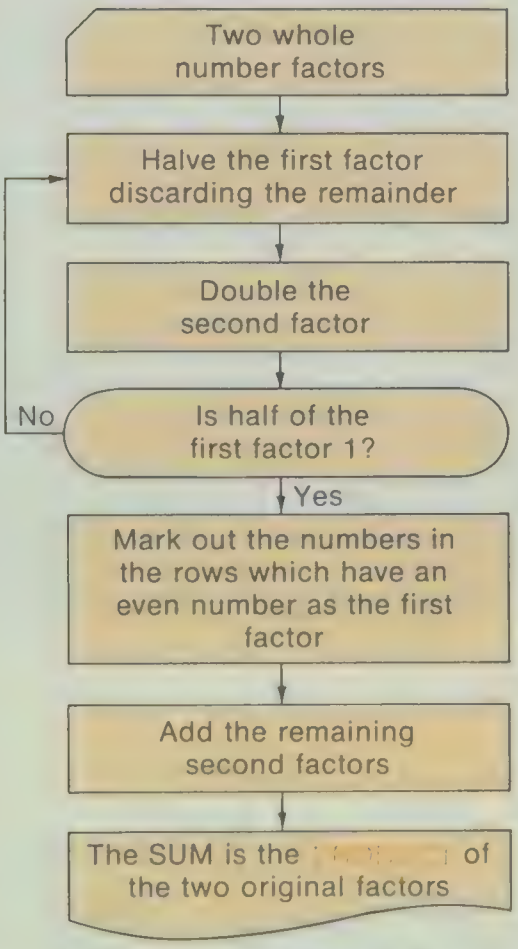
A Find the average yearly rainfall for your area. Use this information to compute the number of kilolitres of water that will fall on each square kilometre. Find the mass of the water that falls on each square kilometre.

B Do you know the speed of light? Can you find how long it takes light from the sun to reach the earth? Look up the speed of light and the distance from the earth to the sun and find the amount of time.

C Do you think there are more fractional numbers than there are whole numbers? You know that between any two whole numbers there are infinitely many fractional numbers. Surprisingly, it can be shown that there are just as many whole numbers as there are fractional numbers. Find out how this can be shown. (See *The Number Story* by Herta T. Freitag and Arthur H. Freitag; Washington, D.C.: National Council of Teachers of Mathematics, 1960.)

PEASANT MULTIPLICATION

An old method of multiplying whole numbers is shown below. It is sometimes called "peasant multiplication." Study the flow chart which explains the steps and the example that accompanies it to show how to find the product 35 · 27.


Two whole number factors
Halve the first factor discarding the remainder
Double the second factor
No
Is half of the first factor 1?
Yes
Mark out the numbers in the rows which have an even number as the first factor
Add the remaining second factors
The SUM is the of the two original factors

First factor	Second factor
35	27
17	54
8	108
4	216
2	432
1	864

Total 945 = 35 · 27

Can you use peasant multiplication to find these products?

1. 25 · 51 **2.** 19 · 43 **3.** 39 · 59 **4.** 24 · 101

MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

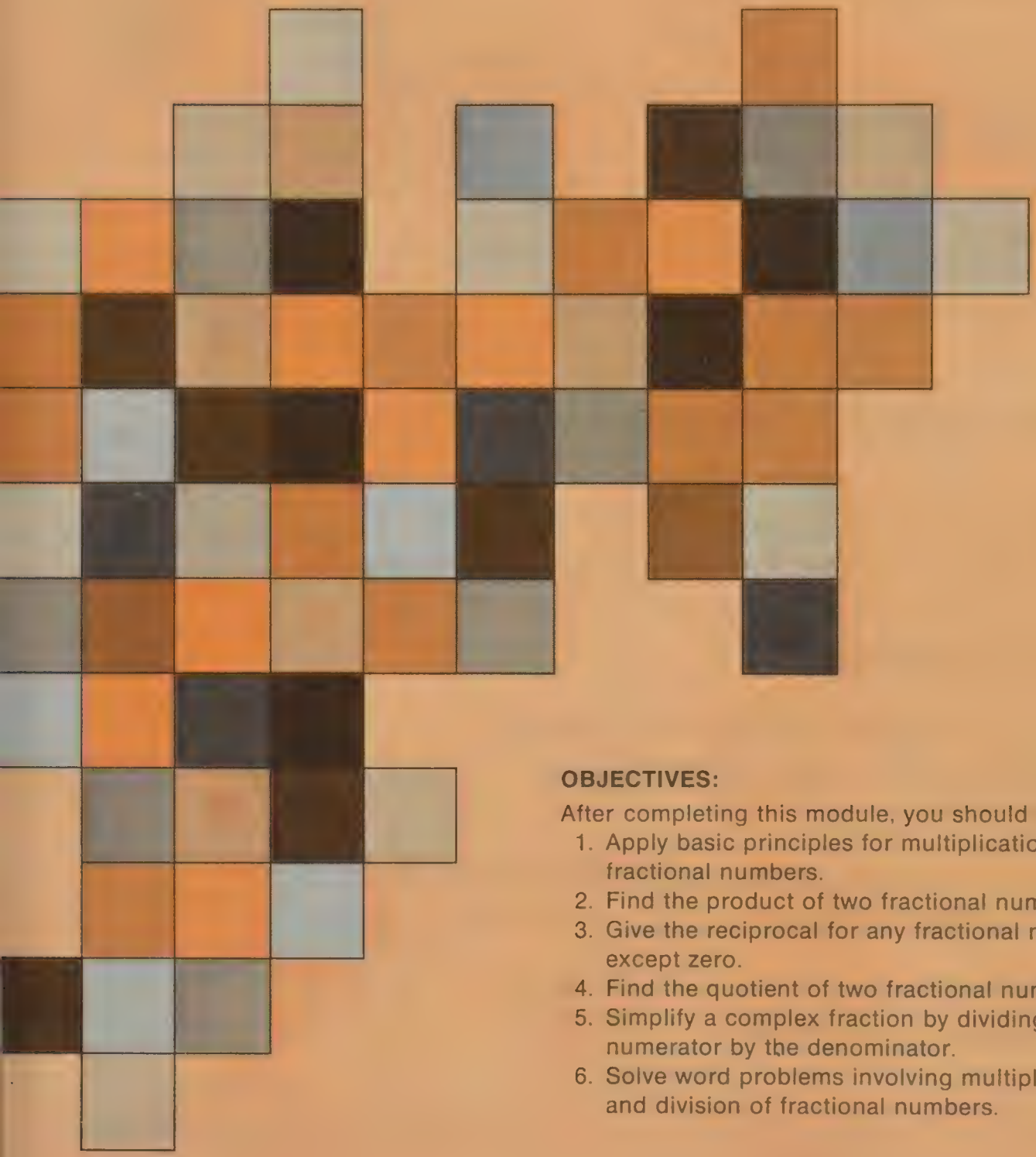
1. $\frac{5}{4}$ or $1\frac{1}{4}$ **2.** $\frac{3}{2}$ or $1\frac{1}{2}$ **3.** 12 **4.** $\frac{13}{12}$ or $1\frac{1}{12}$ **5.** $\frac{11}{24}$

6. A $3\frac{3}{4}$; B $2\frac{1}{2}$; **7.** A $\frac{16}{5}$; B $\frac{53}{6}$ **8.** A $42\frac{1}{12}$; B $73\frac{5}{8}$

9. A $19\frac{3}{10}$; B $23\frac{1}{10}$ **10.** A $\frac{5}{6}$; B $1\frac{1}{6}$ **11.** A 2; B 5; C 4

UNIT B: *The Arithmetic of Fractional Numbers*

MODULE 3: *Multiplying and Dividing Fractional Numbers*



OBJECTIVES:

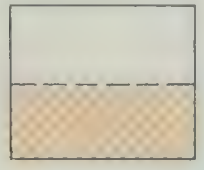
After completing this module, you should be able to:

1. Apply basic principles for multiplication of fractional numbers.
2. Find the product of two fractional numbers.
3. Give the reciprocal for any fractional number except zero.
4. Find the quotient of two fractional numbers.
5. Simplify a complex fraction by dividing the numerator by the denominator.
6. Solve word problems involving multiplication and division of fractional numbers.

Investigating the Ideas

Study examples **A** and **B** below.

A

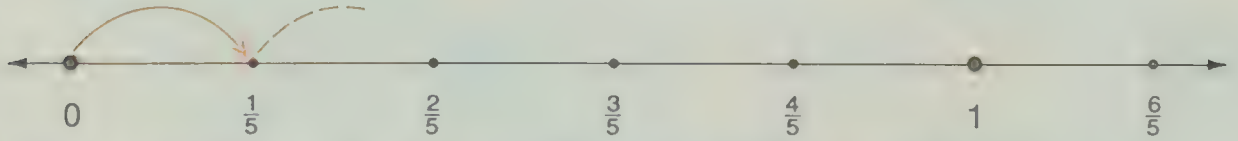


Shade $\frac{1}{2}$ of a region.

Shade $\frac{1}{4}$ of $\frac{1}{2}$ of a region.

What part of the region has been shaded twice?

B



Make 3 jumps of $\frac{1}{5}$ on the number line. Where do you end?

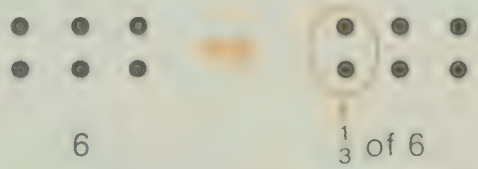
Can you find the answers to questions for A and B?	Make up two more examples using different fractions.

Discussing the Ideas

1. Example A suggests a multiplication equation.
   Since $\frac{1}{4}$ **of** $\frac{1}{2}$ **of the region is** $\frac{1}{8}$ **of a region**, you can write $\frac{1}{4} \cdot \frac{1}{2} = \frac{\text{IIII}}{\text{IIII}}$.

2. Example B suggests another multiplication equation.
   Since 3 jumps of $\frac{1}{5}$ takes you to $\frac{3}{5}$, you can write $3 \cdot \frac{1}{5} = \frac{\text{III}}{\text{IIII}}$.

3. **A** What is $\frac{1}{3}$ of 6?
   B Complete: $\frac{1}{3} \cdot 6 = \frac{6}{3} = \text{IIIIII}$.



4. Fractions such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ are often called **unit fractions** because their numerators are 1. The Investigation and the Discussion above suggest the statements shown at the right. Give a few more examples of multiplication using unit fractions.

If ***a*** and ***b*** are any whole numbers, except zero,

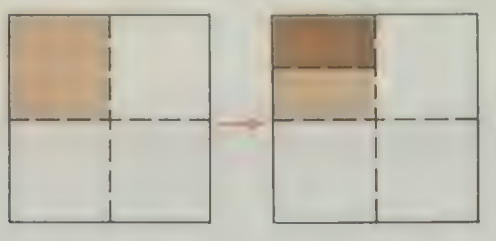
$\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{a \cdot b}$

If ***n*** is any whole number,

$\frac{1}{n} \cdot n = 1$

1. Copy and complete the multiplication equation suggested by each pair of figures.

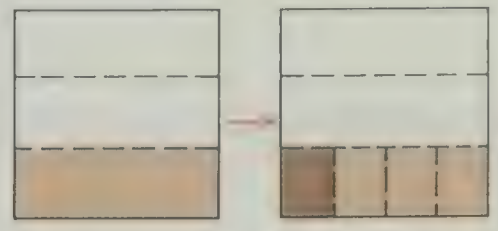
A



$\frac{1}{2} \cdot \frac{1}{4} = \frac{\square}{\square}$

$\frac{1}{4}$ of a region $\frac{1}{2}$ of $\frac{1}{4}$

B

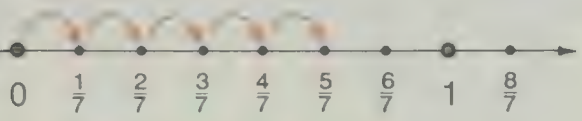


$\frac{1}{4} \cdot \frac{1}{3} = \frac{\square}{\square}$

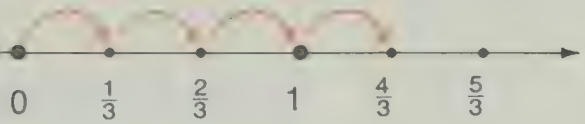
$\frac{1}{3}$ of a region $\frac{1}{4}$ of $\frac{1}{3}$

2. Give the multiplication fact suggested by each number line.

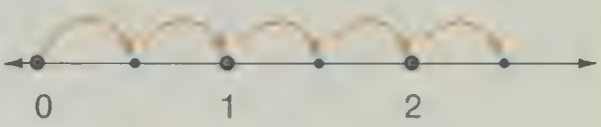
A



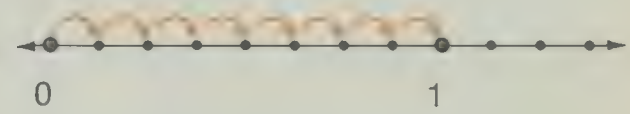
B



C



D



3. Solve the equations.

A $\frac{1}{2} \cdot \frac{1}{3} = n$ **C** $\frac{1}{5} \cdot \frac{1}{5} = n$ **E** $\frac{1}{3} \cdot \frac{1}{6} = n$ **G** $\frac{1}{10} \cdot \frac{1}{10} = n$ **I** $\frac{1}{8} \cdot \frac{1}{4} = n$

B $\frac{1}{4} \cdot \frac{1}{5} = n$ **D** $\frac{1}{2} \cdot \frac{1}{4} = n$ **F** $\frac{1}{5} \cdot \frac{1}{4} = n$ **H** $\frac{1}{10} \cdot \frac{1}{100} = n$ **J** $\frac{1}{100} \cdot \frac{1}{100} = n$

4. Find the products.

A $3 \cdot \frac{1}{4}$ **C** $10 \cdot \frac{1}{10}$ **E** $10 \cdot \frac{1}{100}$ **G** $0 \cdot \frac{1}{3}$ **I** $30 \cdot \frac{1}{5}$

B $5 \cdot \frac{1}{8}$ **D** $4 \cdot \frac{1}{2}$ **F** $7 \cdot \frac{1}{1000}$ **H** $1 \cdot \frac{1}{8}$ **J** $200 \cdot \frac{1}{4}$

5. Write a multiplication equation for each part and then solve it.

EXAMPLE: $\frac{1}{4}$ of 24 ANSWER: $\frac{1}{4} \cdot 24 = \frac{24}{4} = 6$

A $\frac{1}{3}$ of 30 **C** $\frac{1}{2}$ of 14 **E** $\frac{1}{8}$ of 32 **G** $\frac{1}{9}$ of 54 **I** $\frac{1}{10}$ of 20

B $\frac{1}{10}$ of 100 **D** $\frac{1}{6}$ of 54 **F** $\frac{1}{5}$ of 5 **H** $\frac{1}{4}$ of 20 **J** $\frac{1}{25}$ of 100

6. In a carton of eggs, $\frac{1}{4}$ of a dozen remain. If you use $\frac{1}{3}$ of the $\frac{1}{4}$ dozen of eggs, what part of a dozen did you use?

think

A bug tries to climb an 8-metre pole. Each day he climbs up 2 metres. Each night while he sleeps, he slips down 1 metre. When will he reach the top of the pole?

More practice, page S-15, Set 26

Discussing the Ideas

In the table of basic principles $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ represent any fractional number and n represents a whole number.

Commutative principle	Associative principle	Unit fraction principles
$\frac{a}{b} \cdot \frac{c}{d} = \frac{c}{d} \cdot \frac{a}{b}$	$\left(\frac{a}{b} \cdot \frac{c}{d}\right) \cdot \frac{e}{f} = \frac{a}{b} \cdot \left(\frac{c}{d} \cdot \frac{e}{f}\right)$	$\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{a \cdot b}$
One principle	**Distributive principle**	
$\frac{a}{b} \cdot 1 = \frac{a}{b}$	$\frac{a}{b} \cdot \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} \cdot \frac{c}{d}\right) + \left(\frac{a}{b} \cdot \frac{e}{f}\right)$	$\frac{1}{a} \cdot n = n \cdot \frac{1}{a} = \frac{n}{a}$

1. Find the number for **x**. Then find the number for **y**.

   A $\frac{1}{2} \cdot \frac{1}{5} = x$ $\quad \frac{1}{5} \cdot \frac{1}{2} = y$

   B $\frac{1}{3} \cdot \frac{1}{7} = x$ $\quad \frac{1}{7} \cdot \frac{1}{3} = y$

   C $\frac{1}{8} \cdot \frac{1}{9} = x$ $\quad \frac{1}{9} \cdot \frac{1}{8} = y$

   D $\frac{1}{2} \cdot \left(\frac{1}{3} \cdot \frac{1}{4}\right) = x$ $\quad \left(\frac{1}{2} \cdot \frac{1}{3}\right) \cdot \frac{1}{4} = y$

   E $\frac{1}{5} \cdot \left(\frac{1}{2} \cdot \frac{1}{3}\right) = x$ $\quad \left(\frac{1}{5} \cdot \frac{1}{2}\right) \cdot \frac{1}{3} = y$

   F $\frac{1}{2} \cdot \left(\frac{1}{4} \cdot \frac{1}{5}\right)$ x $\left(\frac{1}{2} \cdot \frac{1}{4}\right) \cdot \frac{1}{5}$ y

2. Give the principle that justifies each step of the multiplication problem below.

   A $\left(4 \cdot \frac{1}{3}\right) \cdot \frac{1}{5} = 4 \cdot \left(\frac{1}{3} \cdot \frac{1}{5}\right)$

   B $4 \cdot \left(\frac{1}{3} \cdot \frac{1}{5}\right) = 4 \cdot \left(\frac{1}{15}\right)$

   C $4 \cdot \left(\frac{1}{15}\right) = \frac{4}{15}$

3. Which two principles used together allow you to change the order and grouping of the factors as shown below?

   Instead of: $\left(\frac{2}{3} \cdot \frac{7}{8}\right) \cdot \frac{3}{2}$ $\longrightarrow$ You can write: $\frac{2}{3} \cdot \left(\frac{3}{2} \cdot \frac{7}{8}\right)$

4. **A** If $a \neq 0$, what whole number is $\frac{a}{a}$? **B** If $a \neq 0$, what whole number is $\frac{0}{a}$?

5. Find the products.

   A $\frac{5}{7} \cdot \frac{9}{9}$ **B** $\frac{13}{25} \cdot \frac{47}{47}$ **C** $\frac{73}{42} \cdot \frac{0}{61}$ **D** $\left(\frac{7}{11} \cdot \frac{5}{9}\right) \cdot \frac{0}{23}$ **E** $\frac{31}{31} \cdot \frac{0}{25}$ **F** $\frac{67}{67} \cdot \frac{99}{99}$

6. Complete the equations. Use the distributive principle.

   A $\left(\frac{5}{6} \cdot \frac{1}{5}\right) + \left(\frac{5}{6} \cdot \frac{4}{5}\right) = \frac{5}{6} \cdot \left(\frac{1}{5} + x\right)$ **B** $\frac{3}{4} \cdot \left(\frac{1}{3} + \frac{1}{2}\right) = \left(\frac{3}{4} \cdot \frac{1}{3}\right) + x$

1. Use the commutative principle to find n.
   - **A** Since $\frac{3}{5} \cdot \frac{2}{3} = \frac{2}{5}$, then $\frac{2}{3} \cdot \frac{3}{5} = n$.
   - **B** Since $\frac{1}{2} \cdot \frac{4}{7} = \frac{2}{7}$, then $\frac{4}{7} \cdot \frac{1}{2} = n$.
   - **C** Since $\frac{3}{4} \cdot \frac{6}{5} = \frac{9}{10}$, then $\frac{6}{5} \cdot \frac{3}{4} = n$.
   - **D** Since $\frac{2}{5} \cdot \frac{7}{10} = \frac{7}{25}$, then $\frac{7}{10} \cdot \frac{2}{5} = n$.
   - **E** Since $\frac{8}{3} \cdot \frac{5}{8} = \frac{5}{3}$, then $\frac{5}{8} \cdot \frac{8}{3} = n$.
   - **F** Since $5 \cdot \frac{2}{3} = \frac{10}{3}$, then $\frac{2}{3} \cdot 5 = n$.

2. Solve the equations.
   - **A** $\frac{1}{4} \cdot 1 = n$
   - **B** $1 \cdot \frac{1}{8} = n$
   - **C** $n \cdot 1 = \frac{5}{6}$
   - **D** $\frac{3}{5} \cdot \frac{3}{3} = n$
   - **E** $\frac{5}{2} \cdot \frac{4}{4} = n$
   - **F** $\frac{9}{10} \cdot \frac{10}{10} = n$
   - **G** $\frac{1}{9} \cdot n = \frac{1}{9}$
   - **H** $\frac{0}{3} \cdot \frac{1}{4} = n$
   - **I** $\frac{5}{12} \cdot n = 0$
   - **J** $n \cdot \frac{5}{5} = \frac{7}{8}$

3. Solve the equations.
   - **A** $3 \cdot \left(\frac{1}{2} \cdot \frac{1}{4}\right) = n$
   - **B** $\left(3 \cdot \frac{1}{2}\right) \cdot \frac{1}{4} = n$
   - **C** $\left(3 \cdot \frac{1}{4}\right) \cdot \frac{1}{2} = n$
   - **D** $5 \cdot \left(\frac{1}{3} \cdot \frac{1}{4}\right) = n$
   - **E** $\left(5 \cdot \frac{1}{4}\right) \cdot \frac{1}{3} = n$
   - **F** $\left(5 \cdot \frac{1}{3}\right) \cdot \frac{1}{4} = n$
   - **G** $7 \cdot \left(\frac{1}{2} \cdot \frac{1}{5}\right) = n$
   - **H** $\left(7 \cdot \frac{1}{5}\right) \cdot \frac{1}{2} = n$

4. Solve the equations.
   - **A** $\left(6 \cdot \frac{1}{4}\right) \cdot \frac{1}{3} = 6 \cdot \left(\frac{1}{4} \cdot n\right)$
   - **B** $\left(7 \cdot \frac{1}{5}\right) \cdot n = 7 \cdot \left(\frac{1}{5} \cdot \frac{1}{8}\right)$
   - **C** $\left(n \cdot \frac{1}{2}\right) \cdot \frac{1}{5} = 7 \cdot \left(\frac{1}{2} \cdot \frac{1}{5}\right)$

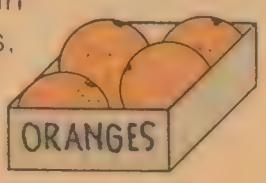
5. Find each product by first finding the product of the unit fractions.
   - **A** $3 \cdot \frac{1}{2} \cdot \frac{1}{3}$
   - **B** $7 \cdot \frac{1}{5} \cdot \frac{1}{6}$
   - **C** $\frac{1}{4} \cdot 5 \cdot \frac{1}{2}$
   - **D** $\frac{1}{10} \cdot \frac{1}{10} \cdot 100$
   - **E** $7 \cdot \frac{1}{3} \cdot \frac{1}{5}$
   - **F** $3 \cdot \frac{1}{10} \cdot \frac{1}{10}$
   - **G** $23 \cdot \frac{1}{10} \cdot \frac{1}{100}$
   - **H** $\frac{1}{4} \cdot 5 \cdot \frac{1}{4}$
   - **I** $\frac{1}{3} \cdot 4 \cdot \frac{1}{7}$
   - **J** $\frac{1}{8} \cdot 24 \cdot \frac{1}{3}$

6. Find the answers in the easiest way possible.
   - **A** $\frac{3}{5} \cdot \left(\frac{3}{4} + \frac{1}{4}\right)$
   - **B** $\frac{6}{7} \cdot \left(\frac{5}{8} + \frac{3}{8}\right)$
   - **C** $\left(\frac{15}{19} + \frac{4}{19}\right) \cdot \frac{17}{23}$
   - **D** $\left(\frac{23}{39} + \frac{16}{39}\right) \cdot \frac{37}{45}$
   - **E** $\left(\frac{5}{7} + \frac{2}{7}\right) \cdot \frac{2}{5}$
   - **F** $\frac{8}{9} \cdot \left(\frac{2}{3} + \frac{1}{3}\right)$
   - **G** $\left(\frac{3}{10} + \frac{7}{10}\right) \cdot \frac{10}{11}$
   - **H** $\left(\frac{4}{15} + \frac{11}{15}\right) \cdot \frac{18}{18}$

7. Solve the equations.
   - **A** $4 \cdot \frac{1}{8} \cdot 2 \cdot \frac{1}{3} = n \cdot \left(\frac{1}{8} \cdot \frac{1}{3}\right)$
   - **B** $3 \cdot \frac{1}{5} \cdot 2 \cdot \frac{1}{7} = n \cdot \left(\frac{1}{5} \cdot \frac{1}{7}\right)$
   - **C** $10 \cdot \frac{1}{3} \cdot 10 \cdot \frac{1}{3} = n \cdot \frac{1}{9}$
   - **D** $6 \cdot \frac{1}{5} \cdot 3 \cdot \frac{1}{4} = (6 \cdot 3) \cdot n$
   - **E** $5 \cdot \frac{1}{2} \cdot 7 \cdot \frac{1}{3} = 35 \cdot n$
   - **F** $11 \cdot \frac{1}{4} \cdot 5 \cdot \frac{1}{2} = n \cdot \frac{1}{8}$
   - **G** $12 \cdot \frac{1}{5} \cdot 3 \cdot \frac{1}{4} = 36 \cdot n$

think

If the number of oranges in a bin in a fruit market are counted by twos, threes, fives, and sevens, there is one orange left over each time. What is the smallest number of oranges that could be in the bin?



Investigating the Ideas

Factors

Products

How many of the products can you find using any of the four factors?	Write an equation for each product that you find.

Discussing the Ideas

1. Which two factors above can you multiply to give the product $\frac{5}{2}$?

2. Which two factors above can you multiply to give the product $\frac{7}{3}$?

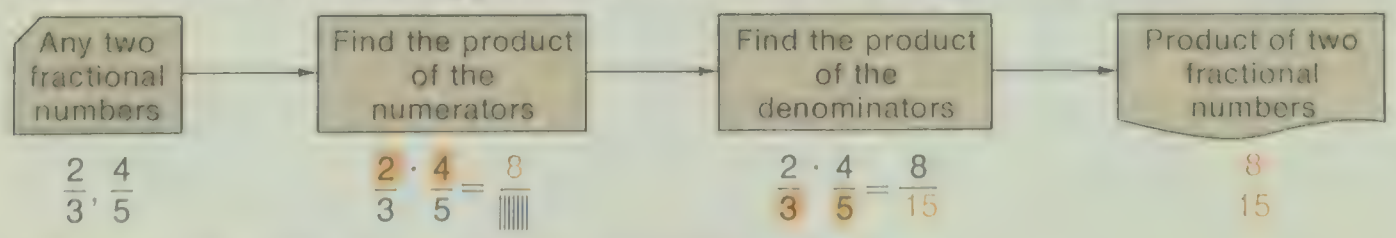
3. Which three factors above can you multiply to give the product $\frac{5}{6}$?

4. By using the basic principles, you can find products such as $\frac{5}{2} \cdot \frac{7}{3}$.

   Can you explain what was done in each step?

$$\frac{5}{2} \cdot \frac{7}{3} = \left(5 \cdot \frac{1}{2}\right) \cdot \left(7 \cdot \frac{1}{3}\right)$$
$$= \left(5 \cdot 7\right) \cdot \left(\frac{1}{2} \cdot \frac{1}{3}\right)$$
$$= 35 \cdot \frac{1}{6}$$
$$= \frac{35}{6}$$

5. The flow chart below suggests a shortcut for finding products.


Any two fractional numbers
Find the product of the numerators
Find the product of the denominators
Product of two fractional numbers
2/3, 4/5
2/3 · 4/5 = 8/
2/3 · 4/5 = 8/15
8/15

Use the flow chart to find these products.

A $\frac{1}{2} \cdot \frac{3}{5}$ **B** $\frac{4}{5} \cdot \frac{2}{7}$ **C** $\frac{1}{8} \cdot \frac{3}{2}$ **D** $\frac{5}{8} \cdot \frac{3}{4}$ **E** $\frac{7}{8} \cdot \frac{10}{11}$ **F** $\frac{5}{6} \cdot \frac{3}{10}$

1. Find the products.

A $\frac{3}{4} \cdot \frac{3}{5}$	**E** $\frac{7}{2} \cdot \frac{5}{3}$	**I** $\frac{53}{17} \cdot \frac{21}{34}$	**M** $\frac{43}{16} \cdot \frac{7}{16}$	**Q** $\frac{17}{10} \cdot \frac{7}{10}$
B $\frac{2}{3} \cdot \frac{4}{5}$	**F** $\frac{7}{4} \cdot \frac{5}{2}$	**J** $\frac{3}{7} \cdot \frac{4}{19}$	**N** $\frac{7}{8} \cdot \frac{5}{6}$	**R** $\frac{23}{100} \cdot \frac{47}{100}$
C $\frac{3}{7} \cdot \frac{5}{6}$	**G** $\frac{5}{6} \cdot \frac{3}{8}$	**K** $\frac{4}{19} \cdot \frac{3}{7}$	**O** $\frac{16}{24} \cdot \frac{27}{36}$	**S** $\frac{5}{21} \cdot \frac{7}{25}$
D $\frac{6}{6} \cdot \frac{4}{3}$	**H** $\frac{3}{11} \cdot \frac{5}{13}$	**L** $\frac{13}{19} \cdot \frac{7}{7}$	**P** $\frac{10}{15} \cdot \frac{50}{100}$	★ **T** $\frac{a}{b} \cdot \frac{c}{d}$

2. The examples below show how products are found by first replacing whole numbers and mixed numerals with improper fractions.

EXAMPLES: **A** $4 \cdot 3\frac{1}{3} = 13\frac{1}{3}$; $\frac{4}{1} \cdot \frac{10}{3} = \frac{40}{3}$ **B** $3\frac{1}{4} \cdot 2\frac{1}{2} = 8\frac{1}{8}$; $\frac{13}{4} \cdot \frac{5}{2} = \frac{65}{8}$

Find the products as in the examples.

A $1\frac{2}{3} \cdot 3\frac{1}{2}$	**D** $6 \cdot 5\frac{3}{4}$	**G** $12\frac{1}{2} \cdot 2\frac{1}{3}$	**J** $16\frac{2}{3} \cdot 1\frac{4}{5}$
B $1\frac{3}{4} \cdot 1\frac{1}{2}$	**E** $8\frac{1}{5} \cdot 6\frac{2}{3}$	**H** $3\frac{5}{6} \cdot 2\frac{2}{3}$	★ **K** $13\frac{3}{5} \cdot 1\frac{3}{17}$
C $2\frac{7}{8} \cdot 1\frac{1}{4}$	**F** $3\frac{1}{3} \cdot 3$	**I** $1\frac{2}{7} \cdot 4$	★ **L** $20\frac{2}{5} \cdot 8\frac{1}{3}$

Before trying Exercise 3, study the shortcut demonstrated in the following example.

STEP 1: Think $\frac{5}{15} = \frac{1}{3}$ → $\frac{\cancel{8}^{1}}{8} \cdot \frac{4}{\cancel{15}_{3}}$

STEP 2: Think $\frac{4}{8} = \frac{1}{2}$ → $\frac{\cancel{8}^{1}}{\cancel{8}_{2}} \cdot \frac{\cancel{4}^{1}}{\cancel{15}_{3}}$

STEP 3: Multiply $\frac{1}{2} \cdot \frac{1}{3}$ → $\frac{\cancel{8}^{1}}{\cancel{8}_{2}} \cdot \frac{\cancel{4}^{1}}{15_{3}} = \frac{1}{6}$

3. Find the products using the shortcut.

A $\frac{3}{8} \cdot \frac{4}{21}$	**F** $\frac{4}{9} \cdot \frac{15}{10}$
B $\frac{5}{4} \cdot \frac{2}{15}$	**G** $\frac{14}{63} \cdot \frac{15}{12}$
C $\frac{8}{21} \cdot \frac{14}{12}$	**H** $\frac{13}{38} \cdot \frac{19}{52}$
D $\frac{9}{20} \cdot \frac{25}{6}$	**I** $\frac{77}{32} \cdot \frac{12}{35}$
E $\frac{5}{7} \cdot \frac{21}{10}$	**J** $\frac{50}{42} \cdot \frac{14}{30}$

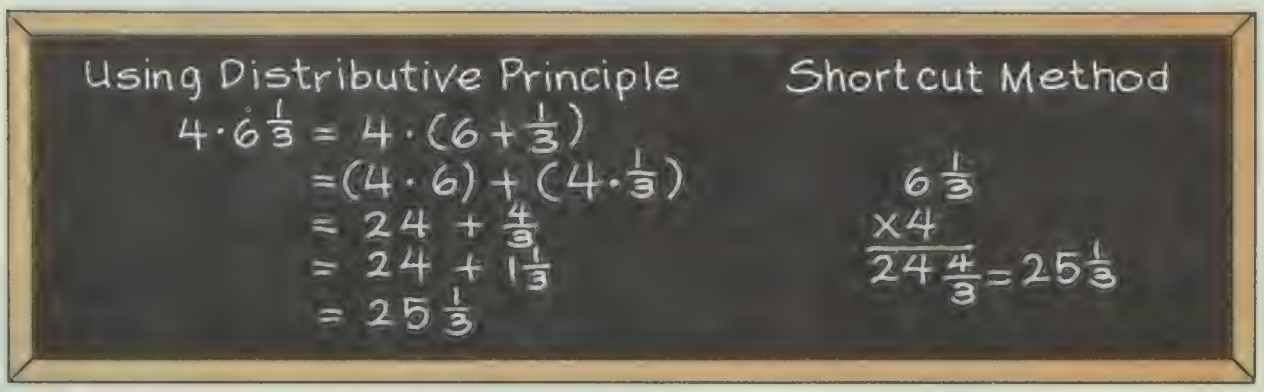
think

Find the smallest number y (between 100 and 125) so that the sum of all its factors is equal to $y + y + y$. (**Hint:** It must be a number with **many** factors.)

More practice, page S-16, Set 27

Investigating the Ideas

The two examples on the chalkboard show how you can find products using mixed numerals.



Can you find the product $8 \cdot 3\frac{2}{5}$ using each method?

Discussing the Ideas

1. What steps of the distributive principle method are done mentally using the shortcut method?

2. Study the method shown below for finding $2\frac{1}{2} \cdot 4\frac{1}{3}$.

$2\frac{1}{2} \cdot 4\frac{1}{3}$	$= (2 + \frac{1}{2}) \cdot 4\frac{1}{3}$	$= 2 \cdot 4\frac{1}{3} + \frac{1}{2} \cdot 4\frac{1}{3}$
STEP 1: $4\frac{1}{3} \times 2\frac{1}{2} = 8\frac{2}{3}$	STEP 2: $4\frac{1}{3} \times 2\frac{1}{2} = 8\frac{2}{3}$, $2\frac{1}{6}$	STEP 3: $4\frac{1}{3} \times 2\frac{1}{2} = 8\frac{2}{3}$, $2\frac{1}{6}$, $10\frac{5}{6}$
$2 \cdot 4\frac{1}{3} = 8\frac{2}{3}$	$\frac{1}{2} \cdot 4\frac{1}{3} = 2\frac{1}{6}$	$8\frac{2}{3} + 2\frac{1}{6} = 10\frac{5}{6}$

Explain how to find $3\frac{3}{4} \cdot 5\frac{1}{2}$ using this method.

3. Explain how to check your multiplication in Exercise 2 by writing both $3\frac{3}{4}$ and $5\frac{1}{2}$ as improper fractions and then finding the product.

$3\frac{3}{4} \cdot 5\frac{1}{2} = \blacksquare$
$\downarrow \quad \downarrow \quad \uparrow$
$\frac{15}{4} \cdot \frac{11}{2} = \blacksquare$

1. Find the products.

A $12\frac{1}{5} \times 4$

B $8 \times \frac{3}{4}$

C $6\frac{1}{2} \times 4\frac{1}{3}$

D $10\frac{1}{6} \times 7\frac{1}{2}$

E $8\frac{1}{8} \times 3\frac{1}{4}$

F $12\frac{1}{5} \times 7\frac{1}{2}$

G $16\frac{3}{4} \times 12\frac{1}{2}$

H $93\frac{1}{3} \times 15\frac{2}{3}$

I $15\frac{1}{3} \times 12\frac{1}{5}$

J $120\frac{1}{10} \times 25\frac{1}{3}$

2. Find the products.

A $4 \cdot 6\frac{1}{4}$

B $7 \cdot 2\frac{1}{2}$

C $3 \cdot 4\frac{1}{6}$

D $\frac{1}{2} \cdot 10\frac{1}{4}$

E $\frac{1}{8} \cdot 12\frac{1}{2}$

F $\frac{1}{3} \cdot 15\frac{1}{2}$

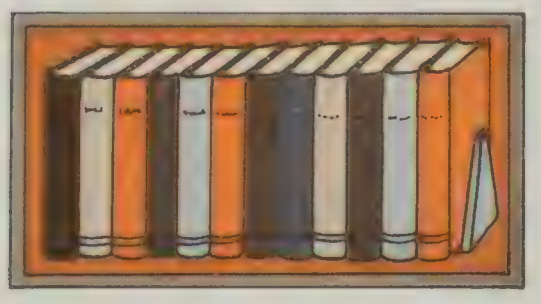
G $\frac{3}{4} \cdot 8\frac{2}{3}$

H $10 \cdot 8\frac{3}{10}$

I $\frac{2}{3} \cdot 9\frac{3}{5}$

J $\frac{3}{5} \cdot 10\frac{5}{6}$

3. There are 12 books on a shelf. Each book is $3\frac{1}{2}$ cm thick. What is the total thickness of the books?



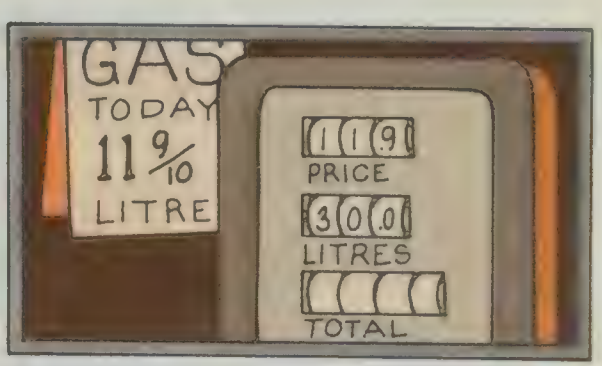
4. Carla earned $18 a certain week doing parttime work. The next week she earned $1\frac{2}{3}$ times as much. How much did she earn during the second week?

5. Jacqueline saw this road sign. If she walks about $1\frac{3}{5}$ kilometres in half an hour how many hours to Paris?

6. A note pad is $10\frac{3}{5}$ cm wide and $15\frac{1}{2}$ cm long. Find the area of a pad by multiplying the length times the width.

7. Alice had $24. She spent half of it in one store and half of her remaining money in another store. How much money did she have left?

8. Raymond lives 3 blocks away from school. Andy lives $4\frac{1}{3}$ times as far from school as Raymond. How far does Andy live from school?



9. Andrea bought 30 litres of gasoline for her car. The price was $11\frac{9}{10}$¢ per litre. How much did she pay for the gasoline?

More practice, page S-16, Set 28

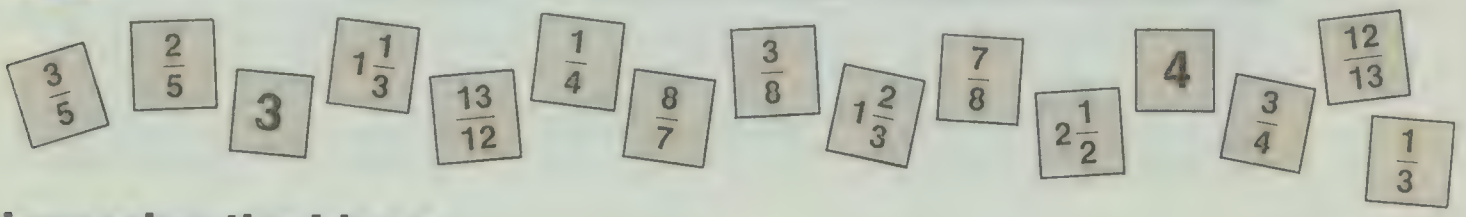
Investigating the Ideas

Some pairs of numbers have a product of 1.

$\square \cdot \square = 1$

$\square \cdot \square = 1$

Using the numbers below, how many pairs can you find that have a product of 1?	Record the pairs that you find.

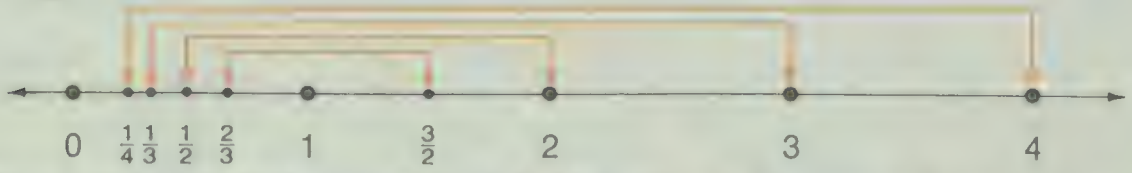


Discussing the Ideas

1. If the product of two fractional numbers is 1, then each number is the **reciprocal** or **multiplicative inverse** of the other.
   - **A** Since $8 \cdot \frac{1}{8} = 1$, what is the multiplicative inverse of $\frac{1}{8}$?
   - **B** How can you find the reciprocal of $\frac{3}{2}$?

2. Do you think that zero has a reciprocal? Explain.

3. The number line shows how reciprocals are related. Study the number line and then explain how you can find the number for each ▨.



   - **A** If a number is between 2 and 3, then its reciprocal will be between ▨ and ▨.
   - **B** Numbers between $\frac{3}{2}$ and 2 have reciprocals between ▨ and ▨.
   - **C** Numbers which are very large have reciprocals which are very close to ▨.
   - **D** Numbers which are very close to 1 have reciprocals which are very close to ▨.
   - **E** ▨ is the only number which is its own reciprocal.

4. Can you describe an easy method for finding the reciprocal of any fractional number?

1. Give the reciprocals of the following numbers.

A $\frac{5}{8}$ **B** $\frac{8}{5}$ **C** $\frac{1}{9}$ **D** 8 **E** $\frac{4}{11}$ **F** $1\frac{7}{8}$ **G** $8\frac{1}{7}$ **H** $\frac{99}{100}$

2. Solve the equations by finding the reciprocal of the given factor.

A $n \cdot \frac{3}{8} = 1$ **C** $n \cdot \frac{9}{4} = 1$ **E** $3\frac{1}{2} \cdot n = 1$ **G** $\frac{1}{10} \cdot n = 1$

B $\frac{5}{6} \cdot n = 1$ **D** $n \cdot \frac{8}{7} = 1$ **F** $7 \cdot n = 1$ **H** $100 \cdot n = 1$

3. Multiply each number by the reciprocal of 2.

A 8 **B** 14 **C** 7 **D** 1 **E** 15 **F** 20

4. Multiply each number by the reciprocal of 3.

A 12 **B** 30 **C** 18 **D** 9 **E** 60 **F** 8

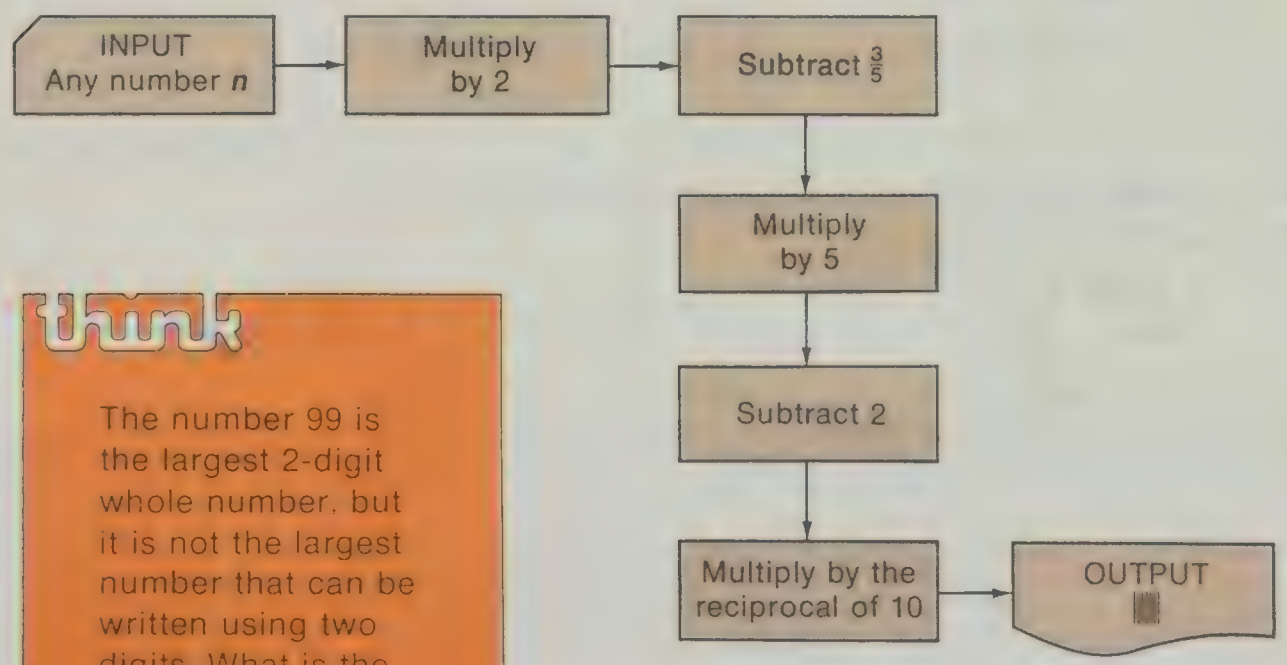
5. Multiply each number by the reciprocal of 5.

A 10 **B** 25 **C** 100 **D** 40 **E** 15 **F** 7

6. Complete the sentence: Multiplying a whole number by the reciprocal of 2 is the same as dividing the number by ? .

7. Use each number as an input number for the flow chart to find the output number.

A 2 **B** 5 **C** $\frac{3}{2}$ **D** $2\frac{1}{4}$ **E** $\frac{5}{8}$ **F** 1



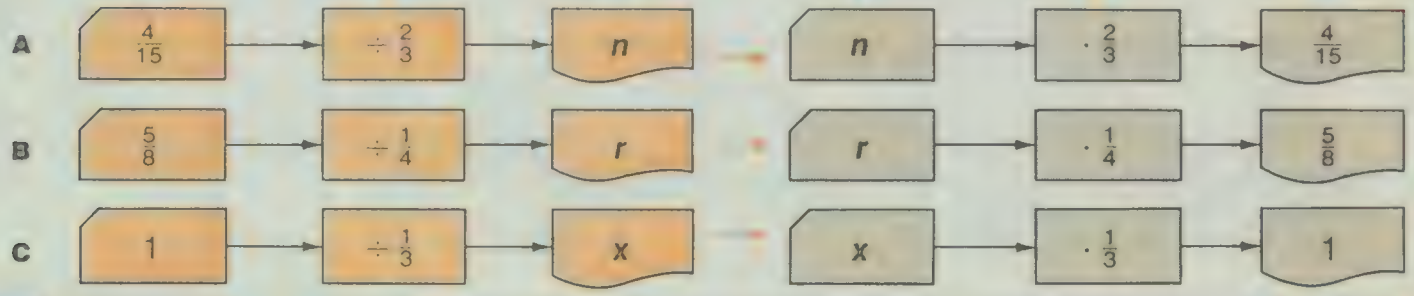
think

The number 99 is the largest 2-digit whole number, but it is not the largest number that can be written using two digits. What is the largest number?

Investigating the Ideas

In order to find the quotients (outputs) for the division flow charts below, you can think about finding the missing factors (inputs) in the inverse flow charts.

DIVISION FLOW CHART INVERSE FLOW CHART



How many of the missing numbers in the flow charts can you find?

Discussing the Ideas

1. In division flow chart A, Mark used a shortcut to find the number for *n*. Can you explain how to use Mark's method for for finding the quotients for flow charts B and C?

Mark

$\frac{4}{15} \div \frac{2}{3} = \frac{4 \div 2}{15 \div 3} = \frac{2}{5}$

2. Explain how to find the number for **x** in the flow chart D.

3. Since $\frac{1}{5} \cdot \frac{3}{2} = \frac{3}{10}$, then $\frac{3}{10} \div \frac{3}{2} = \frac{1}{5}$ and $\frac{3}{10} \div \frac{1}{5} = \frac{3}{2}$.
What division equations could you write for these multiplication equations?

A $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$ **B** $\frac{2}{7} \cdot \frac{1}{3} = \frac{2}{21}$ **C** $\frac{3}{5} \cdot \frac{2}{3} = \frac{6}{15}$ **D** $\frac{3}{7} \cdot \frac{3}{4} = \frac{9}{28}$

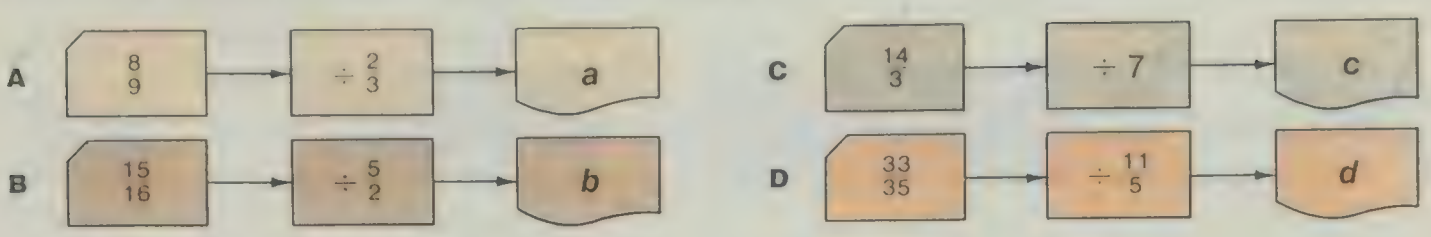
4. Find the missing factor. Then show a division equation so that the quotient will be the missing factor of the multiplication equation.

A $n \cdot \frac{3}{2} = \frac{3}{4}$ **B** $t \cdot \frac{1}{5} = \frac{3}{10}$ **C** $\frac{4}{7} \cdot r = \frac{16}{35}$ **D** $\frac{3}{5} \cdot w = \frac{3}{20}$

5. Explain how to find the quotients by thinking about missing factors.

A $\frac{10}{21} \div \frac{5}{7}$ **B** $\frac{3}{8} \div \frac{3}{4}$ **C** $\frac{15}{28} \div \frac{3}{4}$ **D** $\frac{35}{8} \div \frac{5}{4}$

1. Find the output number for each flow chart.



2. Write division equations for each multiplication equation.

A $\frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$ B $\frac{5}{6} \cdot \frac{1}{3} = \frac{5}{18}$ C $\frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}$ D $\frac{5}{8} \cdot \frac{3}{8} = \frac{15}{64}$

3. Find the missing factor in the first equation. Then find the correct quotient for the screened equation.

A $n \cdot \frac{5}{8} = \frac{15}{32}$ $\frac{15}{32} \div \frac{5}{8} = n$

B $n \cdot \frac{4}{9} = \frac{8}{45}$ $\frac{8}{45} \div \frac{4}{9} = n$

C $\frac{3}{5} \cdot n = \frac{9}{10}$ $\frac{9}{10} \div \frac{3}{5} = n$

D $\frac{1}{2} \cdot n = \frac{3}{8}$ $\frac{3}{8} \div \frac{1}{2} = n$

E $\frac{3}{5} \cdot n = \frac{21}{40}$ $\frac{21}{40} \div \frac{3}{5} = n$

F $n \cdot \frac{3}{2} = \frac{9}{10}$ $\frac{9}{10} \div \frac{3}{2} = n$

G $\frac{3}{10} \cdot n = 1$ $1 \div \frac{3}{10} = n$

H $n \cdot \frac{2}{3} = 1$ $1 \div \frac{2}{3} = n$

4. Find the missing factor. Then write a division equation so that the quotient will be the missing factor of the multiplication equation.

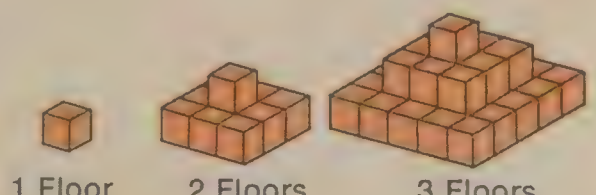
A $\frac{3}{4} \cdot n = \frac{3}{8}$
B $n \cdot \frac{5}{9} = \frac{10}{27}$
C $\frac{3}{4} \cdot n = \frac{9}{20}$
D $\frac{3}{8} \cdot n = \frac{9}{16}$
E $\frac{4}{5} \cdot n = \frac{12}{35}$
F $n \cdot \frac{1}{2} = 4$
G $\frac{1}{10} \cdot n = \frac{1}{2}$
H $\frac{3}{5} \cdot n = \frac{3}{10}$
I $\frac{5}{7} \cdot n = \frac{25}{49}$
J $\frac{4}{7} \cdot n = \frac{9}{7}$
K $\frac{5}{12} \cdot n = \frac{10}{24}$
L $\frac{3}{2} \cdot n = \frac{3}{2}$

5. Find the quotients.

A $\frac{3}{4} \div \frac{1}{2}$
B $\frac{15}{21} \div \frac{3}{7}$
C $\frac{1}{10} \div \frac{1}{5}$
D $\frac{7}{10} \div \frac{7}{10}$
E $\frac{1}{4} \div \frac{1}{2}$
F $\frac{35}{24} \div \frac{5}{4}$
G $\frac{15}{32} \div \frac{3}{8}$
H $\frac{3}{10} \div 1$
I $\frac{4}{18} \div \frac{2}{3}$
J $\frac{27}{100} \div \frac{9}{10}$
K $\frac{18}{25} \div \frac{3}{5}$
L $\frac{54}{111} \div \frac{27}{37}$

think

Each cube represents a room in a hotel constructed according to the pattern shown. How many floors does the hotel have if there are 969 rooms in the hotel?



Discussing the Ideas

1. Study the steps of the example below. The example may help you understand a shortcut for division problems such as $\frac{3}{4} \div \frac{2}{5}$.
Can you explain what is being done in each step of the example?

	Step 1		Step 2		Step 3		Step 4		Step 5
$\frac{3}{4} \div \frac{2}{5}$	$=$	$\frac{3 \cdot (2 \cdot 5)}{4 \cdot (2 \cdot 5)} \div \frac{2}{5}$	$=$	$\frac{(3 \cdot 2 \cdot 5) \div 2}{(4 \cdot 2 \cdot 5) \div 5}$	$=$	$\frac{3 \cdot 5}{4 \cdot 2}$	$=$	$\frac{3}{4} \cdot \frac{5}{2}$	$=$	$\frac{15}{8}$

2. From step 4 in the example above, you can conclude that $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2}$

   A How is the divisor $\frac{2}{5}$ and the second factor related?

   B Can you explain how you might write the division problem as a multiplication problem? What is the quotient?

3. The examples above suggest the general statement at the right concerning division of fractional numbers. Illustrate the statement if $\frac{a}{b} = \frac{2}{7}$ and $\frac{c}{d} = \frac{3}{4}$.

> If $\frac{a}{b}$ and $\frac{c}{d}$ are fractional numbers and $\frac{c}{d} \neq 0$, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

4. How would you complete this sentence?
   To divide a first number by a second number, multiply the first number by the ? of the second number.

5. In order to divide fractional numbers when they are expressed as mixed numerals, you can rewrite the problem using improper fractions. How would you complete each of these problems?

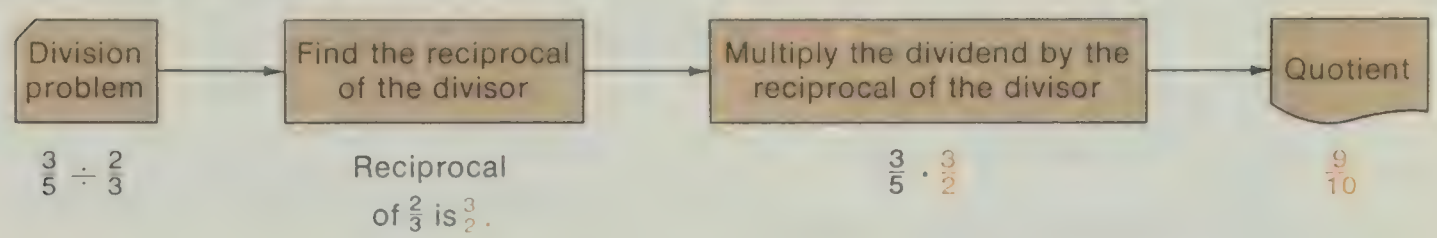
   A $2\frac{1}{2} \div 1\frac{3}{4}$
   $\frac{5}{2} \div \frac{7}{4} = \frac{5}{2} \cdot \frac{4}{7} = ?$

   B $3\frac{7}{8} \div 1\frac{2}{3}$
   $\frac{31}{8} \div \frac{5}{3} = \frac{31}{8} \cdot \frac{\square}{\square} = ?$

   C $4\frac{1}{4} \div 2\frac{1}{3}$
   $\frac{\square}{\square} \div \frac{\square}{\square} = \frac{\square}{\square} \cdot \frac{\square}{\square} = ?$

   D $5 \div 3\frac{1}{3}$
   $\frac{\square}{\square} \div \frac{\square}{\square} = \frac{\square}{\square} \cdot \frac{\square}{\square} = ?$

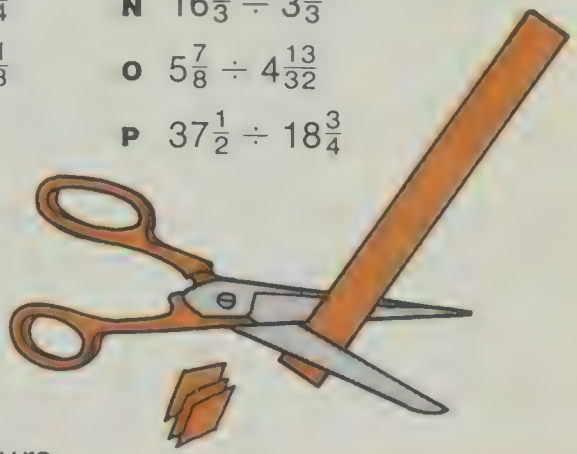
1. Find the quotients. The flow chart for dividing fractional numbers may help you.



A $\frac{4}{5} \div \frac{2}{3}$	**F** $\frac{9}{10} \div \frac{3}{2}$	**K** $10 \div \frac{3}{8}$	**P** $\frac{3}{10} \div \frac{1}{10}$
B $\frac{1}{6} \div \frac{2}{5}$	**G** $\frac{25}{4} \div \frac{5}{8}$	**L** $8 \div \frac{5}{9}$	**Q** $\frac{4}{9} \div \frac{2}{3}$
C $\frac{3}{7} \div \frac{1}{2}$	**H** $\frac{7}{8} \div \frac{3}{4}$	**M** $5 \div \frac{3}{5}$	**R** $\frac{20}{3} \div 5$
D $\frac{5}{12} \div \frac{5}{3}$	**I** $\frac{3}{4} \div \frac{7}{8}$	**N** $\frac{1}{2} \div \frac{1}{4}$	**S** $\frac{5}{8} \div \frac{5}{4}$
E $8 \div \frac{2}{3}$	**J** $3 \div \frac{1}{4}$	**O** $\frac{1}{2} \div \frac{3}{4}$	**T** $\frac{7}{10} \div \frac{3}{5}$

2. Find the quotients. Rewrite the mixed numerals as improper fractions before dividing.

A $2\frac{1}{2} \div \frac{3}{4}$	**E** $2\frac{3}{4} \div 1\frac{1}{2}$	**I** $\frac{9}{10} \div 1\frac{2}{3}$	**M** $1 \div 2\frac{3}{4}$
B $10\frac{1}{2} \div 6$	**F** $6\frac{2}{3} \div 4$	**J** $15\frac{7}{8} \div 5\frac{1}{4}$	**N** $16\frac{2}{3} \div 3\frac{1}{3}$
C $1\frac{2}{5} \div 1\frac{1}{2}$	**G** $9 \div 3\frac{1}{3}$	**K** $100 \div 3\frac{1}{3}$	**O** $5\frac{7}{8} \div 4\frac{13}{32}$
D $2\frac{2}{3} \div \frac{3}{4}$	**H** $7\frac{1}{2} \div 2$	**L** $10 \div 2\frac{1}{2}$	**P** $37\frac{1}{2} \div 18\frac{3}{4}$



3. If you cut a strip of paper 15 centimetres long into pieces $1\frac{1}{2}$ centimetres long, how many pieces will you have?

4. An airplane travelled 1200 kilometres in $1\frac{2}{3}$ hours. What was the average speed of the airplane?

5. Jack wanted to saw a 24-cm strip of plastic into 5 pieces of equal length. If $\frac{3}{10}$ of a cm is lost with each cut, how long will each piece be?

6. Which number in the set $\left\{1, \frac{4}{9}, \frac{9}{4}, \frac{16}{81}, \frac{81}{16}\right\}$ is the solution for $n \div \frac{4}{9} = \frac{4}{9}$?

Think

Two numbers are reciprocals of each other. One of these numbers is four times as large as the other. Determine both numbers.

More practice, page S-17, Set 29

Discussing the Ideas

1. Give the missing numbers in the sentences below.

   A You can find the quotient for $4 \div 9$ by writing $4 \div 9 = \frac{4}{1} \div \frac{9}{1} = \frac{4}{1} \cdot \frac{1}{9} = \frac{\square}{\square}$.

   B The quotient $2 \div 5$ is $\frac{2}{5}$ because $5 \cdot \frac{\square}{\square} = 2$.

2. Let us agree that a quotient such as $\frac{2}{3} \div \frac{5}{8}$ can be represented by a **complex fraction** $\dfrac{\frac{2}{3}}{\frac{5}{8}}$.

   A What is the numerator of the complex fraction?

   B What is the denominator of the complex fraction?

3. Give each quotient as a complex fraction.

   A $\frac{1}{2} \div \frac{2}{3}$ **B** $\frac{3}{4} \div \frac{3}{8}$ **C** $\frac{5}{6} \div \frac{1}{10}$ **D** $\frac{5}{4} \div \frac{4}{3}$

4. Write each complex fraction as a quotient of two fractional numbers.

   A $\dfrac{\frac{2}{5}}{\frac{3}{4}}$ **B** $\dfrac{\frac{1}{2}}{\frac{5}{6}}$ **C** $\dfrac{\frac{9}{10}}{\frac{3}{5}}$ **D** $\dfrac{\frac{17}{32}}{\frac{1}{4}}$

5. You can simplify a complex fraction using division. Complete each equation.

   A $\dfrac{\frac{26}{5}}{\frac{13}{3}} = \frac{26}{5} \div \frac{13}{3} = \frac{26}{5} \cdot \frac{3}{13} = ?$ **B** $\dfrac{\frac{3}{5}}{\frac{2}{7}} = \frac{3}{5} \div \frac{2}{7} = \frac{3}{5} \cdot \frac{7}{2} = ?$

   C $\dfrac{\frac{15}{2}}{\frac{3}{4}} = \frac{15}{2} \div \frac{\square}{\square} = \frac{15}{2} \cdot \frac{\square}{\square} = ?$ **D** $\dfrac{\frac{5}{8}}{\frac{7}{18}} = \frac{\square}{\square} \div \frac{\square}{\square} = \frac{\square}{\square} \cdot \frac{\square}{\square} = ?$

6. The examples below illustrate another method of simplifying a complex fraction. Explain how this method works and complete the equations.

   A $\dfrac{\frac{26}{5}}{\frac{13}{3}} = \dfrac{\frac{26}{5} \cdot \frac{3}{\square}}{\frac{13}{3} \cdot \frac{3}{13}} = \dfrac{\frac{6}{5}}{1} = ?$ **B** $\dfrac{\frac{3}{5}}{\frac{2}{7}} = \dfrac{\frac{3}{5} \cdot \frac{7}{\square}}{\frac{2}{7} \cdot \frac{7}{\square}} = \dfrac{\frac{21}{10}}{1} = ?$

1. Give each quotient as a fraction.

A $4 \div 5$	**D** $11 \div 3$	**G** $17 \div 6$	**J** $\frac{2}{3} \div \frac{1}{5}$
B $7 \div 16$	**E** $19 \div 25$	**H** $42 \div 9$	**K** $\frac{3}{7} \div \frac{7}{3}$
C $8 \div 7$	**F** $23 \div 4$	**I** $13 \div 26$	**L** $\frac{5}{9} \div \frac{10}{19}$

2. Write each quotient as a complex fraction.

A $\frac{5}{6} : \frac{1}{2}$ **B** $\frac{7}{8} \div \frac{3}{4}$ **C** $2 \div \frac{3}{4}$ **D** $\frac{1}{2} \div 4$ **E** $\frac{17}{3} \div \frac{5}{7}$

3. Write each complex fraction as a quotient of two fractional numbers.

A $\dfrac{\frac{3}{5}}{\frac{1}{2}}$ **B** $\dfrac{\frac{1}{8}}{\frac{5}{2}}$ **C** $\dfrac{\frac{1}{3}}{\frac{3}{7}}$ **D** $\dfrac{\frac{3}{10}}{\frac{5}{4}}$ **E** $\dfrac{\frac{3}{4}}{\frac{3}{2}}$

4. We have agreed that the symbol $\dfrac{\frac{2}{3}}{\frac{5}{7}}$ means $\frac{2}{3} \div \frac{5}{7}$.

Use division to simplify each complex fraction.

A $\dfrac{\frac{1}{2}}{\frac{2}{8}}$	**D** $\dfrac{\frac{7}{6}}{\frac{7}{3}}$	**G** $\dfrac{4\frac{1}{2}}{9}$	**J** $\dfrac{37\frac{1}{2}}{100}$	**M** $\dfrac{2\frac{1}{2}}{10}$	**P** $\dfrac{4\frac{2}{5}}{2\frac{3}{4}}$
B $\dfrac{\frac{5}{9}}{\frac{1}{3}}$	**E** $\dfrac{\frac{3}{4}}{\frac{9}{4}}$	**H** $\dfrac{8\frac{1}{3}}{25}$	**K** $\dfrac{3\frac{1}{3}}{13\frac{1}{3}}$	**N** $\dfrac{7}{4\frac{2}{3}}$	**Q** $\dfrac{2\frac{7}{8}}{3\frac{5}{6}}$
C $\dfrac{\frac{9}{10}}{\frac{9}{100}}$	**F** $\dfrac{\frac{3}{4}}{\frac{3}{2}}$	**I** $\dfrac{\frac{5}{6}}{\frac{5}{18}}$	**L** $\dfrac{1\frac{1}{2}}{1\frac{1}{3}}$	**O** $\dfrac{3\frac{1}{5}}{1\frac{3}{5}}$	**R** $\dfrac{3\frac{1}{3}}{\frac{1}{3}}$

Think

Check to see if each equation in Exercises 1 through 4 is correct.
Find two more equations of the same kind for each exercise.

1. $1\frac{1}{2} \cdot 3 = 1\frac{1}{2} + 3$
$1\frac{1}{3} \cdot 4 = 1\frac{1}{3} + 4$
$1\frac{1}{4} \cdot 5 = 1\frac{1}{4} + 5$

2. $1 \cdot \frac{1}{2} = 1 - \frac{1}{2}$
$2 \cdot \frac{2}{3} = 2 - \frac{2}{3}$
$3 \cdot \frac{3}{4} = 3 - \frac{3}{4}$

3. $1\frac{1}{3} \div \frac{2}{3} = 1\frac{1}{3} + \frac{2}{3}$
$2\frac{1}{4} \div \frac{3}{4} = 2\frac{1}{4} + \frac{3}{4}$
$3\frac{1}{5} \div \frac{4}{5} = 3\frac{1}{5} + \frac{4}{5}$

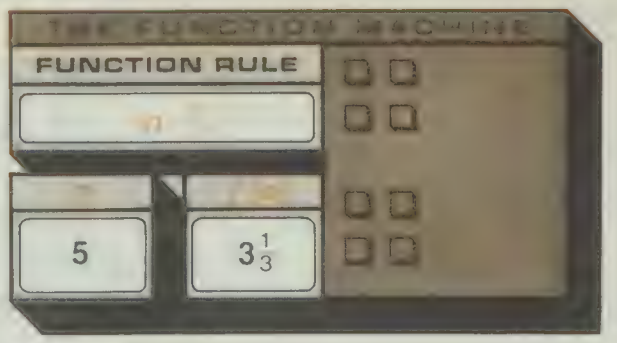
4. $4\frac{1}{2} \div 3 = 4\frac{1}{2} - 3$
$5\frac{1}{3} \div 4 = 5\frac{1}{3} - 4$
$6\frac{1}{4} \div 5 = 6\frac{1}{4} - 5$

More practice, page S-17, Set 30

Fractional Number Functions

The function machine shows the function rule $f(n) = n \cdot \frac{2}{3}$. When the input number is 5, the output number is $5 \cdot \frac{2}{3}$ or $3\frac{1}{3}$.

We write, $f(5) = 3\frac{1}{3}$ and we say "f of 5 equals $3\frac{1}{3}$."



1. Use the function rule $f(n) = n \cdot \frac{2}{3}$ to complete each exercise.

A $f(2) =$ ▒	**D** $f(0) =$ ▒	**G** $f(3\frac{1}{2}) =$ ▒	**J** $f($▒$) = 2\frac{1}{5}$
B $f(3) =$ ▒	**E** $f(\frac{3}{2}) =$ ▒	**H** $f($▒$) = 1$	**K** $f(\frac{2}{3}) =$ ▒
C $f(1) =$ ▒	**F** $f(1\frac{1}{5}) =$ ▒	**I** $f($▒$) = \frac{3}{2}$	**L** $f($▒$) = 100$

In Exercises 2 through 7, complete the function tables.

2.

Function Rule

	n	$f(n)$
A	4	▒
B	$\frac{2}{5}$	▒
C	1	▒
D	$\frac{5}{4}$	▒
E	$1\frac{2}{3}$	▒

3.

Function Rule

	n	$f(n)$
A	$\frac{1}{5}$	▒
B	6	▒
C	▒	$3\frac{1}{2}$
D	▒	1
E	$2\frac{3}{5}$	▒

4.

Function Rule $n \cdot 1\frac{2}{3}$

	n	$f(n)$
A	0	▒
B	$\frac{2}{7}$	▒
C	▒	$2\frac{2}{5}$
D	▒	2
E	$3\frac{3}{4}$	▒

5.

Function Rule

	n	$f(n)$
A	4	▒
B	$\frac{7}{5}$	▒
C	▒	$\frac{1}{2}$
D	▒	$\frac{3}{2}$
E	$4\frac{1}{7}$	▒

★ **6.**

Function Rule

	n	$f(n)$
A	$n \cdot$ ▒	
	$\frac{1}{3}$	$\frac{1}{6}$
	$\frac{2}{5}$	$\frac{1}{5}$
	4	2
	$\frac{8}{3}$	$\frac{4}{3}$
B	$2\frac{3}{5}$	▒

★ **7.**

Function Rule $n \div \frac{1}{2}$

	n	$f(n)$
	$\frac{2}{3}$	$\frac{4}{3}$
A	$\frac{3}{5}$	▒
B	3	▒
C	$3\frac{2}{7}$	▒
D	$4\frac{1}{5}$	▒

Problem Solving–Division

Write a division equation for each exercise and then solve the equation. For some exercises, it may be helpful to write a multiplication equation first. Study the example before proceeding to the exercises.

EXAMPLE: $\frac{2}{5}$ of Janet's age is 6. What is Janet's age?

SOLUTION: Let x represent Janet's age.

Then $\frac{2}{5} \cdot x = 6$.

$x = 6 \div \frac{2}{5} = 6 \cdot \frac{5}{2} = 15$

Janet is 15 years old.

1. $\frac{3}{4}$ of Jim's age is 12. How old is Jim?

2. $\frac{2}{3}$ of a number is $\frac{16}{3}$. What is the number?

3. $\frac{2}{3}$ of a number is $\frac{3}{4}$. What is the number?

4. Harry's height is $\frac{4}{5}$ of Paul's height. Harry is 155 cm tall. How tall is Paul?

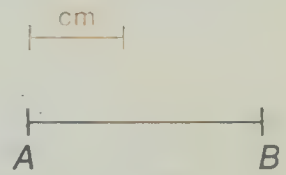
5. A jet flying 903 km/h is flying at about $\frac{6}{7}$ the speed of sound at sea level. What is the speed of sound at sea level?

6. Joe walked $12\frac{7}{10}$ km in $2\frac{1}{2}$ hours. What was his average walking speed in km/h?

7. A car travelled 140 km in $1\frac{3}{4}$ hours. What was the average speed of the car?

8. John can walk about $5\frac{1}{2}$ km/h. About how long will it take him to walk 22 km?

9. A centimetre is only $\frac{2}{5}$ as long as segment AB. How long is the segment?



10. The product of two numbers is $\frac{8}{3}$. If one of the numbers is $\frac{1}{3}$, what is the other number?

11. The quotient of two numbers is $\frac{8}{3}$. The larger number is $\frac{1}{3}$. Find the other number.

12. The product of two numbers is $\frac{2}{3}$. One of the numbers is $\frac{5}{4}$. Find the other number.

13. If the quotient of two numbers is $\frac{2}{3}$ and the smaller number is $\frac{5}{4}$, what is the other number?

THE NORTHWEST TERRITORIES

Area	3 380 620 km²
Population	34 805
Capital (named Jan. 1967)	Yellowknife
Largest Lake	Great Bear Lake (31 790 km²)
Longest River	Mackenzie River (4240 km)

The Northwest Territories extend east from the Yukon to Hudson Bay including the islands of the Bay and north from the 60th parallel of latitude excluding that portion of the mainland of Quebec. Originally including much of the prairie provinces and the northern sections of Ontario and Quebec, the Northwest Territories' present boundaries were established in 1912.

1. The area of the Northwest Territories is about $\frac{1}{3}$ the area of all Canada. Use this fact to estimate the area of Canada. Round your answer to the nearest 100 000 km².

2. The population of the Northwest Territories was about $\frac{1}{4}$ of the population of the Prince Edward Island in 1971. About how many people lived in Prince Edward Island in 1971 to the nearest 1 000?

3. The Great Bear Lake is 12 660 km² larger than Lake Ontario. How large is Lake Ontario?

4. The area of the Northwest Territories is about $6\frac{3}{10}$ times the area of the Yukon. Estimate the area of the Yukon, rounding to the nearest 1 000 km².

5. The 1971 population of the Northwest Territories was $\frac{15}{8}$ as large as the 1946 population. About how many people lived in the Northwest Territories in 1946?

6. The area of the Northwest Territories is 29 820 km² more than twice the area of Quebec. What is the area of Quebec?

7. The longest river in the Northwest Territories is approximately two thirds the length of the Mississippi River. About how long is the Mississippi?

THE PROVINCES

Newfoundland	
Land Area	370 582 km²
Population "73"	537 000

Prince Edward Island	
Land Area	5 658 km²
Population "73"	114 000

Nova Scotia	
Land Area	52 854 km²
Population "73"	802 000

New Brunswick	
Land Area	72 111 km²
Population "73"	648 000

Quebec	
Land Area	1 357 150 km²
Population "73"	6 068 000

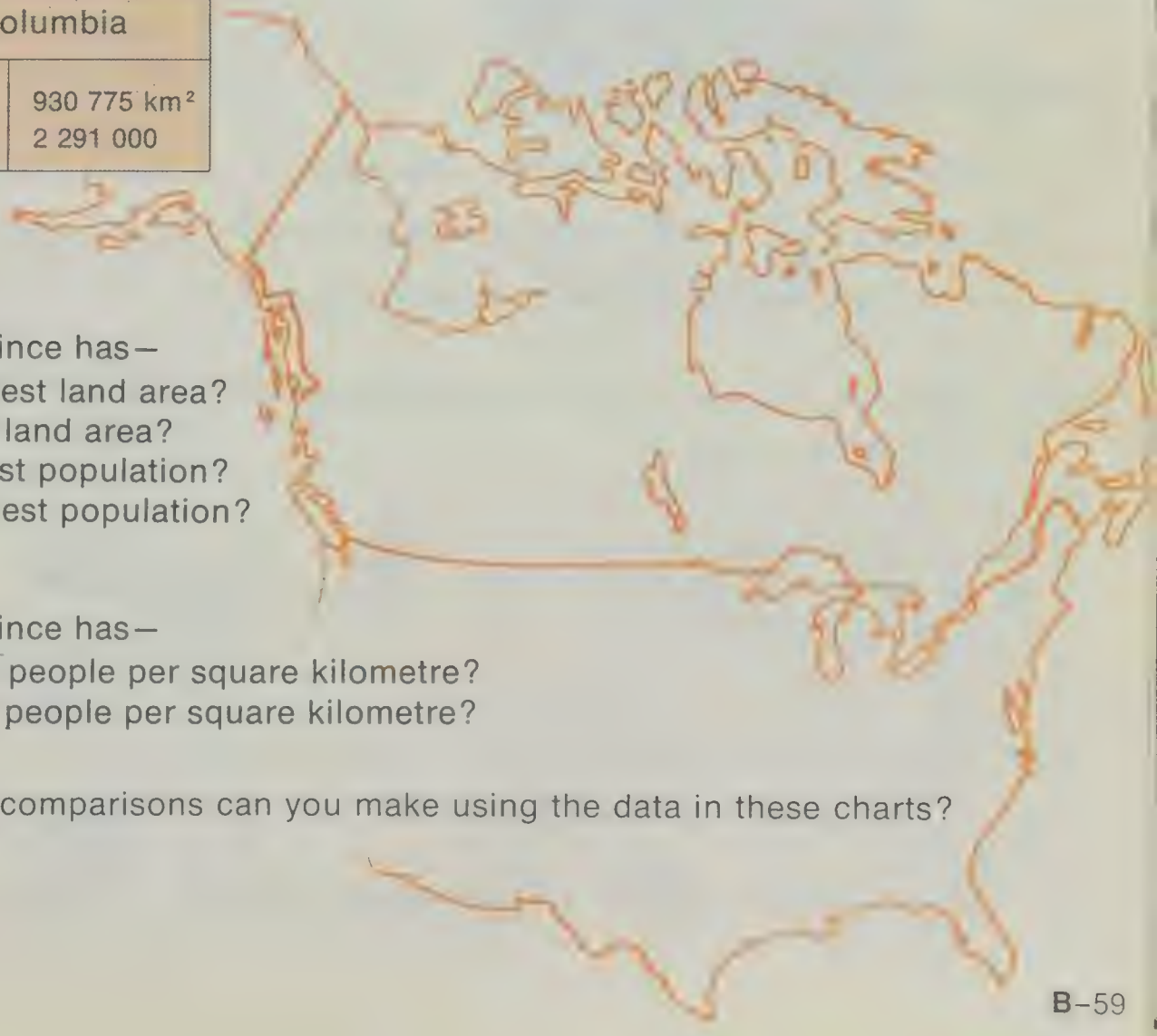
Ontario	
Land Area	891 430 km²
Population "73"	7 893 000

Manitoba	
Land Area	548 640 km²
Population "73"	993 000

Saskatchewan	
Land Area	570 420 km²
Population "73"	910 000

Alberta	
Land Area	644 560 km²
Population "73"	1 671 000

British Columbia	
Land Area	930 775 km²
Population "73"	2 291 000



1. Which province has—
   - A the greatest land area?
   - B the least land area?
   - C the largest population?
   - D the smallest population?

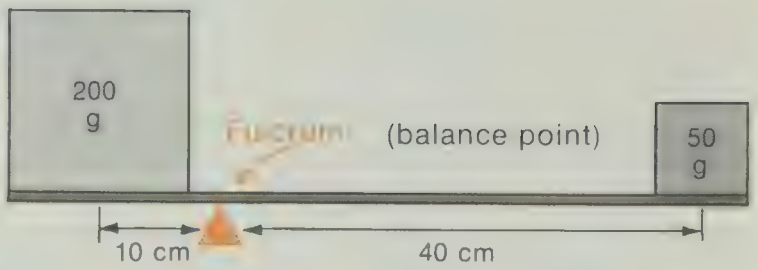
2. Which province has—
   - A the most people per square kilometre?
   - B the least people per square kilometre?

3. What other comparisons can you make using the data in these charts?

Investigating the Ideas

In diagram A, the 200-gram mass balances with the 50-gram mass because the product of one mass and its distance from the fulcrum is equal to the product of the other mass and its distance from the fulcrum.

A



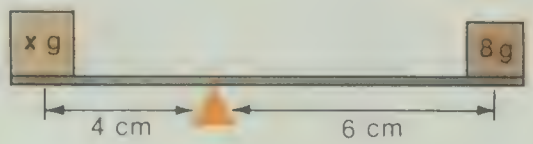
mass · distance = mass · distance

$200 \cdot 10 = 50 \cdot 40$

B

Can you find the missing numbers for diagrams B and C if the masses balance?

C



Discussing the Ideas

1. In order for the masses of diagram B to balance,

   the equation $12 \cdot x = 28 \cdot 15$ must hold.

   How can you find the number for **x** in the equation?

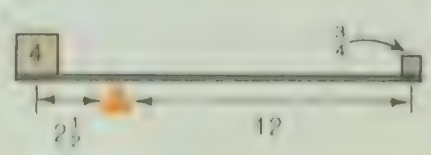
2. **A** What equation can you write for diagram C?
   B What is the mass **x**?

3. Which object in the diagram at the right is heavier? How can you tell?

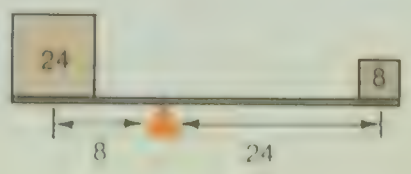
4. In which of these diagrams will the masses balance?

A

B

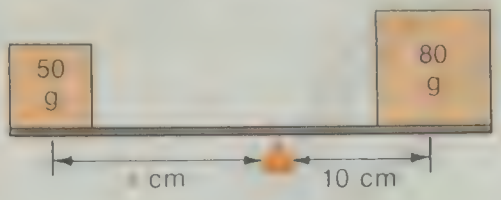


C

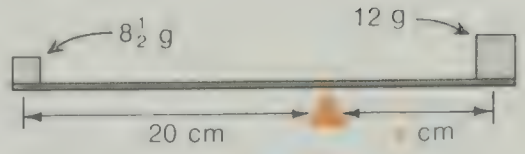


1. Write a[redacted] for the diagram if the masses balance. Then solve the equation for the distance **x**.

A

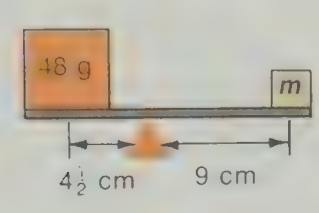


B

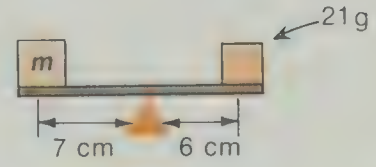


2. The masses shown in each diagram balance. [redacted] for each diagram and then solve it to find the mass ***m***.

A

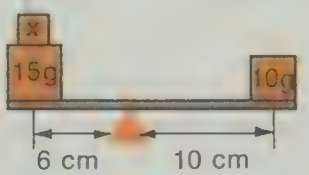


B

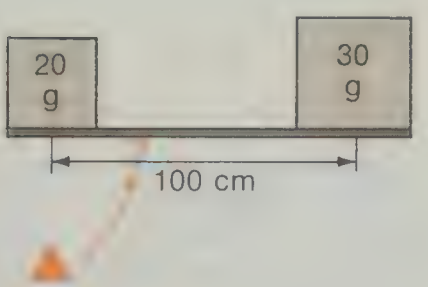


3. Tom weighs 56 kilograms and sits 160 centimetres from the balance point (fulcrum) of a seesaw. Lora weighs 40 kilograms and is able to balance with Tom. How far is Lora from the balance point?

4. What is the mass **x** if the 10-gram mass balances the others?



5. Where must the fulcrum be placed so that the masses will balance?



think

Fill in the empty squares so that they will be **magic squares**.

A

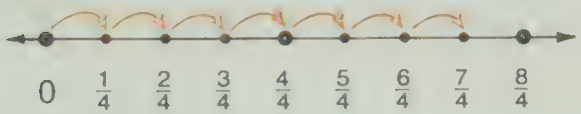
$\frac{1}{2}$		$\frac{1}{6}$
$\frac{1}{12}$	$\frac{5}{12}$	
$\frac{2}{3}$		$\frac{1}{3}$

B

$6\frac{1}{4}$		$11\frac{1}{4}$
	$7\frac{1}{2}$	
		$8\frac{3}{4}$

REVIEWING THE IDEAS

1. Give the multiplication fact suggested by the number line.



2. Solve the equations.

A $\frac{1}{4} \cdot \frac{1}{3} = n$

B $\frac{1}{5} \cdot \frac{1}{2} = n$

C $5 \cdot \frac{1}{6} = n$

D $\frac{1}{7} \cdot 3 = n$

3. Find the part of each number.

A $\frac{1}{4}$ of 16

B $\frac{1}{8}$ of 24

C $\frac{1}{6}$ of 54

D $\frac{1}{3}$ of 27

E $\frac{1}{9}$ of 63

F $\frac{1}{12}$ of 84

4. Solve the equations.

A $\frac{1}{2} \cdot \frac{3}{8} = n \cdot \frac{1}{2}$

B $\frac{5}{6} \cdot y = \frac{5}{6}$

C $\left(\frac{7}{8} \cdot x\right) \cdot \frac{3}{4} = \frac{7}{8} \cdot \left(\frac{5}{3} \cdot \frac{3}{4}\right)$

D $\frac{3}{4} \cdot w = 1$

E $\frac{1}{4} \cdot m = \frac{1}{20}$

F $12 \cdot \frac{1}{2} \cdot \frac{1}{5} \quad 12 \cdot t$

5. Find the products.

A $\frac{2}{3} \cdot \frac{5}{8}$

B $\frac{1}{4} \cdot \frac{3}{7}$

C $\frac{3}{10} \cdot \frac{1}{3}$

D $\frac{7}{8} \cdot \frac{4}{7}$

E $\frac{5}{9} \cdot \frac{3}{10}$

F $\frac{5}{6} \cdot \frac{12}{25}$

6. Find the products.

A $1\frac{1}{2} \cdot 6$

B $8 \cdot 5\frac{1}{4}$

C $3 \cdot 2\frac{2}{3}$

D $\frac{13}{5} \cdot 1\frac{2}{3}$

E $2\frac{1}{3} \cdot 3\frac{1}{2}$

F $2\frac{2}{5} \cdot 4\frac{1}{6}$

7. Give the reciprocal of each number.

A $\frac{11}{13}$

B $\frac{17}{3}$

C 5

D $\frac{1}{6}$

E $3\frac{1}{2}$

F $4\frac{3}{7}$

8. Find the quotients.

A $\frac{3}{5} \div \frac{3}{8}$

B $\frac{5}{7} \div \frac{15}{14}$

C $\frac{27}{17} \div \frac{12}{7}$

D $2\frac{3}{4} \div \frac{4}{5}$

E $5\frac{3}{8} \div 4$

F $4\frac{2}{3} \div 1\frac{3}{4}$

9. Simplify each complex fraction.

A $\dfrac{\frac{3}{4}}{\frac{3}{8}}$

B $\dfrac{\frac{10}{11}}{\frac{5}{22}}$

C $\dfrac{1\frac{1}{4}}{3\frac{1}{2}}$

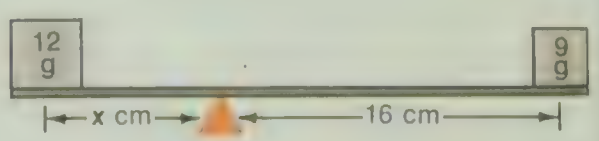
10. A special kind of colored glass used in making stained glass windows costs 10¢ per gram or fraction of a gram. What does a piece weighing $2\frac{3}{4}$ grams cost?

11. A dealer paid $2400 for a car. He added on $\frac{1}{5}$ of this cost in order to determine his selling price. What was the selling price of the car?

12. A car was driven 90 km/h for 1 hour 45 minutes. How far did it go?

13. A man spent $\frac{3}{5}$ of his money for a watch. If the watch cost $60, how much money did he have originally?

14. What is the distance **x** if the masses balance.



TEST YOURSELF

1. Solve the equations.
   A $\frac{1}{2} \cdot \frac{1}{6} = n$ B $5 \cdot \frac{1}{4} = n$

2. How much is
   A $\frac{1}{3}$ of 63? B $\frac{1}{6}$ of 102?

3. Give the number for n.
   A $\frac{3}{4} \cdot \frac{6}{7} = n \cdot \frac{3}{4}$
   B $(\frac{2}{3} \cdot \frac{5}{8}) \cdot \frac{7}{4} = \frac{2}{3} \cdot (n \cdot \frac{7}{4})$

4. Find the products.
   A $\frac{5}{6} \cdot \frac{3}{8}$ B $\frac{2}{3} \cdot \frac{4}{5}$

5. Find the products.
   A $1\frac{1}{2} \cdot 2\frac{2}{3}$ B $12\frac{2}{3} \times 3\frac{1}{2}$

6. What is the reciprocal of $\frac{9}{10}$?

7. What is the reciprocal of $5\frac{3}{4}$?

8. Find the quotients.
   A $\frac{1}{2} \div \frac{3}{4}$ C $2\frac{1}{4} \div 1\frac{7}{8}$
   B $\frac{2}{5} \div \frac{4}{15}$ D $1 \div 3\frac{2}{5}$

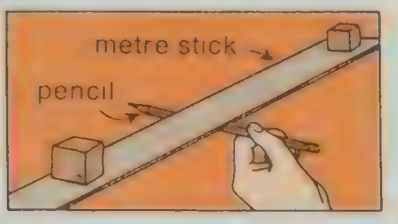
9. Simplify the complex fractions.
   A $\frac{\frac{2}{3}}{\frac{5}{4}}$ B $\frac{\frac{3}{4}}{1\frac{1}{3}}$

10. A car averaged 88 kilometres per hour for $2\frac{3}{4}$ hours. How far did it travel in that time?

RESEARCH PROJECTS

A Choose a stock from the listings in the financial section of a daily newspaper. Keep a record of the closing price of the stock each day for a period of two weeks. Then make a line-segment graph to show the "ups and downs" of your stock. Pretend that you bought 1000 shares of the stock at the beginning of the two-week period and sold them at the end of the period. Find out how much money you would have gained or lost on the stock.

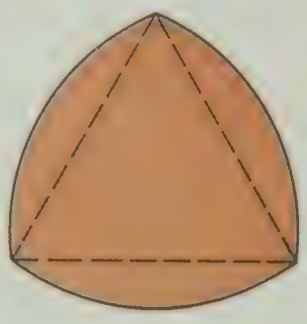
B Make a working model for a few equilibrium problems similar to those on pages B–60 and B–61 of this module. You might use a metre stick to hold objects to be balanced on a fulcrum. A pencil might be used for a fulcrum.



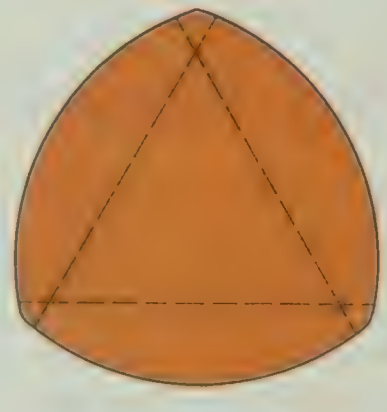
Compare known masses with unknown masses using the model. Do not expect your results to be very accurate using this simple equipment. Your science teacher may have more sophisticated equipment for you to use.

When a flat strip is placed on two circular rollers with the same diameter and pushed, the strip will move along the rollers and remain parallel to the surface on which the rollers move.

It is surprising that there are other rollers which are not circular which have the same property. These noncircular rollers are sometimes called *curves of constant width.* One of the simplest of these curves is called a **Reuleaux Triangle**. The Reuleaux Triangle is constructed from an equilateral triangle. The diagram at the right suggests how to make an Reuleaux Triangle. The center of each arc is the vertex of the triangle opposite that arc.



If you wish to make the curve less pointed and easier to roll, you may extend the sides of the triangle as shown at the right. The small arc at each vertex has its center at the nearer vertex of the triangle.



Make two Reuleaux Triangles that are the same size from cardboard. See if your curves are good noncircular rollers. You can make more permanent models by cutting them out of wood.

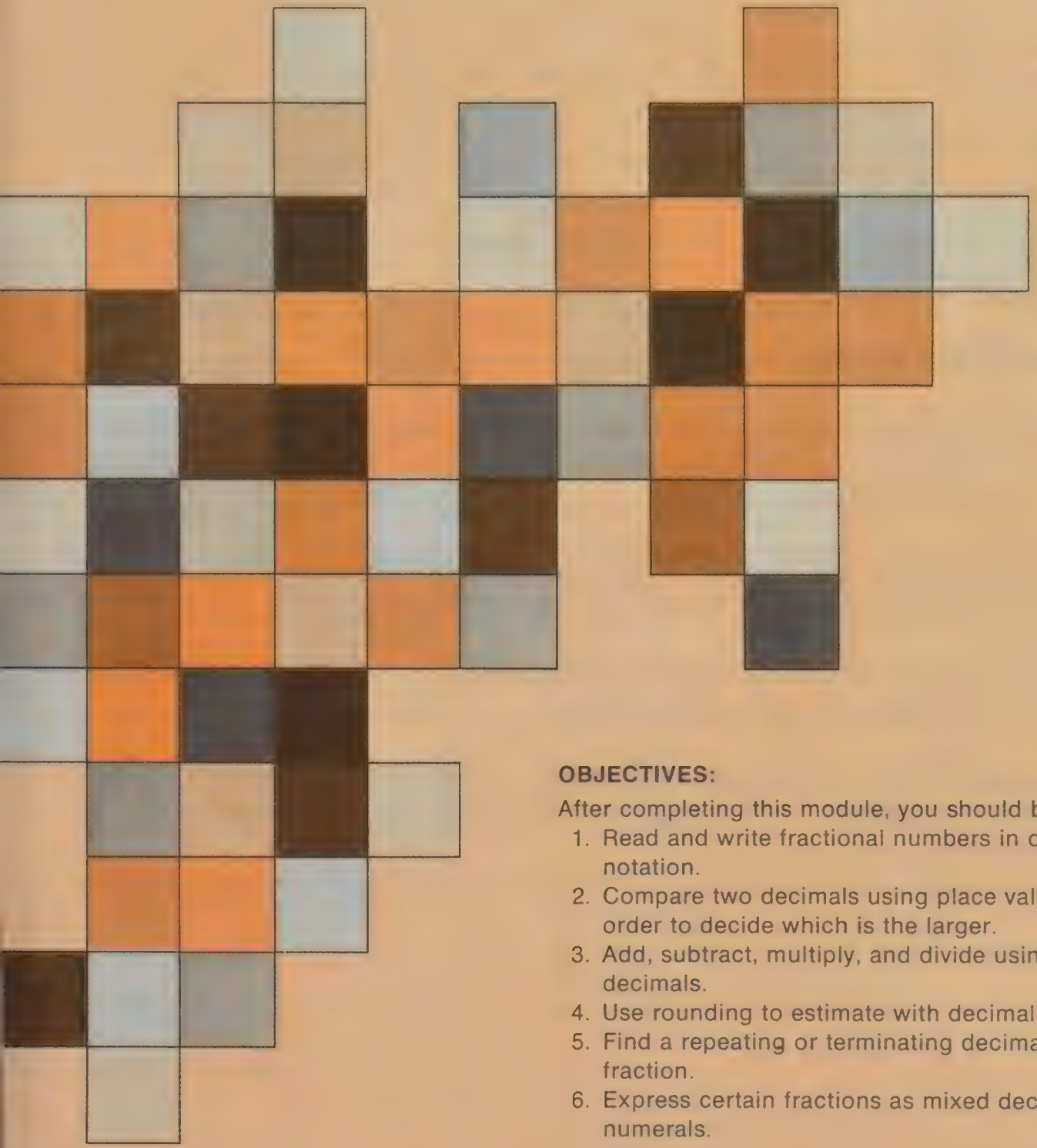
MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

1. A $\frac{1}{12}$; B $\frac{5}{4}$ or $1\frac{1}{4}$ 2. A 21; B 17 3. A $\frac{6}{7}$; B $\frac{5}{8}$
4. A $\frac{5}{16}$; B $\frac{8}{15}$ 5. A 4; B $44\frac{1}{3}$ 6. $\frac{10}{9}$ or $1\frac{1}{9}$ 7. $\frac{4}{23}$
8. A $\frac{3}{4}$; B $\frac{3}{5}$ or $1\frac{1}{5}$; C $\frac{6}{5}$ or $1\frac{1}{5}$; D $\frac{5}{17}$ 9. A $\frac{8}{15}$; B $\frac{9}{16}$ 10. 242 km

UNIT B: *The Arithmetic of Fractional Numbers*

MODULE 4: *Decimals*



OBJECTIVES:

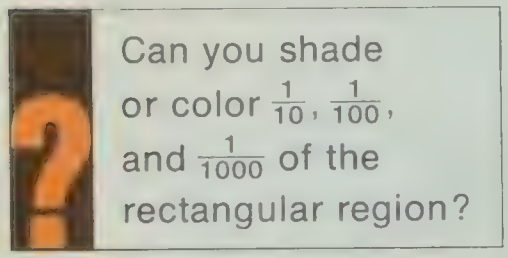
After completing this module, you should be able to:

1. Read and write fractional numbers in decimal notation.
2. Compare two decimals using place value in order to decide which is the larger.
3. Add, subtract, multiply, and divide using decimals.
4. Use rounding to estimate with decimals.
5. Find a repeating or terminating decimal for a fraction.
6. Express certain fractions as mixed decimal numerals.
7. Solve word problems using decimal notation.

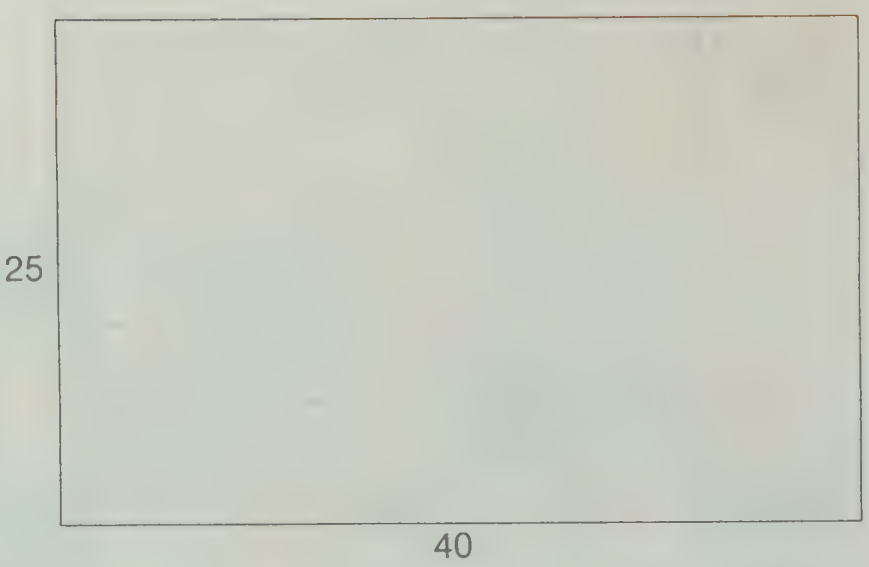
What are Decimals?

Investigating the Ideas

Use a sheet of graph paper to draw a rectangular region that is 40 squares long and 25 squares wide



Can you shade or color $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$ of the rectangular region?



Discussing the Ideas

1. A How many small squares are in the entire rectangular region above?
   B How many squares must you shade or color in order to show $\frac{1}{10}$ of the entire rectangular region?

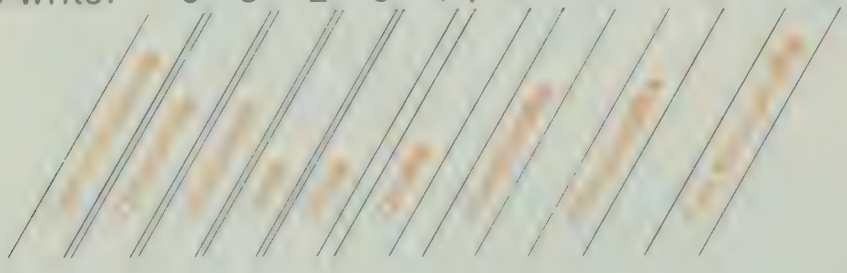
2. Instead of writing the fraction $\frac{1}{10}$, you can write the **decimal** 0.1. For $\frac{1}{100}$, you can write 0.01. What decimal could you write for $\frac{1}{1000}$?

3. A How many of the small squares represent $\frac{237}{1000}$ of the entire region?
   B What decimal can you write for $\frac{237}{1000}$?

4. The diagram below shows how we extend our place-value system to give meaning to decimals. The dot between the ones' and the tenths' place is a **decimal point**.

   For the sum $6\ 3\ 2\ 8\ 4 + \frac{8}{10} + \frac{2}{100} + \frac{3}{1000} +$ [illegible]

   we write: 6 3 2 8 4 . 8 2 3 5



   Give the missing fractions.

   A The 8 in the tenths' place means ? .
   B The 5 in the ten thousandths' place means ? .
   C The 2 in the hundredths' place means ? .

1. Copy the sentence. Give the missing word and number.

   EXAMPLE: 43.3 . The 8 in the [illegible] place represents

   A 476.835: The 4 in the ? place represents ▒.

   B 476.835: The 3 in the ? place represents ▒.

   C 9476.835: The 9 in the ? place represents ▒.

   D 9476.835: The 5 in the ? place represents ▒.

   E 0.4568: The 8 in the ? place represents ▒.

   F 5.87654: The 4 in the ? place represents ▒.

   G 0.123457 The 7 in the ? place represents ▒.

2. Write a decimal for each fraction.

   A $\frac{9}{10}$ B $\frac{17}{100}$ C $\frac{247}{1000}$ D $\frac{3}{100}$ E $\frac{18}{1000}$ F $\frac{1637}{10\,000}$

3. Write each number as shown in the example.

   $3.759 = \frac{7}{10} + \frac{5}{100} + \frac{9}{1000} = \frac{700}{1000} + \frac{50}{1000} + \frac{9}{1000} = 3\frac{759}{1000}$

   We read 3.759 as "three **and** seven hundred fifty-nine thousandths"
   or simply "three **point** seven five nine."

   A 7.52 B 0.364 C 9.452 D 0.2763 E 27.584 F 8.073

4. Write a decimal for each number represented.

   A $8 + \frac{4}{10}$ B $21 + \frac{23}{100}$ C $0 + \frac{69}{100}$ D $8 + \frac{472}{1000}$ E $15 + \frac{407}{1000}$ F $29 + \frac{2152}{10000}$ G $614 + \frac{3}{100}$ H $50\frac{47}{1000}$ I $50\frac{471}{10000}$ J $0 + \frac{37\,019}{100000}$ K $4 + \frac{612}{1000000}$ L $25\frac{5607}{1000000}$

5. Write a mixed numeral for each number.

   A 6.4 B 17.31 C 5.03 D 1.5 E 2.003 F 81.426 G 919.7154 H 40.0352 I 2.1746 J 286.11111 K 27.5680 L 3.000123

6. Give the missing exponent for each ▒.

   A $0.1 = \frac{1}{10} = \frac{1}{10^{▒}}$ B $0.01 = \frac{1}{100} = \frac{1}{10^{▒}}$ C $0.001 = \frac{1}{1000} = \frac{1}{10^{▒}}$

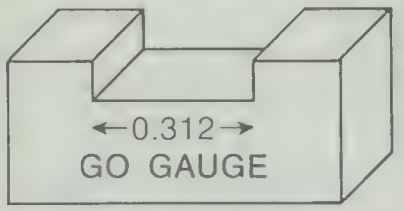
7. Write each number in expanded notation as in the example.

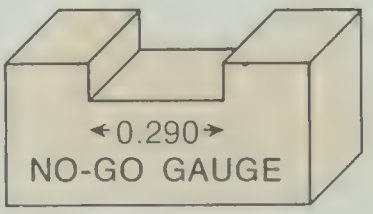
   $634.265 = (6 \cdot 10^2) + (3 \cdot 10^1) + (4 \cdot 10^0) + (2 \cdot \frac{1}{10^{▒}}) + (6 \cdot \frac{1}{10^{▒}}) + (5 \cdot \frac{1}{10^{▒}})$

   A 35.42 B 1.254 C 562.34 D 503.208 E 0.0307 F 27.64 G 8.374 H 973.56 I 860.079 J 0.506

Investigating the Ideas

The two gauges below are to be used to see if the pieces A, B, C, D, and E have the correct width. Each piece must be small enough to slide through the GO GAUGE yet too large to fit into the slot of the NO-GO GAUGE.





Can you find which pieces are the correct size?

Discussing the Ideas

1. Which pieces did you think were too large? Why?

2. Which pieces did you think were too small? Why?

3. A Since $\frac{286}{1000}$ ▒ $\frac{290}{1000}$, how does 0.286 compare with 0.290?
   B How does 0.286 compare with 0.312?
   C Does piece B fit into the slots of both gauges?

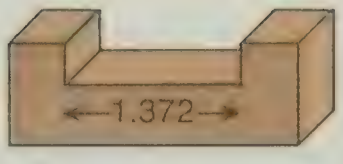
4. A Since $0.3 = \frac{3}{10} = \frac{30}{100} = \frac{300}{1000} = 0.300$, will piece A fit into the GO GAUGE?
   B Will it fit into the NO-GO GAUGE?

5. Which symbol ($<$, $=$, or $>$) goes in each ▒?

   A 0.305 ▒ 0.312 C 0.31 ▒ 0.310 E 0.31 ▒ 0.305
   B 0.305 ▒ 0.290 D 0.3 ▒ 0.31 F 0.290 ▒ 0.29

6. If the pieces A, B, C, D, and E were arranged from the smallest to the largest in width, what would the order be?

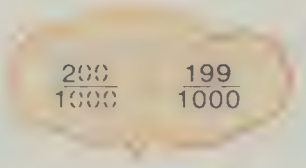
1. Which pieces below will fit into the slot of this gauge?



2. Give the correct symbol ($<$, $=$, or $>$) for each ▨.

A 1.372 ▨ 1.732	C 1.372 ▨ 1.237	E 1.372 ▨ 1.327
B 1.372 ▨ 1.273	D 1.372 ▨ 1.723	F 1.732 ▨ 1.723

3. Give the correct symbol ($<$, $=$, or $>$) for each ▨.

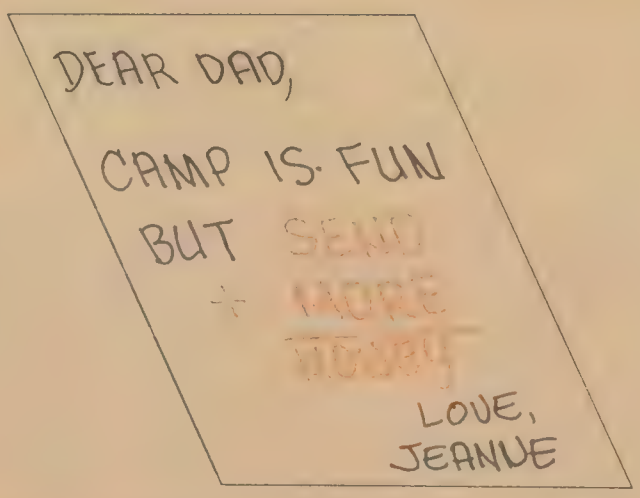


A 0.89 ▨ 0.98	B 0.67 ▨ 0.670	C 0.2 ▨ 0.199
D 1.010 ▨ 0.999	E 0.54 ▨ 0.45	F 8.93 ▨ 8.930
G 5.001 ▨ 4.999	H 0.0988 ▨ 0.1	I $0.9 + 0.1$ ▨ 1
J $0.009 + 0.1$ ▨ 0.010	K $0.999 + 0.001$ ▨ 1	L 0.09999 ▨ 0.1
M 5.654 ▨ 5.0654	N 98.999 ▨ 98.99	O $0.1 + 0.9$ ▨ $0.9 + 1$
P $0.99 + 0.99$ ▨ $0.99 + 0.9$	Q $0.056 + 1$ ▨ $0.06 + 1$	R 0.7362 ▨ 0.73620

think

Jeanne sent her father this message from summer camp.

Can you help Jeanne's father figure out this addition puzzle: SEND + MORE = MONEY? Each letter stands for one of the digits 0, 1, 2, . . . , 9, and no two letters may represent the same digit.



Adding and Subtracting with Decimals

Investigating the Ideas

Three students tried to find the sum 23.42 + 32.5 + 0.671.

John	Karen	Elaine
23.42	23.420	23.42
032.5	32.500	32.5
0.671	0.671	0.671
3.338	56.591	565.91

Can you find which student found the correct sum?

Discussing the Ideas

1. What is wrong with the way John arranged his addition problem?

2. Karen used the fact that 23.42 and 23.420 are **equivalent decimals** because $23.42 = 23\frac{42}{100} = 23\frac{420}{1000} = 23.420$.

   A Explain why 32.5 and 32.500 are equivalent decimals?

   B How did this help Karen to arrange her work for the problem?

   C What is the sum of all the thousandths in Karen's problem?

$$\begin{array}{r} {\scriptstyle 1\ \ \ \ \ \ } \\ 23.420 \\ 32.500 \\ +\ \ 0.671 \\ \hline 56.591 \end{array}$$

$$\frac{420}{1000} + \frac{500}{1000} + \frac{671}{1000} = \frac{\text{IIII}}{1000}$$

3. What did Elaine do that caused her to get an incorrect answer to the problem?

4. The figure shows how a Ken arranged his work for the subtraction problem 57.4 − 8.937.

   A Using fractions, explain why 0.4 can be written as 0.400.

   B Can you explain the regrouping in the example?

   C Explain how to complete the subtraction problem.

   D How can you check your answer to the subtraction problem?

$$1\tfrac{400}{1000} = \tfrac{1400}{1000}$$

$$\begin{array}{r} 57.400 \\ -\ \ 8.937 \\ \hline \end{array}$$

1. Write equivalent decimals in **hundredths** as needed to add or subtract.

   A $2.3 + 4 + 5.36$
   B $14.21 + 0.6 + 7.9$
   C $0.54 + 4.3 + 0.8$
   D $5 + 2.12 + 0.4$
   E $28.4 - 9.28$
   F $1.6 - 0.42$
   G $0.9 - 0.76$
   H $237.43 - 0.967$

EXAMPLE: $1.4 + 2 + 3.14$

SOLUTION:
$$\begin{array}{r} 1.40 \\ 2.00 \\ +\,3.14 \\ \hline 6.54 \end{array}$$

2. Find the sums without changing to hundredths.

   A $8 + 2.3 + 1.54$
   B $3.5 + 0.62 + 11$
   C $4.6 + 0.25 + 0.8$
   D $8.7 + 14 + 1.62$
   E $17.04 + 1.3 + 25$
   F $9 + 0.63 + 0.2$

EXAMPLE: $7 + 1.4 + 0.62$

SOLUTION:
$$\begin{array}{r} 7 \\ 1.4 \\ +\,0.62 \\ \hline \end{array}$$

3. Write equivalent decimals in **thousandths** as needed for finding each sum or difference.

   A $1.412 + 86 + 2.3$
   B $76.2 + 0.34 + 0.079$
   C $2.3 + 17 + 4.265$
   D $13.8 + 7.243 + 11.63$
   E $49.5 - 4.7$
   F $58.267 - 0.19$
   G $91 - 0.143$
   H $3.06 - 0.999$

EXAMPLE: $5.21 - 4.218$

SOLUTION:
$$\begin{array}{r} 5.210 \\ -\,4.218 \\ \hline \end{array}$$

4. Find the sums and differences.

   A $0.48 + 31$
   B $3.2 + 4.7$
   C $0.5 + 3 + 0.02$
   D $2.54 + 37.3$
   E $13.354 + 423.4568$
   F $1.05 + 0.75 + 21.5$
   G $6.563 + 0.278 + 37.937$
   H $4.763 - 0.321$
   I $5.346 + 35.27 + 161.2 + 4.387$
   J $146.2 - 21.462$
   K $1.002 + 0.0102$
   L $10.2 + 0.34 + 4$
   M $90 + 9 + 0.9 + .09$
   N $(3.1416 + 1.4142) - 2.8$
   O $(0.306 - 0.21) + 0.067$
   P $0.01 - 0.001$
   Q $100 - 0.01$
   R $10 - 0.00011$

think

Find the two products in part **A** and the two products in part **B**.

Can you find some other pairs of multiplication problems like these?

A

$$\begin{array}{rr} 32 & 23 \\ \times\,69 & \times\,96 \end{array}$$

B

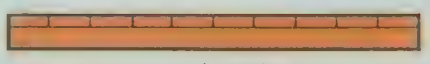
$$\begin{array}{rr} 96 & 69 \\ \times\,46 & \times\,64 \end{array}$$

More practice, page S-18, Set 31

Investigating the Ideas

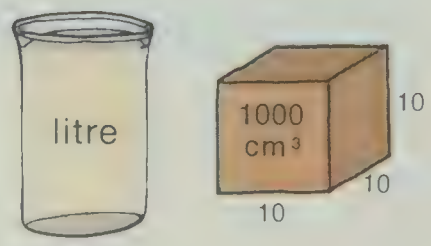
Three of the basic units of the **metric system** are the metre, the kilogram, and the litre.

Unit of length: **Metre**(m)
A metre is 100 centimetres (cm).



Unit of mass: **Kilogram**(kg)
A mass of 1 kilogram is 1000 grams (g).

Unit of capacity: **Litre**(ℓ)
A litre container holds 1000 cubic centimetres (cm^3).



Can you use the information in the chart above to make the estimates below?	Find a way to check your estimates.

1. Estimate your height in metres.
2. Estimate the mass of your mathematics book in kilograms.
3. Estimate the capacity of a soft drink bottle in litres.

Discussing the Ideas

1. Other units in the metric system are related to the basic three. These prefixes and the use of decimals can help you understand their relationship.

milli- means *one thousandth*
centi- means *one hundredth*
deci- means *one tenth*
deca- means *ten*
hecto- means *one hundred*
kilo- means *one thousand*

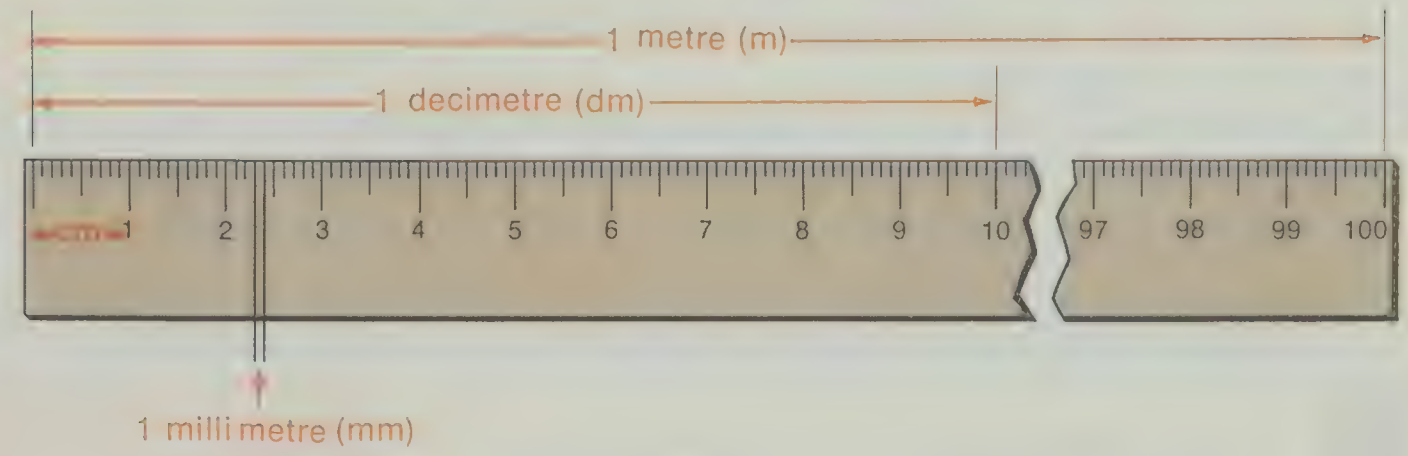
EXAMPLES: A **centi**metre is 0.01 metre; a **kilo**gram is 1000 grams.

A Which means 10 metres? decimetre or decametre
B Which means 0.01 gram? hectogram or centigram
C Which means 0.001 metre? kilometre or millimetre
D Which means 0.1 litre? decilitre or decalitre

2. Explain how to determine the missing numbers.

A 0.3 metre = ▯ decimetres
B 25 centimetres = ▯ metre
C 0.341 litre = ▯ millilitres
D 12 centigrams = ▯ milligrams
E 2000 litres = ▯ kilolitres
F 2.3 centimetres = ▯ millimetres

1. Study this diagram. Then give the missing numbers in each part below using decimal notation.



A 1 m = ▒▒ dm

B 1 m = ▒▒ cm

C 1 m = ▒▒ mm

D 1 dm = ▒▒ m

E 1 cm = ▒▒ m

F 1 mm = ▒▒ m

G 1 cm = ▒▒ mm

H 1 dm = ▒▒ cm

I 1 mm = ▒▒ cm

J 1 cm = ▒▒ dm

K 8 dm = ▒▒ m

L 25 cm = ▒▒ m

M 59 cm = ▒▒ m

N 4 mm = ▒▒ m

O 29 mm = ▒▒ cm

2. Give the missing numbers.

A 1 kilogram = ▒▒ g

B 1 g = ▒▒ kg

C 1 hectolitre = ▒▒ ℓ

D 1 decilitre = ▒▒ ℓ

E 1 ℓ = ▒▒ ml

F 1 ℓ = ▒▒ hectolitre (hl)

3. The body of an average man contains 5.7 litres of blood. The body of an average woman contains about 3.3 litres. How much greater is the blood capacity of a man?

4. Barbara weighed 36.29 kilograms. Caroline weighed 45.8 kilograms. How much more did Caroline weigh?

5. William is 1.72 metres tall. Henry is 1.59 metres tall.
   A How much taller is William?
   B How much taller in centimetres is William?

6. The brain of a man weighs about 1.1 kilograms. The brain of a woman weighs about 0.05 kilograms less. What is the weight of a woman's brain?

Investigating the Ideas

Decimals are symbols for fractions whose denominators are powers of ten. If you can multiply using this type of fraction, then you should be able to understand how to multiply using decimals.

A	B	C
$\frac{1}{10} \times \frac{1}{10} = a$	$\frac{1}{10} \times \frac{1}{100} = a$	$\frac{1}{10} \times \frac{1}{1000} = a$
$0.1 \times 0.1 = b$	$0.1 \times 0.01 = b$	$0.1 \times 0.001 = b$
tenths · tenths = ?	tenths · hundredths = ?	tenths · thousandths = ?

Can you give a fraction for ***a***, a decimal for ***b***, and a word for the blank in each example above?

Discussing the Ideas

1. Study the examples. Then give the product as a decimal.

A $\frac{8}{10}$ $\frac{3}{10}$ → $0.8 \times 0.3 =$

THINK:
$8 \times 3 = 24$
tenths · tenths = hundredths
Product: ?

B $\frac{42}{100}$ $\frac{27}{100}$ → $0.42 \times 0.27 =$

THINK:
$42 \times 27 = 1134$
hundredths · hundredths = ten thousandths
Product: ?

2. Multiplying numbers represented by decimals is much like multiplying whole numbers. Study example A. Then tell where the decimal point should be placed in example B.

A First find 76 · 3, then think:

7.6 tenths (one-place decimal)
times
× 0.3 tenths (one-place decimal)
equals
228 hundredths (two-place decimal)

Place decimal point here

B First find 942 · 31, then think:

94.2 tenths (one-place decimal)
times
× 0.31 hundredths (two-place decimal)
equals
942
2826
29202 thousandths (three-place decimal)

3. Can you state a rule for placing the decimal point in a product of two decimals?

1. Find the products.

A 0.4×0.3	C 0.8×0.6	E 0.9×0.08	G 0.7×0.300	I 0.07×0.0004
B 0.5×0.05	D 0.5×0.07	F 0.08×0.06	H 0.9×0.006	J 0.003×0.005

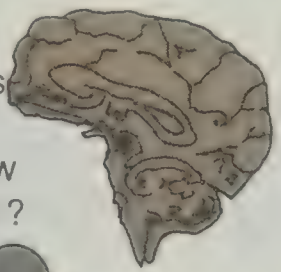
2. Find the products.

A 56.3×2	B 43.5×0.2	C 3.6×4.5	D 4.5×9.8	E 0.15×66
F 0.68×1.8	G 5.9×0.77	H 0.41×0.26	I 0.507×4.9	J 0.379×47.6
K 9.763×0.07	L 8.462×0.76	M 0.346×0.571	N 0.408×0.930	O 4.67×0.394

SHORT STORIES—The Human Body

1. The amount of water in a person's body is 0.58 times body weight. If body weight is 69.2 kilograms, how many kilograms of water?

2. A man's brain is 0.02 of his body weight. If his body weight is 64 kilograms, how much does his brain weigh?

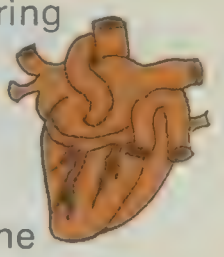


3. Man's muscles are 0.4 of his body weight. If a man weighs 61.3 kilograms, how much do his muscles weigh?

4. We breathe about 12 kilolitres of air per day. The oxygen into the blood is 0.05 of the total air. How many litres of oxygen into the blood each day?

5. Human bones: 0.18 of body weight. Body weight: 67.9 kilograms. What is the weight of the bones?

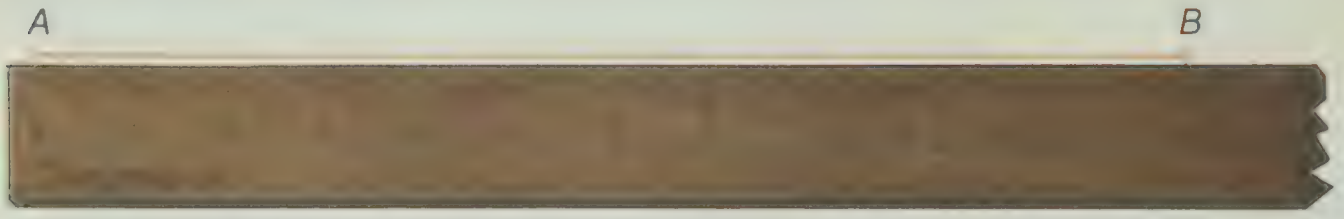
6. The human heart pumps 45 litres of blood per minute during strenuous exercise. During relaxation, it pumps 0.125 as many litres. How many litres per minute does the heart pump during relaxation?



7. Man takes 18 breaths per minute. When resting, he takes in 0.75 litres of air in each breath. When doing light work, 1.62 litres. When doing heavy work, 2.14 litres. How many litres of air used if a man rested 15 minutes, worked lightly for 15 minutes, and then worked hard for 15 minutes?

More practice, page S-18, Set 32

Investigating the Ideas



1. How long is a segment that is ten times as long as AB?
2. How long is a segment that is one hundred times as long as AB?

Can you use a centimetre ruler to draw segments that are 0.1 and 0.01 as long as $\overline{AB}$?

Discussing the Ideas

1. **A** How many centimetres long is the segment that is 0.1 as long as $\overline{AB}$?
   B What is 0.1×12?

2. **A** How many centimetres long is the segment that is 0.01 as long as AB?
   B What is 0.01×12?

3. It is important for you to be able to multiply a decimal by 0.1, 0.01, 0.001, and so forth, as well as by 10, 100, and 1000. Study part **A** in the table below. Try to find the products. You may discover a shortcut for finding them.

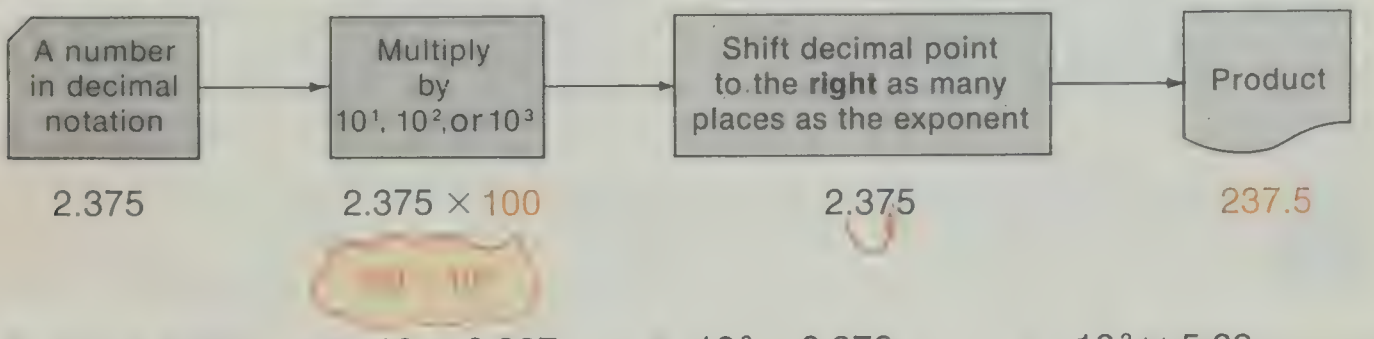
	× 1000	× 100	× 10	Number	× 0.1	× 0.01	× 0.001
A				3.71			
B				42.193			
C				0.04			
D				264.38			
E				6394.7125			

4. Can you describe a shortcut for multiplying a decimal by each of the factors 1000, 100, 10, 0.1, 0.01, and 0.001?

1. Find the products.

A 10×8.5	D 100×5.03	G 1000×5.468	J 10×9.358
B 10×0.64	E 100×2.7	H 1000×3.14	K 100×93.58
C 10×9.037	F 100×29.1	I 1000×0.82	L 1000×9.358

2. Find the products. The flow chart may help you use a shortcut.

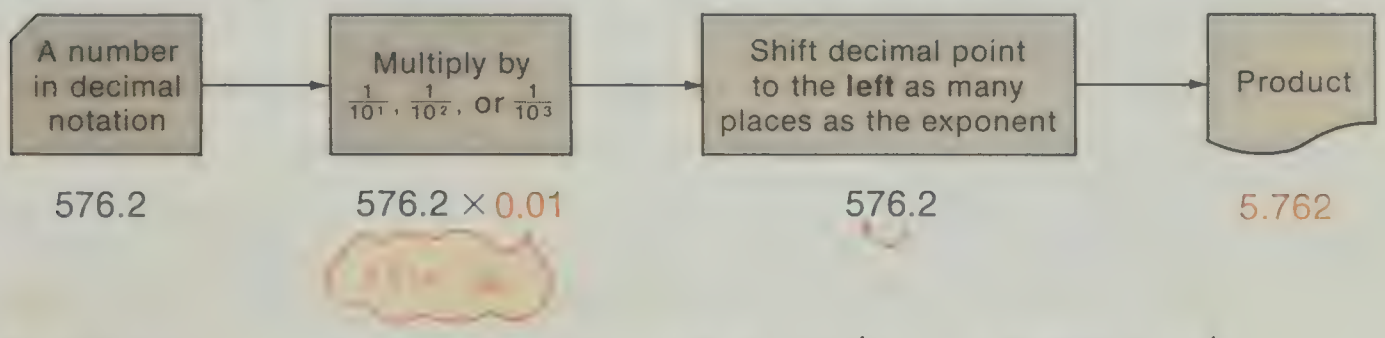


A $10^1 \times 45.62$	D 10×0.937	G $10^3 \times 0.376$	J $10^2 \times 5.63$
B 0.496×10^2	E 8.462×10^2	H 1000×0.84	K 8.891×10
C 8.437×10^3	F $10^2 \times 0.762$	I 53.762×10^3	L $10^3 \times 0.0057$

3. Find the products.

A 0.1×8.3	D 0.01×327	G 0.001×964.3	J 0.1×1.05
B 0.1×25.17	E 0.01×42.6	H 0.001×87.56	K 0.01×3.9
C 0.1×432.08	F 0.01×5.79	I 0.001×6684.2	L 0.001×8.2

4. Find the products. The flow chart may help you use a shortcut.



A 156.2×1	C 8427.4×0.01	E $8.3 \times \frac{1}{10^1}$	G $846.4 \times \frac{1}{10^3}$
B 7.26×0.01	D 839.7×0.001	F $67.7 \times \frac{1}{10^2}$	H 7.4×0.001

think

What is the greatest amount of money you can have (using pennies, nickels, dimes, quarters, and half dollars) and still not be able to give someone change for a dollar?

More practice, page S-19, Set 33

Investigating the Ideas

You can often use estimation to find approximations for products involving decimals.

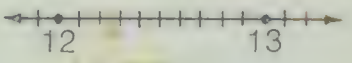
MULTIPLICATION PROBLEMS	ESTIMATED PRODUCT
A 6.87×12.239	56
B 33.75×2.93	63
C 8.367×6.72	84
D 629.6×0.09	99

Can you find which of the estimated products is nearest to the exact product for each of the multiplication problems?

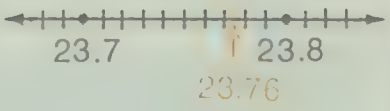
Discussing the Ideas

1. Explain how you decided which estimated product was nearest to the answer for each multiplication problem.

2. Rounding decimals is much like rounding whole numbers.
   - A To **round** 6.87 **to the nearest whole number**, you can think: "Is 6.87 nearer to 6 or to 7?" Which do you think it is?
   - B Which whole number is nearest 12.239, 12 or 13?
   - C Which two whole numbers can be used to estimate 6.87×12.239?



3. Often you must be able to round a decimal to the nearest tenth or to the nearest hundredth.
   - A Is 23.76 nearer to 23.8 or 23.7?
   - B What is 23.76 rounded to the nearest tenth?



4. Estimate the product shown by rounding each number to the nearest tenth.

$$\begin{array}{r} 3.196 \\ \times\ 0.296 \\ \hline \end{array}$$

5. Estimate the product by rounding the first factor to the nearest hundredth and the second factor to the nearest whole number.

$$0.07926 \times 3.129$$

$$0.08 \times 3$$

1. Round each number to the nearest whole number. **When a number is halfway between two numbers, round it to the larger number.**

   A 63.42 **B** 341.849 **C** 9.50 **D** 5.4999 **E** 60.099

2. Round each factor to the nearest whole number. Then use the rounded factors to estimate the product.

   A 5.9×4.67
   B 8.3×7.1
   C 5.697×9.83
   D 7×30.9573
   E 7.85×29.87
   F 9.39×29.32
   G 59.42×98.943
   H 0.94×202.78

   EXAMPLE: 4.73×8.49
   Rounded factors → 5×8
   Estimated product → 40

3. Round the first factor to the nearest tenth. Round the second factor to the nearest whole number. Use the rounded factors to estimate the product.

   A 0.297×8.2
   B 0.189×7.34
   C 0.4297×9.771
   D 0.783×29.8
   E 0.5013×5.23
   F 1.199×2.909

   EXAMPLE: 0.348×9.24
   Rounded factors → 0.3×9
   Estimated product → 2.7

4. Round the first factor to the nearest hundredth. Round the second factor to the nearest whole number. Then estimate the product.

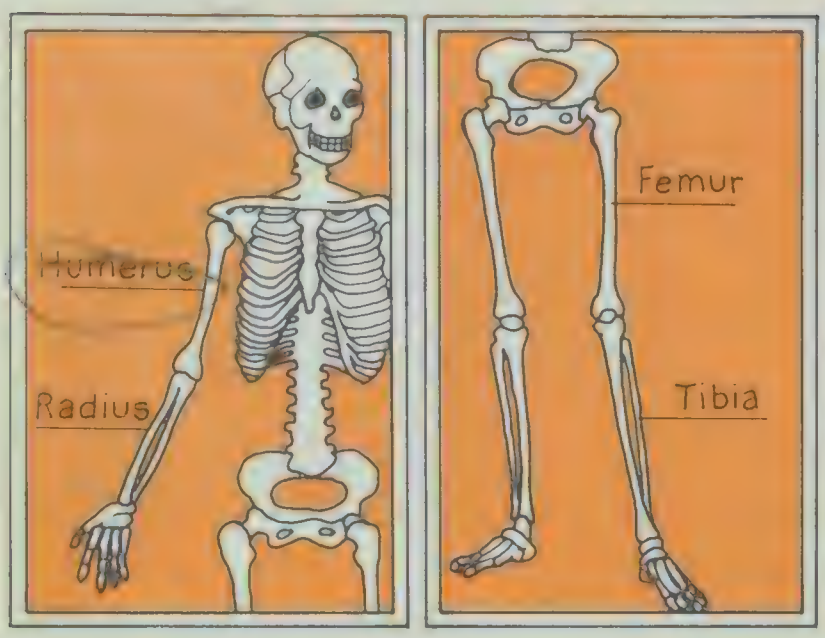
   A 0.034×7.2
   B 0.06419×7.9
   C 0.3023×6.33
   D 0.07895×3.29
   E 0.02507×9.3
   F 0.075×6.5

   EXAMPLE: 0.0576×7.21
   Rounded factors → 0.06×7
   Estimated product → 0.42

5. Estimate the products. Round the factors so that you can estimate the products quickly.

   A 10.001×54.39
   B 10.1×99.98
   C 10.113×153.71
   D 0.512×10.0037
   E 100.11×0.5032
   F 102×0.0305
   G 45.21×98.532
   H 5.324×1004.376
   I 45.113×20.974
   J 3.5432×9998.879
   K 0.101×59.91
   L 0.201×59.91
   M 0.021×48.04
   N 0.021×486.31
   O 0.009×462

ESTIMATING BODY HEIGHTS



Height (centimetres)	
Male	
(2.89 × length of humerus)	+ 70.64
(3.27 × length of radius)	+ 85.93
(1.88 × length of femur)	+ 81.31
(2.38 × length of tibia)	+ 78.66
Female	
(2.75 × length of humerus)	+ 71.48
(3.34 × length of radius)	+ 81.22
(1.95 × length of femur)	+ 72.85
(2.35 × length of tibia)	+ 74.78

Anthropologists, using a single bone from a human skeleton, can estimate quite accurately the height of a man or woman who lived many centuries ago. For example, if an anthropologist finds a 27.9-cm radius for a caveman, he can estimate the height of the caveman as follows: $(3.27 \times 27.9) + 85.93 = 177.163$ Rounding the answer, the man was about 177 cm or 1.77 meters tall.

1. A 45-cm femur was found. About how tall was the man?

2. The length of a humerus bone from a woman who lived 20 000 years ago is 32.3 cm. Estimate the woman's height to the nearest cm.

3. The skeleton of a Neanderthal man who lived about 50 000 years ago contained a tibia 38 cm long. Estimate the man's height.

4. Measure, as accurately as you can, the length of your radius and humerus and then use the table to find your height. How does your result compare with your actual height?

5. Measure your femur and tibia as accurately as you can. Then use the table to determine your height.

6. One student's height in centimetres was about 20 times the length of his middle finger measured to the nearest tenth of a centimetre. How much does your height differ from this comparison?

★ 7. Measure the length of your foot to the nearest tenth of a centimetre. Can you derive a formula that will give you an estimate of your height by using the length of your foot?

Planet Masses

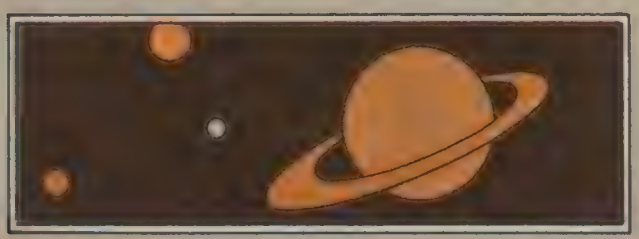
The table compares the mass of the planets with the mass of Earth. When we say that the **mass** of Neptune is 17.20 compared with the mass of Earth, think of a giant balance in which 17.20 "earths" were needed to balance one "Neptune."

Mass of each planet compared with Earth	
Earth	1.000
Mercury	0.0543
Mars	0.1069
Venus	0.8136
Pluto	0.18
Uranus	14.54
Neptune	17.20
Saturn	95.3
Jupiter	318.35

The mass of Earth has been estimated to be about 6×10^{24} kilograms or 6 septillion kilograms.

1. A Round the number in the table for Neptune to the nearest whole number.
   B Use the rounded number to estimate the mass of Neptune.

2. Round the appropriate number in the table to the nearest whole number to estimate the mass of Saturn.



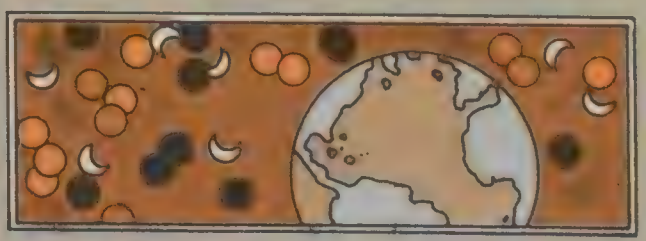
3. Round the appropriate number in the table to the nearest tenth to estimate the mass of Venus.

4. Round the appropriate number in the table to the nearest hundredth to estimate the mass of Mars.

5. Round the appropriate number in the table to the nearest hundredth to estimate the mass of Mercury.

6. About how many planets the size of Uranus would it take to "balance" with Saturn?

7. About how many planets the size of Mercury would it take to "balance" with Earth?

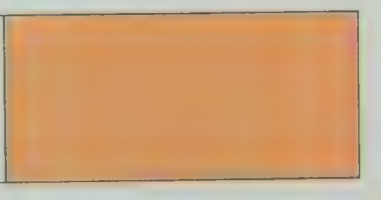


☆ 8. The mass of the moon is only 0.012 as that of Earth. About how many moons would be needed to "balance" the mass of Earth?

Investigating the Ideas

Dividing numbers named by decimals is much like dividing whole numbers. Often you can use estimation and multiplication to help you place the decimal point correctly in the quotient.

Can you place the decimal point correctly in the three quotients below and prove that you are correct?



A $5\overline{)12.85}$ $\quad 257$

B $0.9\overline{)4.716}$ $\quad 524$

C $3.4\overline{)92.14}$ $\quad 271$

Discussing the Ideas

1. What can you do to check to see if you placed the decimal point correctly in the quotients above?

2. A Where does the decimal point go in this quotient? $8\overline{)58.48}$ $\quad 731$

   B If the divisor is a whole number, can you give a rule for placing the decimal point in the quotient?

3. Every division problem involving decimals can be replaced by a division problem that has a whole number divisor and yet has the same quotient as the original problem.
   Explain how the diagram shows that the quotient for $0.9\overline{)0.3924}$ is the same as the quotient for $9\overline{)3.924}$.

$$0.9\overline{)0.3924} \rightarrow \boxed{\frac{0.3924}{0.9} = \frac{0.3924 \times 10}{0.9 \times 10} = \frac{3.924}{9}} \rightarrow 9\overline{)3.924} \quad 0.436$$

4. Explain each step of the example below.

Division problem	Think: 100 × divisor.	Think: 100 × dividend.	Think about the division problem.	Complete the dividing.
$0.07\overline{)2.569}$	$0.07\overline{)2.569}$	$0.07\overline{)2.569}$	$7\overline{)256.9}$	$0.07\overline{)2.569}$ $\quad 37.7$

1. Find the quotients.

A $2\overline{)67.6}$	**C** $4\overline{)15.16}$	**E** $63\overline{)0.504}$	**G** $58\overline{)21.46}$
B $7\overline{)23.59}$	**D** $9\overline{)58.5}$	**F** $29\overline{)17.4}$	**H** $84\overline{)21.00}$

2. Study the example to see how zeros must be annexed in some problems. Then find each quotient.

EXAMPLE: $0.09\overline{)47.7}$

$$\begin{array}{r} 530. \\ 09\overline{)47.7} \\ 45 \\ \hline 27 \\ 27 \\ \hline 0 \\ 0 \\ \hline 0 \end{array}$$

A $0.4\overline{)29.6}$	**I** $0.008\overline{)3.568}$	**Q** $0.23\overline{)0.851}$
B $5.8\overline{)0.406}$	**J** $3.45\overline{)24.15}$	**R** $5.6\overline{)3416}$
C $0.5\overline{)0.675}$	**K** $0.029\overline{)0.493}$	**S** $0.34\overline{)2.856}$
D $0.7\overline{)305.9}$	**L** $61.8\overline{)5.562}$	**T** $0.029\overline{)28.71}$
E $0.2\overline{)538}$	**M** $0.06\overline{)3.708}$	**U** $54.2\overline{)27.642}$
F $0.6\overline{)0.5616}$	**N** $0.08\overline{)756.8}$	**V** $0.41\overline{)1.599}$
G $0.09\overline{)47.7}$	**O** $0.05\overline{)0.1385}$	**W** $8.7\overline{)5.568}$
H $0.25\overline{)0.625}$	**P** $0.07\overline{)15.218}$	**X** $0.643\overline{)263.63}$

3. Study the example. Then find each quotient, rounded to the nearest hundredth. Annex zeros, as in the example, when you need them.

EXAMPLE: $2.3\overline{)1.669}$

$$\begin{array}{r} 0.725 \\ 2.3\overline{)1.669} \\ 161 \\ \hline 59 \\ 46 \\ \hline 130 \\ 115 \\ \hline 15 \end{array}$$

We say: "the quotient, rounded to the nearest hundredth is .73."

A $8\overline{)3.0}$	**G** $0.58\overline{)0.4683}$	
B $7\overline{)32.54}$	**H** $0.004\overline{)20.593}$	
C $0.9\overline{)42.345}$	**I** $9\overline{)4}$	
D $0.05\overline{)2.4730}$	**J** $0.41\overline{)0.3457}$	
E $6.1\overline{)31.76}$	**K** $2.9\overline{)1.672}$	
F $0.39\overline{)2654}$	**L** $52\overline{)0.3854}$	

think

Draw an array of nine dots like the one at the right. Place your pencil at one dot, and without lifting your pencil from the paper and without retracing, draw four line segments so that each dot lies on one of the segments.

More practice, page S-19, Set 34

SPACE TRAVEL

1. Satellite orbiting Earth:
   28 080 kilometres in an hour.
   Averaged how many kilometres per second?

2. Rocket to the moon: About 104 kilometres in 9 seconds.
   How many kilometres per second (to the nearest tenth)?

3. Diameter of Mars: About 0.53 times the diameter of Earth.
   Diameter of Earth: 12 754 kilometres.
   What is the diameter of Mars (to the nearest hundred kilometres)?

4. Rocket to leave Earth:
   11.2 kilometres per second.
   Rocket to leave Mars: 0.6 kilometres per second less than half for Earth.
   How fast does the rocket travel to leave Mars?

5. Objects weigh about 6 times as much on Earth as on the moon.
   Boy on Earth: 42.5 kilograms.
   Dog on Earth: 1.9 kilograms.
   What is the total "moon weight" of the boy and dog?

6. Weight of an object on the moon is 0.16 times Earth weight.
   Moon weight of astronaut:
   11.6 kilograms.
   What is his Earth weight?

7. Rock sample on the moon:
   1.2 kilograms.
   What is the Earth weight?
   (See Exercise 6.)

8. A "year" on Mars: 1.9 Earth years.
   How many Mars years is 18.05 Earth years?

9. Speed needed to leave the moon: 2.4 kilometres per second.
   A How many kilometres per hour?
   B Speed needed to leave Saturn is 14.7 times as fast as for the moon.
   How fast in kilometres per hour?

10. A "day" on the moon:
    About 27.3 Earth days.
    A day on Saturn is 10.5 hours.
    How many times greater is the moon day (to the nearest hundredth)?

11. Time needed to orbit Earth:
    About 95.6 minutes.
    How many complete orbits in 8 hours?

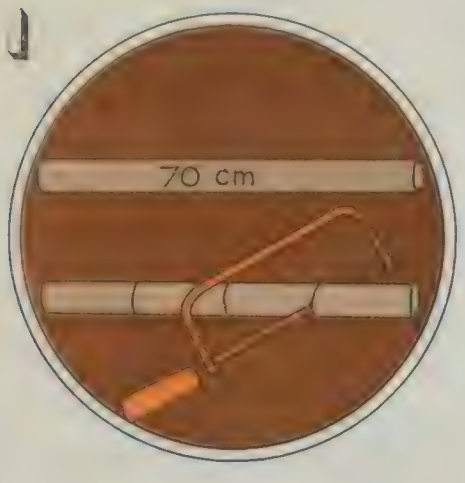
12. Weight of an object on Mars is 0.39 times its weight on Earth.
    Astronaut's weight on Mars:
    30.42 kilograms.
    What is his Earth weight?

13. Steak: 1.12 kilograms.
    Potatoes: 4.5 kilograms.
    Flour: 2.25 kilograms.
    Grapes: 1 kilogram.
    Find the total "Mars weight" of the groceries.
    (See Exercise 12.)

Wordless Problems

Each picture suggests a problem. Study the picture carefully.
Then write and solve your own problem for the picture.

1



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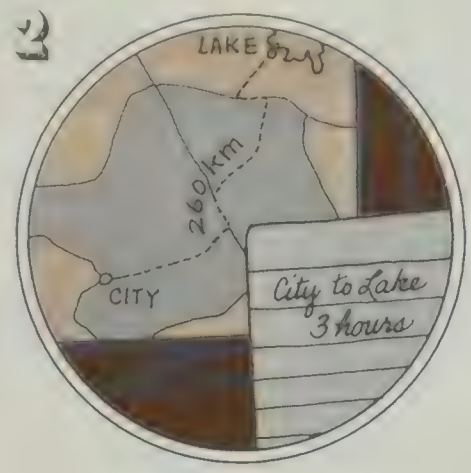

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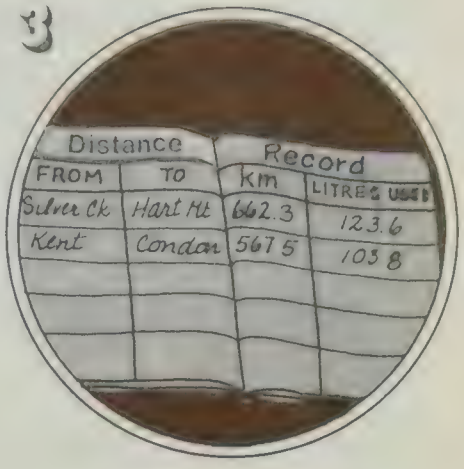

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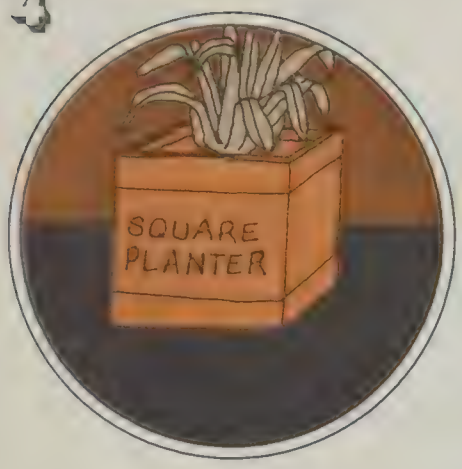

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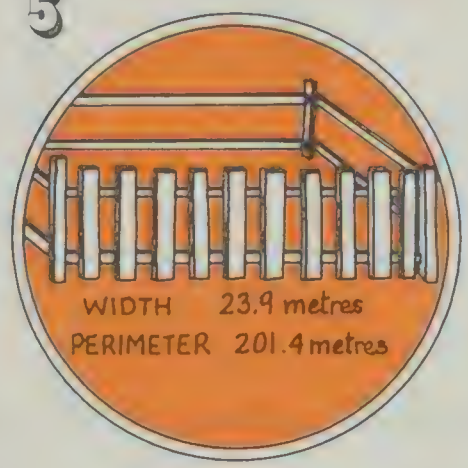

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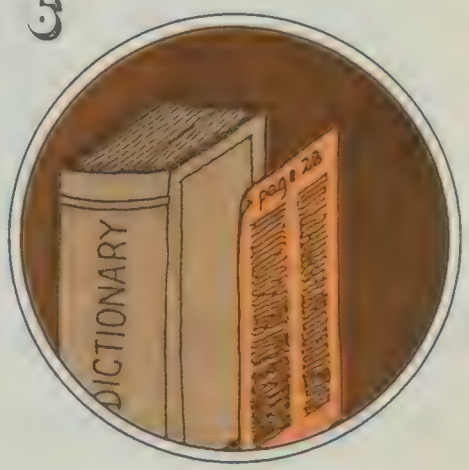

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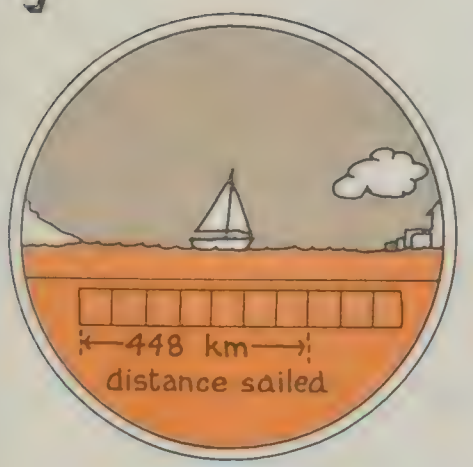


8

9

Investigating the Ideas

When you are given a decimal, it is usually easy to write a fraction that names the same number. But, when you are given a fraction whose denominator is not a power of ten, the problem is more difficult.

Study the two methods below.

Decimal		Fraction
0.17		$\frac{17}{100}$
?		$\frac{3}{8}$

MULTIPLICATION METHOD

$$\frac{3}{8} = \frac{3 \cdot 125}{8 \cdot 125} = \frac{375}{1000}$$

Therefore, $\frac{3}{8} = \frac{375}{1000} = 0.375$

DIVISION METHOD

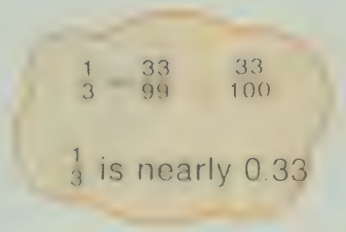
$$\frac{3}{8} = 3 \div 8 = 8\overline{)3.000}$$

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Can you use one of these methods to find decimals which name the same number as $\frac{7}{20}$, $\frac{11}{25}$, and $\frac{13}{16}$?

Discussing the Ideas

1. Explain how to use each of the methods to find a decimal for $\frac{4}{5}$.
2. Explain how to use each of the methods to find a decimal for $\frac{7}{8}$.
3. Which method do you think is easier?
4. Explain how to use each method to find the decimal for $\frac{3}{25}$.
5. This picture shows how Mike thought in order to find an approximate decimal for $\frac{1}{3}$. Do you think Mike has found a decimal that represents the same number as the fraction $\frac{1}{3}$? Explain your answer.



6. Can you use either method shown above to find a decimal that represents the same number as the fraction $\frac{1}{3}$? Explain your answer.

1. Give the number for n. Then give the correct decimal.

A $\frac{23}{50} = \frac{n}{100} = ?$ D $\frac{23}{200} = \frac{n}{1000} = ?$ G $\frac{3}{4} = \frac{n}{100} = ?$

B $\frac{4}{5} = \frac{n}{10} = ?$ E $\frac{9}{500} = \frac{n}{1000} = ?$ H $\frac{17}{125} = \frac{n}{1000} = ?$

C $\frac{7}{25} = \frac{n}{100} = ?$ F $\frac{64}{250} = \frac{n}{1000} = ?$ I $\frac{23}{80} = \frac{n}{10\,000} = ?$

2. Find the quotients. In each problem, continue dividing until the remainder is zero. Then give the correct decimal for the fraction.

A $8\overline{)5}$ $\frac{5}{8} = ?$ C $2\overline{)27}$ $\frac{27}{2} = ?$ E $40\overline{)9}$ $\frac{9}{40} = ?$

B $25\overline{)6}$ $\frac{6}{25} = ?$ D $16\overline{)1}$ $\frac{1}{16} = ?$ F $80\overline{)7}$ $\frac{7}{80} = ?$

3. Write a decimal for each number. Use any method you choose.

A $\frac{3}{5}$	E $\frac{7}{20}$	I $\frac{29}{40}$	M $\frac{7}{200}$	Q $\frac{372}{500}$
B $\frac{1}{4}$	F $\frac{19}{25}$	J $\frac{43}{80}$	N $\frac{9}{400}$	R $\frac{98}{200}$
C $\frac{7}{8}$	G $\frac{27}{50}$	K $\frac{99}{150}$	O $\frac{569}{500}$	S $\frac{120}{150}$
D $\frac{11}{16}$	H $\frac{3}{2}$	L $\frac{147}{160}$	P $\frac{4}{25}$	★ T $\frac{964}{1250}$

4. Study the examples. Then write a decimal that represents a number equal to or approximately equal to the number given. The symbol $\approx$ means "is approximately."

EXAMPLES: A $\frac{37}{99} \approx \frac{37}{100} = 0.37$

B $\frac{56}{300} = \frac{56 \div 3}{300 \div 3} \approx \frac{19}{100} = 0.19$

C $\frac{43}{65} = \frac{4300}{6500} = \frac{4300 \div 65}{6500 \div 65} \approx \frac{66}{100} = 0.66$

A $\frac{17}{50}$	F $\frac{73}{999}$	K $\frac{17}{90}$	P $\frac{4672}{6000}$	U $\frac{79}{101}$
B $\frac{7}{99}$	G $\frac{63}{501}$	L $\frac{412}{2000}$	Q $\frac{72}{13}$	V $\frac{38}{499}$
C $\frac{12}{200}$	H $\frac{41}{300}$	M $\frac{614}{1001}$	R $\frac{1}{13}$	W $\frac{840}{5000}$
D $\frac{23}{600}$	I $\frac{41}{700}$	N $\frac{46}{250}$	S $\frac{27}{75}$	X $\frac{63}{99}$
E $\frac{7}{49}$	J $\frac{11}{30}$	O $\frac{217}{3000}$	T $\frac{14}{37}$	Y $\frac{19}{33}$

★ 5. In each exercise, solve the equation for n. Then find a fraction for x.

A $7 \times 0.42857 = n$
$0.42857 \approx x$

B $19 \times 0.8421 = n$
$0.8421 \approx x$

C $9 \times 0.22222 = n$
$0.22222 \approx x$

More practice, page S-20, Set 35

Investigating the Ideas

The decimal for $\frac{3}{4}$ **terminates.**

$\frac{3}{4} = 0.75$

$$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ 28 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

The decimal for $\frac{6}{11}$ **repeats** endlessly!

$$\begin{array}{r} 0.545\ldots \\ 11\overline{)6.0000\ldots} \\ 55 \\ \hline 50 \\ 44 \\ \hline 60 \\ 55 \\ \hline 50 \\ 44 \\ \hline 6 \\ \vdots \end{array}$$

Can you use division to show which of the fractions have **terminating** or **repeating** decimals?

A. $\frac{1}{4}$ B. $\frac{5}{6}$ C. $\frac{7}{8}$ D. $\frac{4}{7}$

Discussing the Ideas

1. If the decimal for a fraction terminates, what must be the last remainder in the division process?

2. If a fraction has a repeating decimal representation, what can you say about the remainders in the division process after the repeating part begins?

3. A Study the example for finding a decimal for $\frac{1}{3}$. Will the remainder ever be zero?

Dividing tenths	Dividing hundredths	Dividing thousandths
$0.3, R\,1$	$0.33, R\,1$	$0.333, R\,1$
$3\overline{)1.0}$	$\rightarrow\ 3\overline{)1.00}$	$\rightarrow\ 3\overline{)1.000}\ \ldots$

B What kind of decimal names the fraction for $\frac{1}{3}$?

4. The three dots in $0.33\ldots$ indicate that the threes repeat indefinitely. Another way of representing a repeating decimal is to write a bar over the group of digits that repeats.

$\frac{1}{3} = 0.33\ldots = 0.\overline{3}$ $\quad \frac{5}{11} = 0.454545\ldots = 0.\overline{45}$ $\quad \frac{1}{27} = 0.037037\ldots = 0.\overline{037}$

Can you use both the dot and the bar notations to express the repeating decimals that you found in the Investigation?

1. Use division to find the repeating decimal that represents each fractional number.

A $\frac{2}{3}$	C $\frac{7}{11}$	E $\frac{5}{27}$	G $\frac{4}{11}$	I $\frac{7}{15}$	K $\frac{1}{7}$	M $\frac{5}{12}$
B $\frac{1}{6}$	D $\frac{1}{9}$	F $\frac{5}{6}$	H $\frac{2}{37}$	J $\frac{5}{13}$	L $\frac{5}{9}$	★ N $\frac{5}{17}$

2. Round each repeating decimal that you found in Exercise 1 to the nearest hundredth.

3. Use bar notation to represent each repeating decimal.

A 0.232323 . . .	D 0.20672067 . . .	G 0.010101 . . .
B 0.4122222 . . .	E 63.7777 . . .	H 0.434343 . . .
C 0.617617 . . .	F 5.61222222 . . .	I 0.70517051 . . .

4. Use the "three dot" notation to indicate these repeating decimals.

A $0.\overline{7}$	C $7.48\overline{}$	E $0.8\overline{3}$	G $0.\overline{7892}$	I $0.\overline{46}$
B $0.\overline{21}$	D $0.\overline{213}$	F $56.24\overline{9}$	H $3.\overline{0}$	J $8.6\overline{3}$

5. Find a repeating decimal for each fraction.

A $\frac{1}{9}$	B $\frac{1}{99}$	C $\frac{1}{999}$	D $\frac{1}{9999}$	E $\frac{1}{99\,999}$

6. Use the results of Exercise 5 to give a repeating decimal for each fraction. Try to do this without dividing.

A $\frac{7}{9}$	B $\frac{8}{99}$	C $\frac{5}{999}$	D $\frac{2}{9999}$	E $\frac{1000}{99\,999}$

★ 7. Write a fraction that represents the same number as $0.\overline{347}$. Check by dividing.

★ 8. Find the repeating decimal that represents $\frac{1}{81}$. What interesting fact do you notice about this decimal?

think

In the magic square to the right, the sum of each row, column, diagonal, and the four corners is the same.

1. To complete the magic square, use each of the numbers 1 through 16 just once.
2. What is the sum?

1			15
	13		6
7		16	9
14	11		

Investigating the Ideas

Often a combination of a decimal and a fraction called a mixed decimal numeral is written for a fraction. The examples in the table show how you can find several mixed decimal numerals for the fraction $\frac{5}{12}$.

$12\overline{)5.0}$ $\underline{48}$ 2 $\frac{5}{12} = 0.4\frac{1}{6}$	$12\overline{)5.00}$ $\underline{48}$ 20 $\underline{12}$ 8 $\frac{5}{12} = 0.41\frac{2}{3}$	$12\overline{)5.000}$ $\underline{48}$ 20 $\underline{12}$ 80 $\underline{72}$ 8 $\frac{5}{12} = 0.416\frac{2}{3}$
4 and $\frac{1}{6}$ **tenths**	41 and $\frac{2}{3}$ **hundredths**	416 and $\frac{2}{3}$ **thousandths**

Can you give mixed decimal numerals for $\frac{5}{7}$ in **tenths**, **hundredths**, and **thousandths**?

Discussing the Ideas

1. What mixed decimal numeral in tenths does the division problem suggest?

$$13\overline{)8.0} \quad \underline{7.8} \quad 2 \qquad \frac{8}{13} = 0.$$

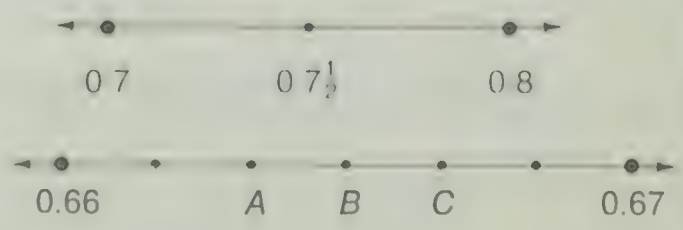
2. How could you find a mixed decimal numeral in hundredths for $\frac{8}{13}$?

3. The example shows how you can find a fraction for a mixed decimal numeral.

   Explain how you could find a fraction for $0.12\frac{1}{2}$.

EXAMPLE:

4. You can think of $0.7\frac{1}{2}$ as a name for the point on the number line that is halfway between the points for 0.7 and 0.8. Can you give mixed decimal numerals for points *A*, *B*, and *C*?



1. Write a mixed decimal numeral for each number name.

A 7 and $\frac{2}{3}$ tenths

B 16 and $\frac{1}{4}$ hundredths

C 37 and $\frac{1}{2}$ thousandths

D 66 and $\frac{2}{3}$ hundredths

E 785 and $\frac{1}{5}$ thousandths

F 83 and $\frac{5}{6}$ hundredths

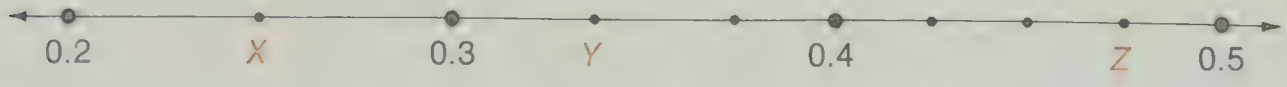
2. Copy and complete the table.

	FRACTION	MIXED DECIMAL NUMERALS		
		tenths	hundredths	thousandths
	$\frac{1}{3}$	$0.3\frac{1}{3}$	$0.33\frac{1}{3}$	$0.333\frac{1}{3}$
A	$\frac{5}{6}$			
B	$\frac{3}{16}$			
C	$\frac{4}{9}$			
D	$\frac{13}{6}$			
E	$\frac{3}{7}$			
F	$\frac{5}{18}$			

3. Write a fraction for each mixed decimal numeral.

A $0.1\frac{1}{4}$ **B** $0.06\frac{1}{2}$ **C** $0.83\frac{1}{3}$ **D** $0.66\frac{2}{3}$ **E** $0.031\frac{3}{4}$ **F** $0.37\frac{1}{2}$

4. Give a mixed decimal numeral for each lettered point on the number line.

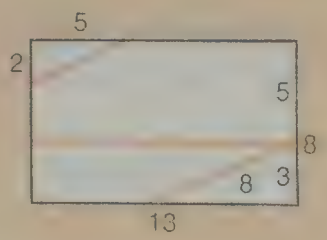


5. Write a fraction for each mixed decimal numeral of Exercise 4.

think

Cut out an 8 by 13-cm rectangle and cut it into pieces as shown in the figure. Reassemble the pieces to form what seems to be a 5 by 21 cm rectangle.

1. What is the area of the 8 by 13-cm rectangle?
2. What is the area of the 5 by 21-cm rectangle?
3. Can you explain where the extra square centimetre is?



More practice, page 5-11, Set 36

scientific notation—DECIMALS

A number other than a power of ten is represented using **scientific notation** when it is written as the product of a **number between 1 and 10 and a power of ten**. In the table, each number in Column A is represented in Column B using scientific notation.

A Number		B Number between 1 and 10	×	Power of ten
864	=	8.64	×	10^2
3785.3	=	3.7853	×	10^3
57	=	5.7	×	10^1
89600	=	8.96	×	10^4

For a power of ten such as 1 000 000, we might write 1×10^6

1. Find the number for *n* in each row of the table. →

Number		Number between 1 and 10	×	Power of ten
A 37	=	*n*	×	10^1
B 467	=	4.67	×	*n*
C 6843	=	*n*	×	10^3
D *n*	=	4.756	×	10^3
E 4800	=	*n*	×	10^3
F 96 000	=	9.6	×	*n*
G *n*	=	5.83726	×	10^2
H 9463.21	=	*n*	×	10^3
I 580 000	=	5.8	×	*n*
J *n*	=	3.7	×	10^6

2. Represent the numbers in scientific notation.
   - A 700
   - B 900
   - C 8000
   - D 70 000
   - E 600 000
   - F 100 000

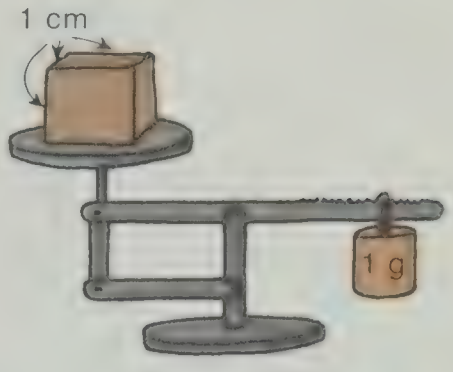
3. Represent the numbers in scientific notation. Each number is a period of time given in seconds.
   - A Hour: 3600
   - B Day: 86400
   - C Month: 2 550 000
   - D Solar year: 31 556 900
   - E Average life of man: 2 000 000 000
   - F Revolution of Pluto about the sun: 7 820 000 000
   - G Half-life of radium 226: 52 400 000 000
   - H One rotation of the Milky Way: 6 000 000 000 000 000

4. Write each numeral in the usual way.
   - A 4.32×10^3
   - B 5.4×10^4
   - C 6.16×10^2
   - D 7.0×10^5
   - E 8.2×10^{10}
   - F 1.53×10^8
   - G 5.9×10^7
   - H 1.02×10^0

More practice, page S-21, Set 37

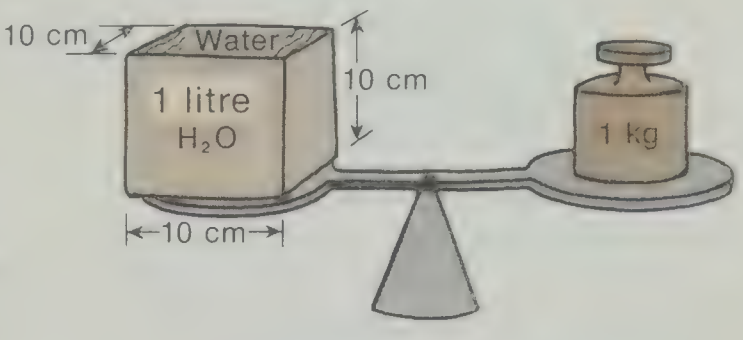
Comparing Units within the Metric System

In the metric system there is an important relationship between some of the units for length (metre), mass (gram), and volume (litre). The problems below deal with this relationship.

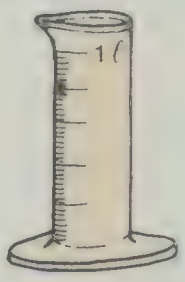


1. The mass of a cube of water that is 1 centimetre on each edge (1 cubic centimetre) is 1 gram. You probably drink about 1800 cubic centimetres of water each day. How many grams of water is this?

2. Use the diagram at the right to answer the questions.
   - **A** How many grams is in a litre of water (H_2O)?
   - **B** How many cubic centimetres are in a litre?
   - **C** One gram of water is what part of a litre of water?

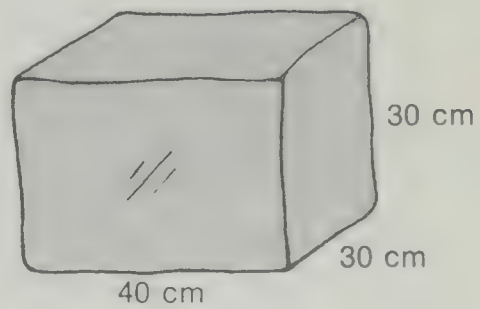


3. A cubic centimetre is 1 millilitre (ml); that is 0.001 litre. What is the mass in grams of 25 millilitres of water?



4. The mass of a given volume of sea water is about 1.03 times that of an equal volume of fresh water. What is the mass in grams of one litre of sea water?

5. The mass of a given volume of gasoline is only 0.68 times that of an equal volume of water.
   - **A** How many grams is one litre of gasoline?
   - **B** How many litres of gasoline would be needed to make one kilogram?



6. Ice has a mass that is about 0.92 times that of an equal volume of water. What is the mass of the block of ice shown?

7. Gold is about 19.3 times as massive as water. How many millilitres (to the nearest tenth) of gold would have the same mass as one litre of water?

REVIEWING THE IDEAS

1. In the decimal **0.347516**, which digit tells how many
   - **A** hundredths
   - **B** tenths
   - **C** millionths
   - **D** ten thousandths
   - **E** thousandths
   - **F** hundred thousandths

2. Write a decimal for $5 + \frac{376}{1000}$.

3. Write a mixed numeral for 72.0936.

4. Write a decimal for
   $7 \cdot 100 + 5 \cdot 10 + 3 \cdot 1 + 2 \cdot \frac{1}{10} + 8 \cdot \frac{1}{100}$

5. Write 26.3745 in expanded notation (exponent form).

6. Give the correct symbol ($<$, $=$, or $>$) for each ▒.
   - **A** 0.9 ▒ 0.900
   - **B** 3.09 ▒ 3.90
   - **C** 6.2370 ▒ 6.0237
   - **D** 7.099 ▒ 7.10

7. Find the sums.
   - **A** $2.6 + 7.1 + 7.39$
   - **B** $3.7 + 59.642 + 463.24 + 0.803$

8. Find the differences.
   - **A** $97.5 - 36.9$
   - **B** $23.7 - 9.341$

9. Find the products.
   - **A** 0.7×4.9
   - **B** 0.03×0.0057
   - **C** 42.8×0.006
   - **D**
     $$\begin{array}{r} 3.59 \\ \times\ 4.3 \\ \hline \end{array}$$

10. Find the products.
    - **A** 10×5.736
    - **B** 100×0.856
    - **C** $10^3 \times 5.428$
    - **D** 0.1×87.54
    - **E** $\frac{1}{10^2} \times 956$
    - **F** 0.001×87.6

11. Round **57.5649** to the nearest
    - **A** whole number
    - **B** tenth
    - **C** hundredth
    - **D** thousandth

12. **A** Estimate the product 7.96×49.843.
    B Find the product in part **A**. How close was your estimate?

13. Find the quotients.
    A $8\overline{)2.776}$ **B** $0.7\overline{)0.526}$ **C** $0.04\overline{)1.9}$

14. Use the information in the chart to answer the following questions.

Day	Kilometres travelled	Litres used	Cents per litre
Sat.	477.6	53.1	11.9
Sun.	256.4	30.5	11.9

    - **A** Find the total number of kilometres travelled.
    - **B** Find the total number of litres used.
    - **C** What was the total cost of the gasoline for the weekend?
    - **D** How many kilometres per litre (to the nearest tenth) did the car average on Saturday?

15. Write a decimal for each number.
    A $\frac{4}{5}$ **B** $\frac{17}{20}$ **C** $\frac{9}{32}$

16. **A** Represent $\frac{9}{11}$ as a repeating decimal.
    B Round the decimal in part **A** to the nearest hundredth.

17. Give a mixed decimal numeral in hundredths for $\frac{4}{7}$.

18. The distance from the Earth to the sun is 149 000 000 kilometres. Express this distance in scientific notation.

19. Give the correct number for each ▒.
    - **A** 1 litre = ▒ millilitres
    - **B** 1 millilitre of water weighs ▒ gram.
    - **C** 10 millimetres = ▒ centimetre
    - **D** 1 litre of water weighs ▒ kilogram.

TEST YOURSELF

1. A Write a fraction for 0.217.
   B Write a decimal for $5\frac{37}{100}$.

2. Give the correct symbol (<, =, or >) for each ▒.
   A 3.732 ▒ 3.722 B 0.07 ▒ 0.01

3. Find the sum 2.78 + 16.5 + 4.295

4. Find the differences.
   A 8.62 − 1.95 B 3.1 − 1.92

5. Which is the best estimate for the product 38.705 × 19.1?
   A 550 B 650 C 750

6. Find the products.
   A 0.02×1.6
   B 100×27.4
   C 0.001×37429
   D
   $$\begin{array}{r} 42.3 \\ \times 0.29 \\ \hline \end{array}$$

7. Find the quotients.
   A $7\overline{)2.24}$ B $0.54\overline{)0.4212}$

8. Find a repeating decimal for $\frac{3}{11}$.

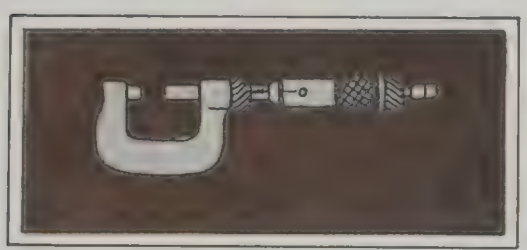
9. Express $\frac{9}{16}$ as a mixed decimal numeral in hundredths.

10. The equator of the Earth is about 40000 kilometres long. This is about 3.14 times as long as the diameter of the Earth.

   A What is the diameter of the Earth? Round your answer to the nearest hundred kilometres.
   B Express your rounded answer in part A in scientific notation.

RESEARCH PROJECTS

A A micrometer is an instrument used measuring small dimensions. Find a micrometer and see if you can figure out how to use it. Then use it to measure the following items:
   A The thickness of a pencil.
   B The thickness of a sheet of notebook paper.
   C The thickness of a hair.



B Learn how to multiply and divide with decimals using a slide rule. Most slide rules come with a pamphlet explaining their use.

C Find out some of the history behind the metric system of measurement. Prepare a display of various metric units. Sufficiently emphasize how the prefixes of the metric units are used.

D Figure out how "decimals" are used in bases other than ten and be able to explain their use to your class.

For example, $3.2_{(6)} = 3\frac{1}{3}_{(ten)}$

$1.23_{(5)} = 1.52_{(ten)}$

THE ② OPERATION

You are familiar with the four operations, **addition, multiplication, subtraction,** and **division** as well as the **basic principles** for addition and multiplication. Just for fun, let's invent a new operation called "squareaddition." Study the examples to figure out how to "squareadd." We might call the answer to a squareaddition problem a "squm."

Examples
$2 \overset{2}{+} 1 = 5$
$2 \overset{2}{+} 4 = 20$
$10 \overset{2}{+} 10 = 200$

1. Find the "squms."

A $3 \overset{2}{+} 1$ **C** $7 \overset{2}{+} 7$ **E** $1 \overset{2}{+} 1$ **G** $2 \overset{2}{+} 6$

B $4 \overset{2}{+} 5$ **D** $15 \overset{2}{+} 25$ **F** $9 \overset{2}{+} 20$ **H** $7 \overset{2}{+} 10$

2. A Does $2 \overset{2}{+} 5 = 5 \overset{2}{+} 2$?

B Does $(2 \overset{2}{+} 1) \overset{2}{+} 3 = 2 \overset{2}{+} (1 \overset{2}{+} 3)$?

C Does $8 \overset{2}{+} 0 = 8$?

D Do you think that the commutative, associative, and zero principles hold true for "squareaddition"?

3. A Find $3 \cdot (2 \overset{2}{+} 4)$.

B Find $(3 \cdot 2) \overset{2}{+} (3 \cdot 4)$.

C Does $3 \cdot (2 \overset{2}{+} 4) = (3 \cdot 2) \overset{2}{+} (3 \cdot 4)$?

D Does the distributive principle seem to hold true with respect to $\overset{2}{+}$?

4. Solve the equations.

A $3 \overset{2}{+} n = 34$ **B** $x \overset{2}{+} 4 = 17$ **C** $2 \cdot y \overset{2}{+} 3 = 109$

5. Invent a new operation of your own. See if a classmate can figure it out. Check to see if the basic principles hold true.

MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

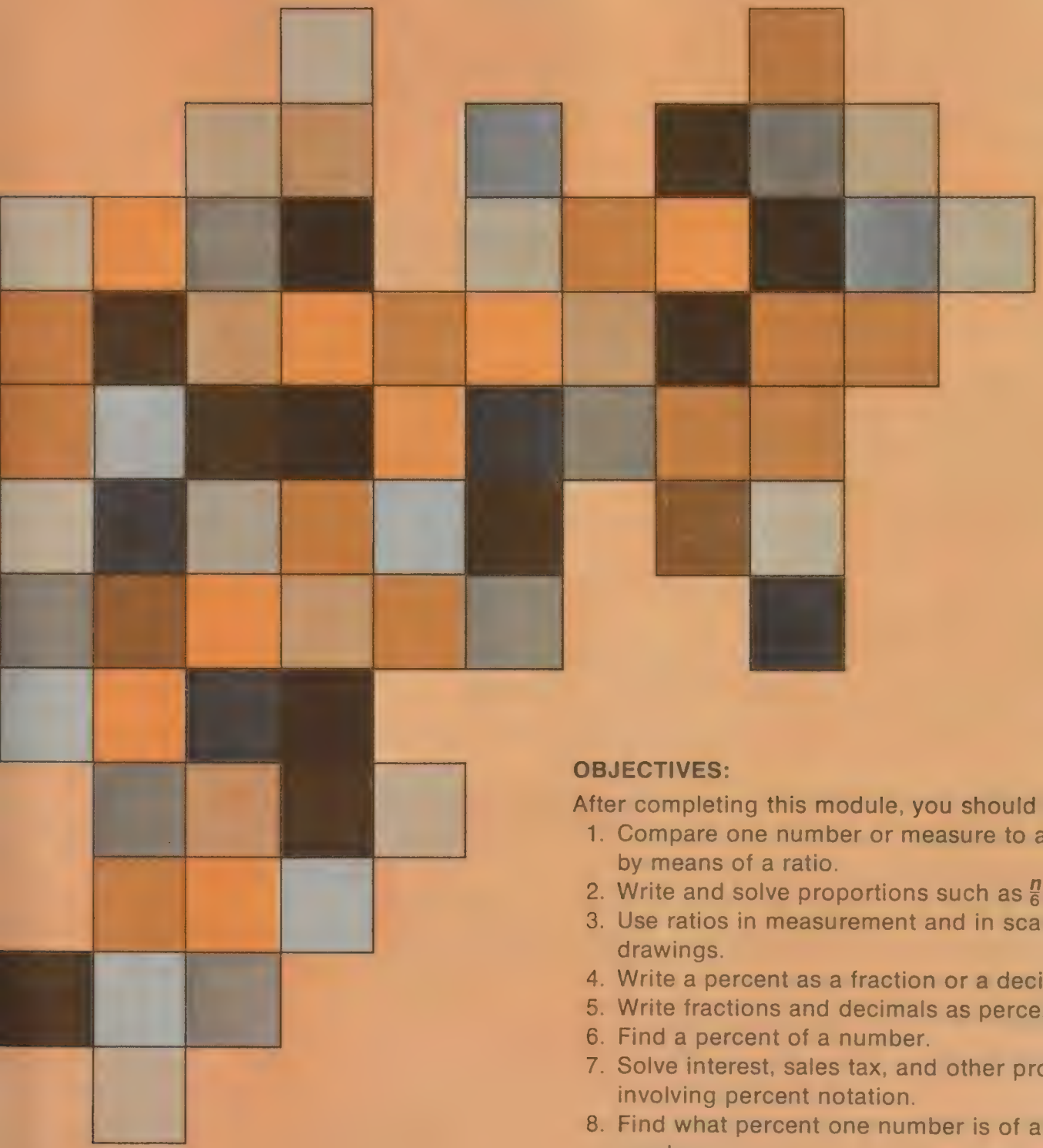
1. A $\frac{217}{1000}$; **B** 5.37 **2. A** >; **B** < **3.** 23.575 **4. A** 6.67; **B** 1.18

5. C [illegible] **6. A** 0.032 **7. A** 0.30

B 0.78 **8.** 0.27 **9.** $0.56\frac{1}{4}$ **10. A** 12700 km; **B** 1.27×10^4

UNIT B: *The Arithmetic of Fractional Numbers.*

MODULE 5: *Ratio, Proportion, and Percent*



OBJECTIVES:

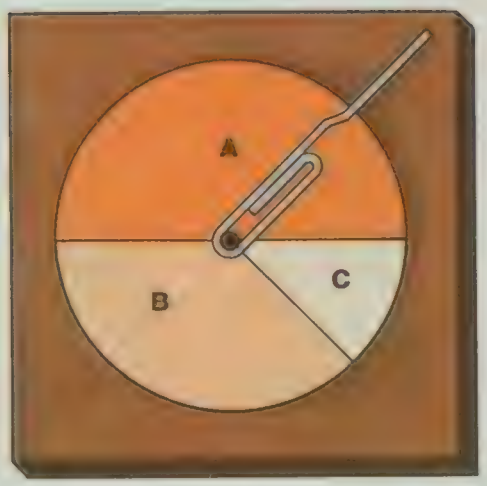
After completing this module, you should be able to:

1. Compare one number or measure to another by means of a ratio.
2. Write and solve proportions such as $\frac{n}{6} = \frac{20}{24}$.
3. Use ratios in measurement and in scale drawings.
4. Write a percent as a fraction or a decimal.
5. Write fractions and decimals as percents.
6. Find a percent of a number.
7. Solve interest, sales tax, and other problems involving percent notation.
8. Find what percent one number is of another number.
9. Make estimates using percent.

Investigating the Ideas

Make a spinner like the one shown at the right.
Region A covers $\frac{1}{2}$ of the dial, region B covers $\frac{1}{3}$, and region C covers $\frac{1}{6}$.

Spin the pointer 60 times and make a tally of the results.



Can you write fractions that compare the number of times the pointer stopped in each region to the total number of spins?

Region	Tally
A	
B	
C	

Discussing the Ideas

1. Did the pointer stop in region A for about $\frac{1}{2}$ of the number of trials?

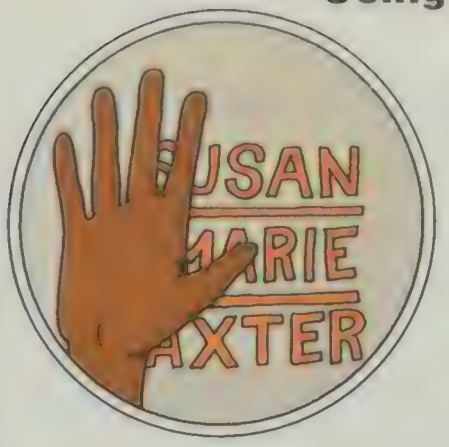
2. Linda made this tally sheet for the experiment.

Region	Tally
A	
B	
C	

   A. How many times did the pointer stop in region A?
   B. What **fractional** part of the trials did the pointer stop in region A?
   C. Repeat the question for regions B and C.

3. In order to compare two numbers, you can often use a **ratio**. For example, Linda could have said, "The ratio of the number of times the pointer stopped in region A to the total number of trials was **32 to 60**." You can write the ratio "32 to 60" as 32:60 or $\frac{32}{60}$.
   What other ratios could Linda have written for her results in the Investigation?

Find the following ratios.



1. The number of boys to the number of girls in your class today.

2. The number of people in your class who wear glasses to those who do not wear glasses.

3. The number of windows in your mathematics classroom to the number of doors.

4. The number of letters in your first name to the number of letters in your last name.

5. The number of letters in your middle name to the number of letters in your first name.

6. The number of eyes you have to the numbers of fingers you have.

7. The length of a millimetre to the length of a centimetre.

8. The length of a centimetre to the length of a metre.

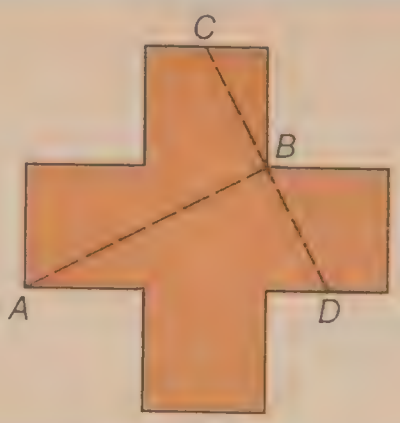
9. The number of vowels in the alphabet to all the letters in the alphabet.

10. The number of prime numbers less than 50 to the number of odd numbers less than 50.

11. The number of provinces in Canada whose name ends in the letter "a" to the number of provinces.

Think

Trace the cross and the dotted lines. Cut out your tracing and then cut along the dotted lines. Now fit four pieces together to make a square.



Investigating the Ideas

Study the team records in the chart.

Can you decide which team has the best record?

Team	Games won	Games played	Win ratio
EXPOS	18	24	$\frac{18}{24}$
CUBS	15	20	$\frac{15}{20}$
PIRATES	10	16	$\frac{10}{16}$
CARDINALS	16	20	$\frac{16}{20}$

Discussing the Ideas

1. Explain how you decided which team had the best record.

2. The Expos won 18 out of 24 games or $\frac{18}{24}$ of their games.
   - A What is the lowest-terms fraction for $\frac{18}{24}$?
   - B Do you think it is correct to say, "The Expos won 3 out of every 4 games that they played?"

3. The ratio $\frac{18}{24}$ in **simplest terms** is $\frac{3}{4}$. What are the win ratios for the Cubs and the Pirates in simplest terms?

4. The ratios 18:24 and 3:4 are **equal ratios** because $\frac{18}{24} = \frac{3}{4}$.

   A statement that two ratios are equal such as $\frac{18}{24} = \frac{3}{4}$ is called a **proportion**.
   - A Could the ratios $\frac{18}{24}$ and $\frac{15}{20}$ form a proportion?
   - B What does this tell you about the Expos and the Cubs?
   - C Could the ratios $\frac{10}{16}$ and $\frac{15}{20}$ form a proportion?

5. In each proportion, we call two of the numbers **means** and the other two extremes
   - A Which numbers are the means and which numbers are the extremes in the two proportions at the right?
   - B Complete this statement:
     In a proportion, the product of the means is equal to the product of the ? .

Means (middle numbers)

3 : 6 = 4 : 8

Extremes (end numb

$\frac{2}{3} = \frac{10}{15}$

Means Extremes

1. Express each ratio in simplest terms.

   A $\frac{6}{18}$ **B** 8 to 10 **C** 25:15 **D** $\frac{15}{25}$ **E** 36 to 20

2. In each proportion, tell which pair of numbers is the means and which is the extremes.

   A $\frac{2}{5} = \frac{6}{15}$ **B** $1:4 = 7:28$ **C** $\frac{5}{3} = \frac{20}{12}$ **D** $5:8 = 200:320$

3. For each proportion, show that the product of the means is equal to the product of the extremes.

   A $2:3 = 4:6$ **B** $1:2 = 12:24$ **C** 5 to 3 = 15 to 9 **D** 10 to 8 = 5 to 4

4. Write a proportion using the four numbers in the equation $9 \cdot 4 = 3 \cdot 12$. Can you find more than one proportion?

5. A ballplayer's ratio of the number of hits to the number of times at the bat is 4 to 14.

   A Give two different symbols for the ratio which compares the number of hits with the number of times at bat.

   B Write a proportion using the ratios in part **A**.

6. Out of a total 42 cars in a parking lot, 15 were sports cars.

   A What was the ratio of the number of sports cars to all the other cars?

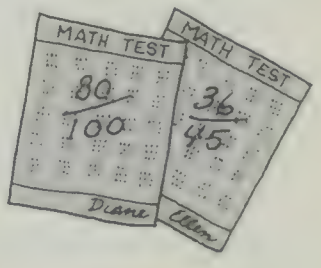
   B What is this ratio in simplest terms?

   C Write a proportion using the ratios of parts **A** and **B**.

7. On a test, Diane said that she got $\frac{36}{45}$ of the problems correct. Ellen said that she got $\frac{80}{100}$ of the problems correct.

   A Express each ratio in simplest terms.

   B Can the two ratios form a proportion?



think

A date such as February 12, 1924, can be abbreviated as 2/12/24. This is a **product date** because $2 \cdot 12 = 24$. March 24, 1972, was also a product date because $3 \cdot 24 = 72$.

1. Which dates in the year 1980 are product dates?

2. Are the dates of any days of the present year product dates?

Solving Problems with Proportions

Investigating the Ideas

Can you find the solution to this problem?

In Jamestown Junior High School the ratio of girls to boys in the seventh grade is 3 to 5. If there are 80 boys in the seventh grade, how many girls are in the seventh grade?

Discussing the Ideas

1. Explain your solution to the problem in the Investigation.

2. You can write a proportion to help you solve the problem. Explain each step in the example, then give the number for n.

Let n represent the number of girls in the seventh grade.

$$\frac{3}{5} = \frac{n}{80}$$

$$5 \cdot n = 3 \cdot 80$$

$$5 \cdot n = 240$$

$$n =$$

3. What is the total number of students in the seventh grade?

4. Explain how you would solve this proportion to find the number for n.

$$\frac{n}{4} = \frac{20}{16}$$

5. Read the problem below.

   At a picnic, the ratio of adults to children was 3 to 5. If there were 24 adults, how many children were there?

   A If you let n = number of children what ratio compares the number of adults to the number of children?

   B Which proportion is correct for the problem?

   A $\frac{3}{5} = \frac{n}{24}$ **B** $\frac{3}{5} = \frac{24}{n}$

   C Solve the correct proportion for n.

1. Solve the following proportions for n.

A $\frac{n}{8} = \frac{12}{16}$ **B** $\frac{n}{25} = \frac{9}{15}$ **C** $2:3 = n:15$ **D** $5:3 = 10:n$ **E** $\frac{1}{5} = \frac{n}{100}$ **F** $\frac{3}{n} = \frac{75}{100}$ **G** $\frac{2}{n} = \frac{50}{75}$ **H** $\frac{3}{1} = \frac{n}{47}$

In Exercises 2 through 9, write a proportion for each problem and then solve it.

2. For a class party, Jeanne bought 10 doughnuts for every 3 students. How many doughnuts did she buy if there are 24 students in the class?

3. The ratio of field goals scored in a basketball game to field goals attempted was 2:5. Eighty field goals were attempted. How many field goals were scored?

4. A car can travel 7 kilometres on one litre of gasoline. How many litres would be needed to travel 266 kilometres?

5. The ratio of kilometres driven to litres of gasoline used was 27 to 4. How far was it possible to drive on 72 litres of gasoline?

6. The ratio of distance in centimetres shown on a map to the actual distance in kilometres is 1 to 50. If the map distance between two cities is 9 centimetres, how many kilometres is it from one city to the other?

7. A baseball player has made 2 hits for every 7 times at bat. If the player has been at bat 210 times, how many hits has he made?

8. In a group of students, there are 7 girls for every 5 boys. Determine **x** if

A there are 14 girls for every **x** boys
B there are 35 girls for every **x** boys
C there are **x** girls for every 15 boys
D the ratio of girls to boys is **x** to 50
E the ratio of girls to boys is 42 to **x**
F the ratio of boys to girls is **x** to 119

★ 9. The ratio 5:3 compares the population of City A with the population of city B. If a total of 8000 persons live in the two cities, how many live in each city?

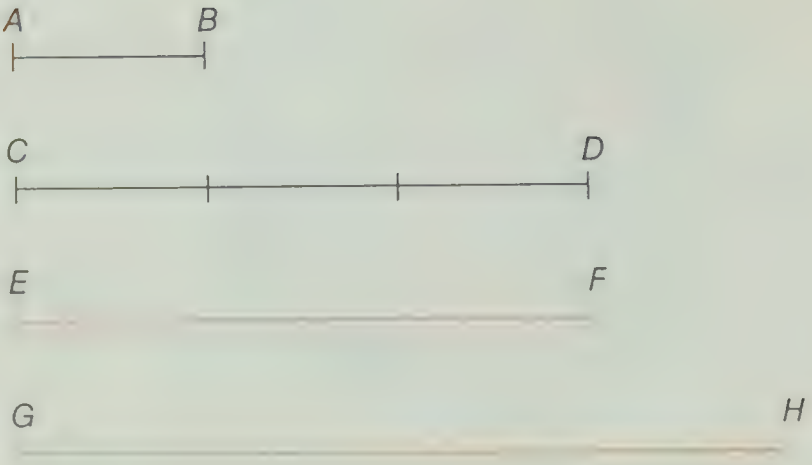
City *A*

City *B*

More practice, page S-22, Set 38

Investigating the Ideas

The ratio of the length of $\overline{AB}$ to the length of $\overline{CD}$ is 1:3.



Can you find the ratio of the length of $\overline{EF}$ to the length of $\overline{GH}$?

Discussing the Ideas

1. What unit did you choose to measure the lengths $\overline{EF}$ and $\overline{GH}$? Do you think that the ratio of the lengths of the segments depends upon the unit that you chose?

2. When you use a ratio to compare two measures, it is helpful to express both measures with the same unit. Study part **A**. Use the relationships between the different units to give the following ratios in the simplest-terms

A 1 m = 100 cm $\rightarrow \frac{1\text{ m}}{1\text{ cm}} = \frac{100}{1}$

B 1000 g = 1 kg $\rightarrow \frac{1\text{ kg}}{1\text{ g}} =$ [illegible]

C 10 mm = 1 cm $\rightarrow \frac{1\text{ mm}}{1\text{ cm}} =$ [illegible]

D 1 min = 60 s $\rightarrow \frac{1\text{ s}}{1\text{ min}} =$ [illegible]

E 1 h = 60 min $\rightarrow \frac{1\text{ h}}{1\text{ min}} =$ [illegible]

F 1 day = 24 h $\rightarrow \frac{1\text{ h}}{1\text{ day}} =$ [illegible]

3. To find the simplest-terms ratio for "30 minutes to 2 hours." you can write

$$\frac{30\text{ min}}{2\text{ h}} = \frac{30\text{ min}}{2 \cdot 60\text{ min}} = \frac{30}{120} = \frac{1}{4}$$

Explain how to find the simplest-terms ratio for each of the following:

A 5 cm to 20 mm

B 24 mm to 4 cm

C 700 g to 1.2 kg

D 40 s to 3 min

E 250 mg to 1 g

F 12 days to 2 weeks

1. Give the appropriate value for **x**. Then give the ratio in simplest terms.

A $\frac{3\text{ m}}{50\text{ cm}} = \frac{3 \cdot x}{50} = \frac{\blacksquare}{\blacksquare}$

B $\frac{3\text{ cm}}{7\text{ mm}} = \frac{3 \cdot x}{7} = \frac{\blacksquare}{\blacksquare}$

C $\frac{7\text{ km}}{850\text{ m}} = \frac{7 \cdot x}{850} = \frac{\blacksquare}{\blacksquare}$

D $\frac{200\text{ m}}{4\text{ km}} = \frac{200}{4 \cdot x} = \frac{\blacksquare}{\blacksquare}$

E $\frac{85\text{ mm}}{15\text{ cm}} = \frac{85}{15 \cdot x} = \frac{\blacksquare}{\blacksquare}$

F $\frac{420\text{ cm}}{8\text{ m}} = \frac{420}{8 \cdot x} = \frac{\blacksquare}{\blacksquare}$

2. In each part, give the indicated ratios in simplest terms.

A 5 cm to 1 m

B 2 m to 10 cm

C 1 mm to 1 cm

D 5 mm to 5 cm

E 6 cm to 4 m

F 20 cm to 4 m

G 2 m to 50 cm

H 2 m 45 cm to 50 cm

I 1 km 420 m to 200 m

3. Express each measure with the same unit, then give the ratio in simplest terms.

A $\frac{3\text{ min}}{40\text{ s}}$

B $\frac{3\text{ h}}{20\text{ s}}$

C $\frac{3\text{ day}}{75\text{ min}}$

D $\frac{8\text{ mm}}{1\text{ cm}}$

E $\frac{400\text{ m}}{2\text{ km}}$

F $\frac{500\text{ ml}}{4\ \ell}$

G $\frac{1\text{ mg}}{1\text{ g}}$

H $\frac{100\text{ g}}{1\text{ kg}}$

I $\frac{2500\text{ g}}{5\text{ kg}}$

J $\frac{3\text{ day}}{8\text{ wk}}$

K $\frac{2\text{ cm}}{11\text{ mm}}$

L $\frac{44\text{ min}}{3\text{ h}}$

4. Give the indicated ratios in simplest terms.

A $\frac{4\text{ cm }8\text{ mm}}{7\text{ cm }2\text{ mm}}$

B $\frac{1\text{ h }20\text{ min}}{3\text{ h }40\text{ min}}$

C $\frac{3\text{ m }20\text{ cm}}{5\text{ m}}$

D $\frac{1\text{ wk }3\text{ days}}{4\text{ wk }2\text{ days}}$

E $\frac{1\text{ m }23\text{ cm }4\text{ mm}}{4\text{ m }32\text{ cm }1\text{ mm}}$

F $\frac{1\text{ km }234\text{ m }56\text{ cm }7\text{ mm}}{7\text{ km }654\text{ m }32\text{ cm }1\text{ mm}}$

think

There is a shortcut for squaring numbers which are multiples of 5. First find the number for n in each example at the right. Then, without using pencil or paper, determine each product below.

$15 \cdot 15 = 10 \cdot 20 + n$

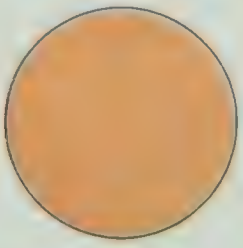
$25 \cdot 25 = 20 \cdot 30 + n$

$55 \cdot 55 = 50 \cdot 60 + n$

1. $65 \cdot 65$ **2.** $75 \cdot 75$ **3.** $85 \cdot 85$ **4.** $95 \cdot 95$

Investigating the Ideas

The figure below shows a **scale drawing** of a certain kind of ball.



Scale: $\frac{1}{10}$ actual size

Kind of Ball	Approximate diameter
Golf ball	4.3 cm
Baseball	7.4 cm
Softball	10.2 cm
Volley ball	21.0 cm
Basketball	24.3 cm

Can you use your centimetre ruler and the information in the table to decide which kind of ball the scale drawing represents?

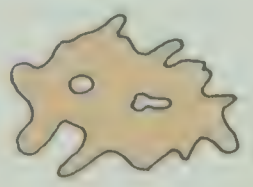
Discussing the Ideas

1. Explain how you decided which ball the scale drawing represented?

2. The **scale** for the drawing of the ball gives the ratio of the diameter of the drawing to the actual diameter of the ball.
   - A What is this ratio?
   - B The diameter of the actual ball is how many times that in the drawing?

3. A scale on a map reads 1 cm = 5 km. If the distance between two points on the map measured 3.8 centimetres. how can you find the actual distance between the two points? What is the distance in kilometres?

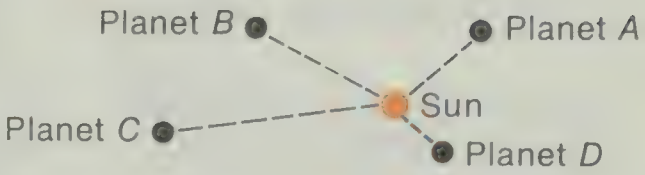
4. This is a drawing of a microscopic creature called an amoeba.
   - A How many times larger is the drawing than an actual amoeba?
   - B The drawing of the amoeba is about 2.5 cm long. What is the actual size of an amoeba?

Amoeba 

Scale: 1 cm = $\frac{1}{10}$ mm

1. The table gives distances from some of the planets to the sun.

Planet	Distance (km)
Mercury	58 000 000
Venus	108 000 000
Earth	150 000 000
Mars	228 000 000
Jupiter	777 000 000
Saturn	1 426 000 000



Scale: 1 cm = 100 000 000 km

Tell which planet in the table is represented by

A Planet *A* B Planet *B* C Planet *C* D Planet *D*

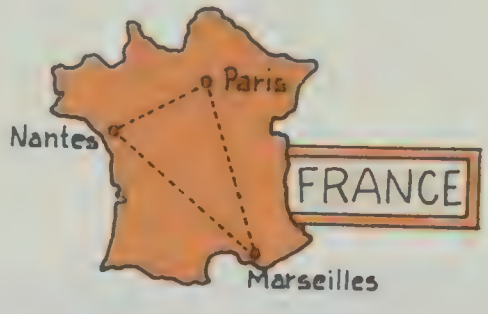
2. If the planet Jupiter were shown on the scale drawing in Exercise 1, about how many centimetres away from the point for the sun would it be?

3. The map at the right is a scale drawing of France.

A What is the distance from Paris to Nantes?

B What is the distance from Nantes to Marseilles?

C What is the distance from Paris to Marseilles?



Scale: 1 cm = 320 km

4. A scale drawing of a whale as it might appear in a dictionary is shown at the right.

A How many millimetres long is the drawing?

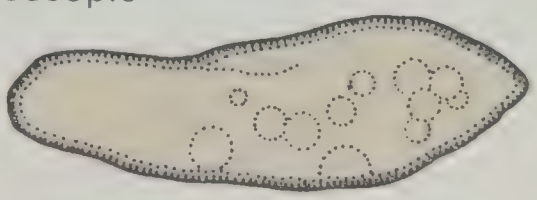
B Estimate the length of the whale in metres.

$\frac{1}{1000}$

5. The drawing at the right shows a microscopic creature called a paramecium.

A How many millimetres long is the drawing?

B Find the actual length of the paramecium to the nearest tenth of a millimetre.



Scale: 250 times actual size

6. Measure the length and width of your classroom. Choose a scale and make a scale drawing of the room.

The Meaning of Percent

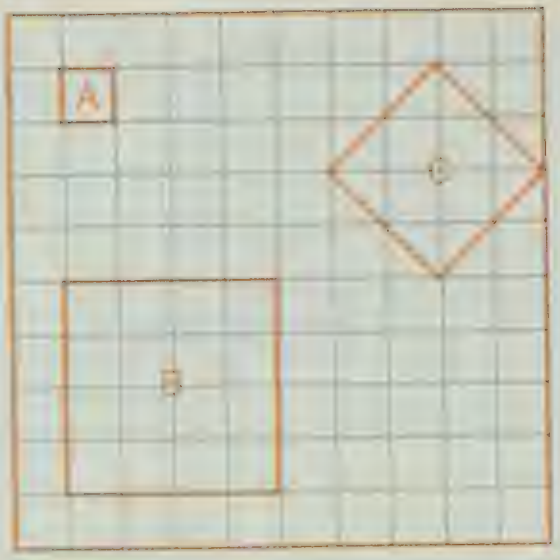
Investigating the Ideas

Square region A covers $\frac{1}{100}$ of the large square region.

Square region B covers $\frac{16}{100}$ of the large square region.

What part of the large square region does region C cover?

Outline a 10 by 10-unit square on graph paper.



Can you draw square regions that will cover $\frac{49}{100}$ and $\frac{50}{100}$ of the large square region?

Discussing the Ideas

1. What is the size of some other square regions that you could draw inside the 10 by 10 square region?

2. Fractions with denominators of 100 are easily expressed in terms of **percents**. For example, instead of saying

   "Square region A covers $\frac{1}{100}$ of the large square region."

   You could say:

   "Square region A covers 1 percent of the large square region."

   You can write: "1 percent" as "1%". 1% means $\frac{1}{100}$ or 0.01.

   What percent of the large square do square regions B and C cover?

3. You can think of 6% as meaning "6 out of 100" or $\frac{6}{100}$ or 0.06.
   - **A** How can you think of 41%?
   - **B** What fraction means the same as 41%?

4. Explain how to find a fraction for each percent.

   A 8% **B** 27% **C** 2% **D** 90% **E** 50% **F** 40%

5. Since $0.23 = \frac{23}{100}$, what percent can you write for 0.23?

1. Write a percent for each fraction. EXAMPLE: $\frac{50}{100} = 50\%$

A $\frac{17}{100}$	**C** $\frac{14}{100}$	**E** $\frac{10}{100}$	**G** $\frac{1}{100}$	**I** $\frac{66}{100}$	**K** $\frac{100}{100}$
B $\frac{29}{100}$	**D** $\frac{11}{100}$	**F** $\frac{9}{100}$	**H** $\frac{97}{100}$	**J** $\frac{75}{100}$	**L** $\frac{125}{100}$

2. Give a decimal for each percent. EXAMPLE: 35% = 0.35

A 23%	**C** 25%	**E** 12%	**G** 9%	**I** 2%	**K** 95%
B 94%	**D** 20%	**F** 10%	**H** 3%	**J** 1%	**L** 102%

3. Give a percent for each decimal. EXAMPLE: [illegible]

A 0.27	**C** 0.66	**E** 0.11	**G** 0.04	**I** 0.40	**K** 1.00
B 0.35	**D** 0.60	**F** 0.08	**H** 0.01	**J** 0.3	**L** 1.16

4. Write an equivalent fraction with a denominator of 100. Then give the percent for the fraction.

EXAMPLE: $\frac{1}{4} = \frac{25}{100} = 25\%$

A $\frac{1}{2}$	**C** $\frac{1}{5}$	**E** $\frac{1}{25}$	**G** $\frac{3}{10}$	**I** $\frac{7}{20}$	**K** $\frac{9}{10}$
B $\frac{1}{10}$	**D** $\frac{1}{20}$	**F** $\frac{3}{4}$	**H** $\frac{4}{25}$	**J** $\frac{2}{5}$	**L** $\frac{11}{50}$

5. Give the number for each ▒ in Examples B, C, and D in the table.

	Statement	Ratio	Fraction (hundredths)	Decimal	Percent
A	3 of 25 boys are on the football team.	3:25	$\frac{12}{100}$	0.12	12%
B	There are 3 boys for every 5 students.	3:5	$\frac{60}{100}$	0.60	▒%
C	I saved $7 out of $20.	7:20	$\frac{▒}{100}$	0.▒	▒%
D	Mike made 5 free throws in 5 tries.	5:5	$\frac{▒}{100}$	0.▒	▒%

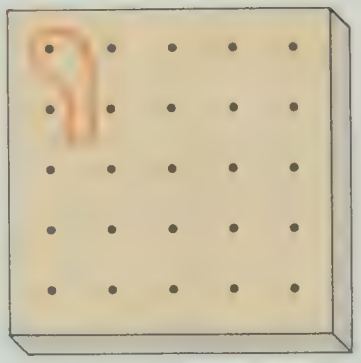
think

The ratio of boys to girls in a group of children was 3 to 5. Then 24 girls left the group and 24 more boys joined it. The ratio of boys to girls became 5 to 3. How many boys and girls were in the original group?

More practice, page S-23, Set 39

Investigating the Ideas

Use a geoboard like this one for the Investigation.



Can you place a rubber band around 16% of the number of nails on the geoboard?

Discussing the Ideas

1. How did you find the number of nails to enclose on the geoboard?

2. Study the method that Jim used to find 16% of the number of nails on the geoboard.

   Use Jim's method to find 24% of the number of nails on the geoboard.

JIM

16% of 25 means 16% × 25.

16% = 0.16

$$0.16 \times 25 \longrightarrow \begin{array}{r} 25 \\ \times 0.16 \\ \hline 150 \\ 25 \\ \hline 4.00 \end{array}$$

3. Could you enclose 10% of the nails on the geoboard with a rubber band? Explain your answer.

4. Joan had $3.00. She spent 40% of her money.
   - A How can you find how much money she spent?
   - B How can you find how much money she had left?

5. A sports coat was originally priced at $60. During a sale, it was marked "25% off."
   - A What does "25% off" mean?
   - B How would you find how much you would have to pay for the coat during the sale?

1. Find the percent of each number.

A 20% of 100	D 50% of 124	G 1% of 100
B 35% of 100	E 10% of 850	H 3% of 100
C 25% of 40	F 23% of 200	I 5% of 80

2. Find the products. Replace each percent by a decimal before multiplying.

A 25% × 18	D 35% × 244	G 12% × 82.3
B 52% × 278	E 78% × 549	H 45% × 2.36
C 80% × 16	F 1% × 1569	I 2% × 139

3. Find the percent of each amount of money. Round your answer to the nearest cent.

EXAMPLE:
Find 15% of $27.98

$$\begin{array}{r} \$27.98 \\ \times 0.15 \\ \hline 13990 \\ 2798 \\ \hline \$4.1970 \text{ or } \$4.20 \end{array}$$

A 74% of $1000	F 24% of $1250
B 33% of $75	G 25% of $36.84
C 40% of $89.95	H 11% of $1.95
D 10% of $96.47	I 8% of $320.49
E 1% of $234.19	J 99% of $67.95

Solve each problem.

4. A winter coat was originally priced at $75. During a sale it was marked "30% off."
   A How much less is the sale price?
   B For what amount will it be sold?

5. Margaret earned $785 one month. 22% of her earnings were deducted for taxes. How much money was deducted for taxes?

6. Between 1960 and 1970, the population of a certain city increased 20%. If the 1960 population was 60 000, what was the 1970 population?

7. A salesman sold a car for $3400. He earned a commission of 8%. How much money did he earn for this sale?

8. A suit of clothes originally priced at $100 was reduced by 30%. Later the suit was reduced by 20% of the reduced price. What was the final price of the suit?

9. After spending 60% of her money for a sweater, Irene had $8.00 left. How much did she have originally?

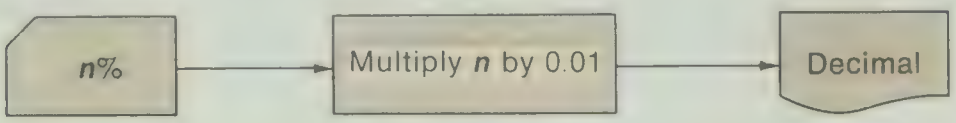
Investigating the Ideas

In problems using percents, you may often need to write a decimal for a percent.

Since percent means "per hundred," you can think of

$n\%$ as meaning $n \cdot \frac{1}{100} = \frac{n}{100}$ or n .01

The flow chart below shows the steps for writing a decimal for a percent.



27% 27 · 0.27

Can you use the flow chart to help you write a decimal for each percent below?

A 38% **B** 3% **C** 12.5% **D** 175% **E** 0.8%

Discussing the Ideas

1. Can you describe a rule for finding a decimal for a percent that is suggested by the flow chart?

2. **A** Use the flow chart to find the decimal for 200%.
   B Jan said, "I earned 200% of my last week's earnings." What do you think Jan meant?
   - **A** 200 times her last week's earnings.
   - **B** 20 times her last week's earnings.
   - **C** 2 times her last week's earnings.

3. Explain each step for finding a decimal for $\frac{1}{2}\%$. Give the decimal for the screen.

$\frac{1}{2}\% = 0.5\% = 0.5 \cdot 0.01 =$ ▒▒▒

4. Study Maria's method for finding $3\frac{1}{2}\%$ of 40. Explain how you would find $1\frac{1}{2}\%$ of 64.

$3\frac{1}{2}\% = 3.5\% = 0.035$

$$\begin{array}{r} 40 \\ \times 0.035 \\ \hline 200 \\ 120 \\ \hline 1.400 \end{array}$$

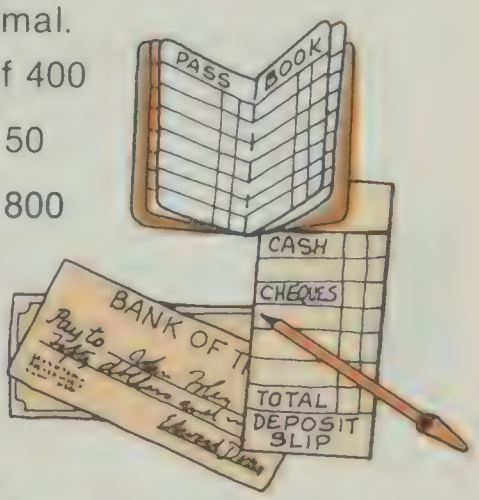
Maria

1. Write each percent as a decimal.

A 11%	E 123%	I 150%	M .03%	Q 34.2%	U $87\frac{1}{2}\%$
B 24%	F 1%	J 400%	N 12.35%	R 3.42%	V $2\frac{1}{2}\%$
C 81%	G 7%	K 250%	O 42.6%	S 0.5%	W $12\frac{1}{2}\%$
D 101%	H 4%	L 1000%	P 12.5%	T 0.1%	★X $3\frac{1}{4}\%$

2. Find the products. First replace each percent by a decimal.

A 35% of 84	D 1000% of 25	G $37\frac{1}{2}\%$ of 400
B 300% of 55	E 240% of 6.5	H $8\frac{1}{2}\%$ of 50
C 125% of 48	F 6% of 45.3	I $6\frac{1}{2}\%$ of 800



3. Alvin had 64 customers on his paper route. He got several new customers and now has 125% as many as before. How many customers does he have now?

4. Marge earned $12 one week from babysitting. The next week she only earned $62\frac{1}{2}\%$ as much. How much money did she earn the second week?

5. Mrs. Johnson owed a bill of $54. She was charged $1\frac{1}{2}\%$ of this amount for a service charge.
   - A How much did she have to pay for the service charge?
   - B What was the total amount she had to pay?

CHARGE-IT-ALL Bank card statement	
Charges to date	$54.00
Service charge $1\frac{1}{2}\%$	
TOTAL	

6. Mr. Allen earned $240 one week. 12.5% of earnings were held for taxes. How much was withheld for taxes?

7. Lindbergh's flight from New York to Paris in 1927 took about 33 hours 30 minutes. In 1961, Major W. R. Payne flew from New York to Paris in a Hustler jet bomber in about 9.9% of this time. What was Major Payne's time?

think

There are 12 dentists in a town. In one class of 26 students, each student goes to one of these dentists.

1. Is it possible that some dentist has none of these students as a patient?
2. Is it possible that each student goes to a different dentist?
3. Show that at least three students go to the same dentist.

More practice, page S-24, Set 40

Investigating the Ideas

The seven colored regions at the right are the pieces of the tangram puzzle.

What percent of the complete square region do you estimate the small square to be?

Can you trace the puzzle, cut out the pieces, and find a way to check your estimate?

Discussing the Ideas

1. The two large triangular regions cover half of the large square of the tangram puzzle.
   - A What percent of the complete square region do they cover?
   - B What percent of the entire square region does one of the large triangular regions cover?

2. A What percent of one of the large triangular regions does the middle sized triangular region cover?
   - B What percent of the complete square region does the middle sized triangular region cover?

3. What percent of the complete square region does one of the small triangular regions cover? How can you decide?

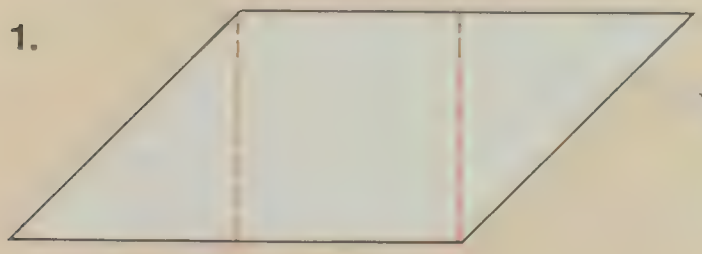
4. Explain how to find the percent of the complete square region covered by the small square.

5. Find what percent of the large square region is covered by the parallelogram.

6. Can you use some of the pieces and form a square that is 25% as large as the complete square region?

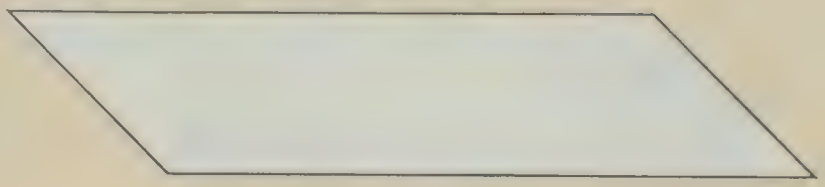
Cover each region with your tangram pieces. Then give the percent of the entire puzzle that the pieces would cover.

1.



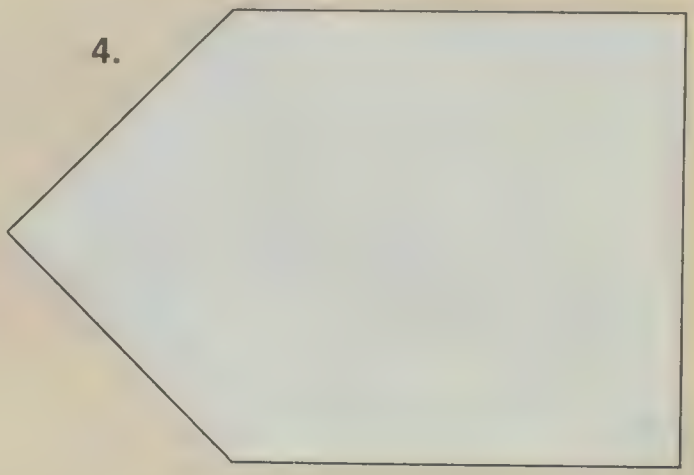
Answer : 25%

2.



3.

4.



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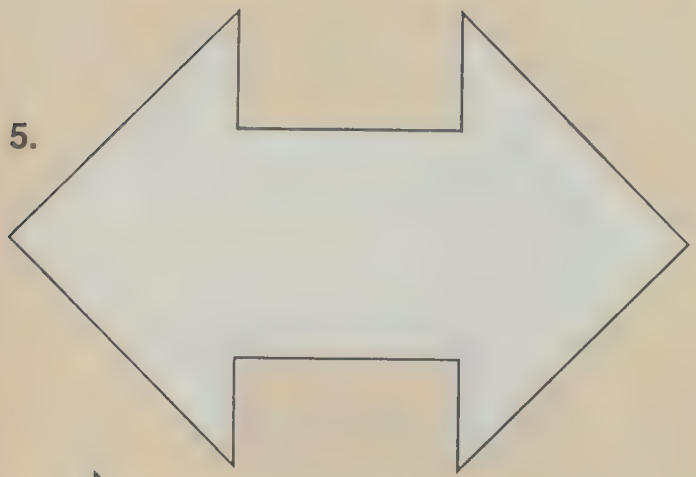

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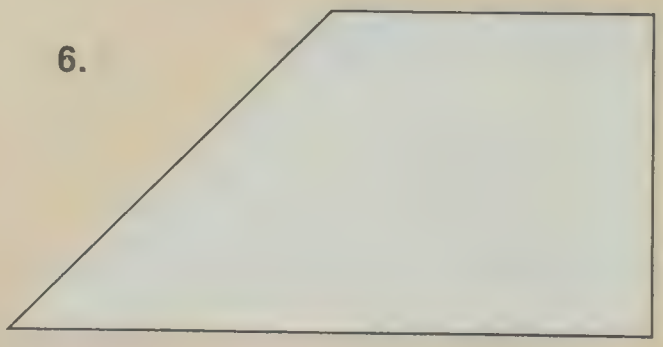

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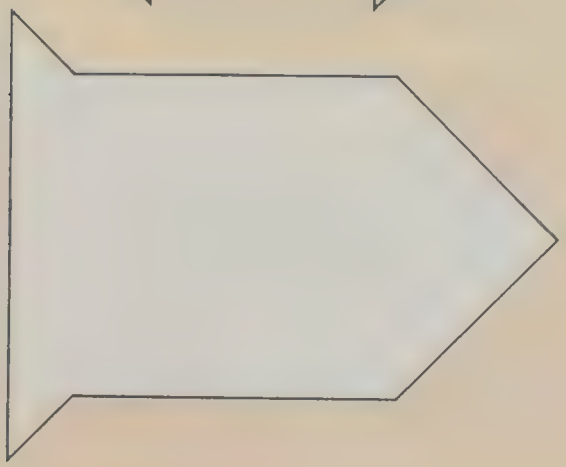

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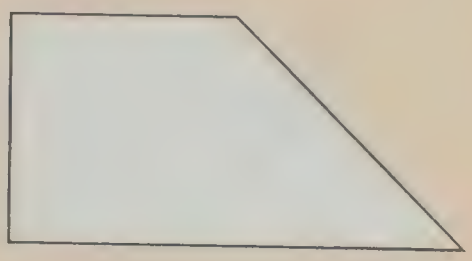
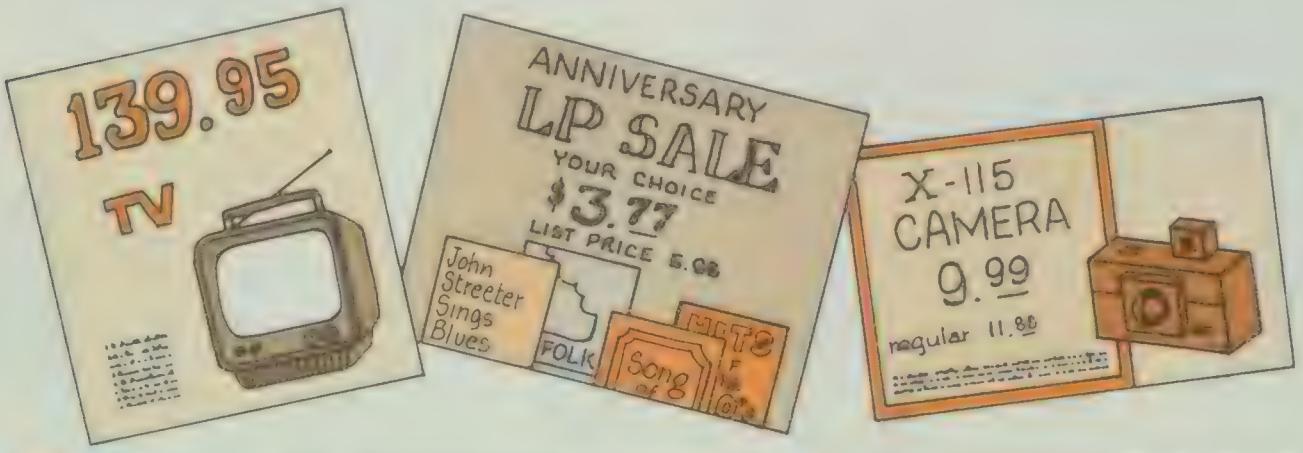


Investigating the Ideas

In most provinces you must pay a sales tax on certain items that you buy. The sales tax rate is usually expressed as a percent of the amount purchased and may vary from 5% to 10%.

Do you know the sales tax rate in your community?

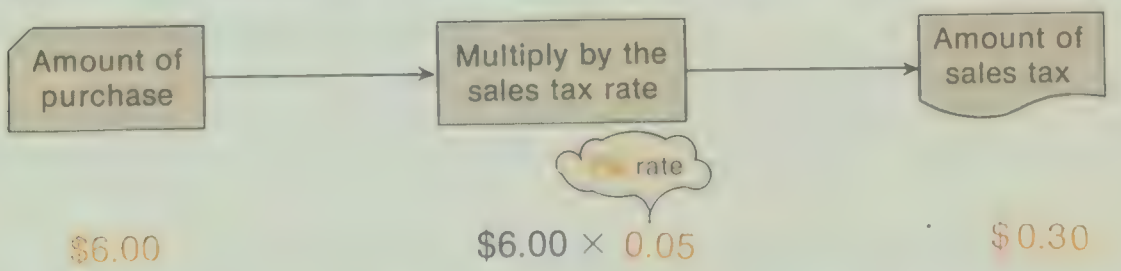


Can you choose an item from an advertisement like those above and find out how much sales tax you would have to pay on it?

Discussing the Ideas

1. Explain how you found the amount of sales tax on the item that you choose.

2. Study the flow chart below for computing the amount of sales tax.



How can you find the **total amount** that you must pay for the item costing $6.00 when the sales tax is 5%?

3. Explain how you would find the amount of sales tax and the total amount of the purchase if the item costs $18.75 and the local sales tax rate is 7%.

1. A new automobile costs $4200. The sales tax rate is 7%. How much sales tax must be paid on the car?

2. A suit of clothes was priced at $80. The sales tax rate was 5%.
   - A How much tax must be paid on the suit?
   - B What is the total cost of the suit including the sales tax?

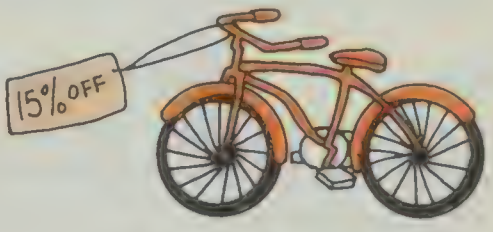
3. What is the total cost of a pair of boots priced at $18.50 if the sales tax rate is 7%?

4. An electric blender costs $32.50. The sales tax rate is 6%. What is the total cost of the blender to the nearest cent?

5. Mrs. Wright had a grocery bill of $23.40. She had to pay a 5% sales tax on all the non-food items. These items totalled $4.80
   - A What was the amount of sales tax?
   - B What was the total cost of the groceries?

6. In a certain city, the sales tax rate was $5\frac{1}{2}$%. How much tax must be paid on a purchase of $100? On a purchase of $500.

7. Pamela bought a $30-dress that was marked "20% off." The sales tax rate was 6%. What was the total amount she had to pay for the dress?



8. Roger bought a new 10-speed bicycle. Its price was $120 but he got 15% off on a sale. The sales tax rate was 8%. What was the total cost of the bicycle?

think

A used car dealer bought a car from a wholesaler for $600 and marked up the price 50% from what he had paid for it. When the car did not sell in two months' time, he had a 40%-off sale. How much money did he make or lose on the car?

Investigating the Ideas

Suppose that you had this much money.

Can you find how much your money would earn if you put it into a savings account in a local bank for one year?

Discussing the Ideas

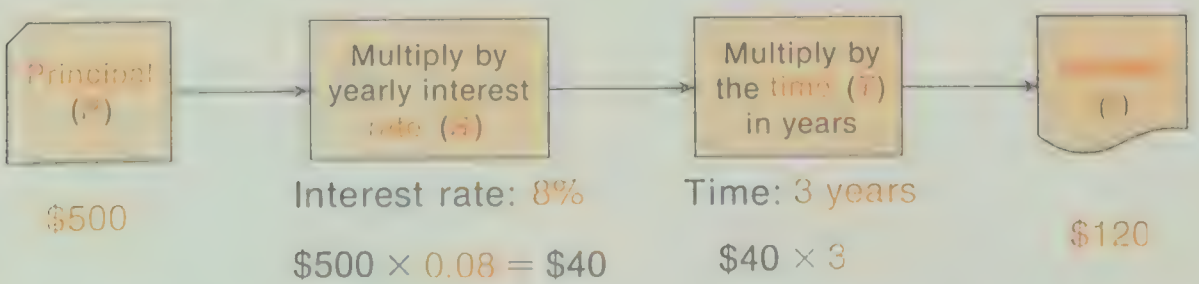
1. The money that a bank pays you for the use of your money (**principal**) is called **interest**.
   How much interest would you earn on the amount of money in the Investigation? How did you find it?

2. The **interest rate** is usually expressed as a percent and is often a **yearly** rate.
   What are some yearly interest rates at some local banks?

3. It is helpful to think of interest as so many dollars per $100 saved or borrowed.
   For example, if you borrow $500 for one year at a yearly rate of 8%, the interest charge is $8.00 per $100
   How much interest must you pay if you borrowed $500?

4. If you borrowed $500 at 8% yearly interest rate for **3 years**, you would have to pay 3 times as much interest as for one year. How much interest would you have to pay for 3 years?

5. The flow chart will help you understand the interest formula $I = P \cdot R \cdot T$.



Can you use the flow chart to find the interest on $1000 at a yearly rate of 6% for 6 months ($\frac{1}{2}$ year)?

1. Find the interest for one year given the following amounts and rates.

   A \$200 at 6% per year
   B \$600 at 7% per year
   C \$500 at 5% per year
   D \$800 at 4% per year
   E \$300 at 4% per year
   F \$200 at 8% per year
   G \$700 at 2% per year
   H \$600 at 9% per year
   I \$400 at $6\frac{1}{4}$% per year
   J \$300 at $5\frac{1}{2}$% per year
   K \$1000 at $8\frac{3}{4}$% per year
   L \$1200 at $\frac{3}{4}$% per year

2. Find the interest using the formula $I = P \cdot R \cdot T$.

   A \$355 at 5% per year for 2 years
   B \$800 at 7% per year for 4 years
   C \$4200 at 8% per year for 3 years
   D \$21,500 at 4% per year for 10 years
   E \$450 at 6% per year for $1\frac{1}{2}$ years
   F \$620 at 8% per year for 4 mon

3. Sometimes interest rates are expressed as **monthly rates** rather than yearly rates. For example, if the monthly interest rate is 2%, this is equivalent to a yearly interest rate of $12 \times 2\%$ or 24% per year.

   Express each monthly rate as a yearly rate.

   A 1% B $1\frac{1}{2}$% C 3% D $2\frac{1}{2}$% E $1\frac{1}{4}$% F $\frac{3}{4}$%

4. Give the monthly rates equivalent to the following yearly rates.

   A 12% B 18% C 6% D 8% E 9% F 30%

5. Mr. Nickols borrowed \$300 for one year from his bank. The interest rate was 1% per month. How much interest did Mr. Nickols have to pay at the end of one year?

6. Suppose that you wanted to borrow \$1000 for one year. Bank A has a 10% yearly interest rate. Bank B has a $\frac{3}{4}$% monthly interest rate.

   A Which bank would charge you less interest?
   B How much less interest would you have to pay that bank?

7. Carol had \$150 in her savings account. The yearly interest rate was $4\frac{1}{2}$%.

   A How much interest did her money earn in one year?
   B How much money would Carol have in her account after the interest was added to her savings?

8. Mr. Marsh borrowed \$22500 from a bank to buy a house. He paid a $7\frac{1}{4}$% interest rate on the amount he borrowed during the first year. How much interest did he pay the first year?

Finding What Percent One Number is of Another

Investigating the Ideas

Study the score board below.

SCHOOL	GAMES WON	GAMES PLAYED
Rockville Junior High	9	20
Jamestown Junior High	6	15
Brownsburg Junior High	7	16

Can you find which school has won the greater part of their games?

Discussing the Ideas

1. How did you find which school had won the greater part of their games?

2. To compare the school's records, you could compare the fractions $\frac{9}{20}$, $\frac{6}{15}$, and $\frac{7}{16}$. This is often easier to do if you use decimals. Find a decimal in hundredths for each fraction.

   A $\frac{9}{20} \rightarrow 20\overline{)9.00}$ B $\frac{6}{15} \rightarrow 15\overline{)6.00}$ C $\frac{7}{16} \rightarrow 16\overline{)7.00}$

3. Express each decimal in Exercise 2 as a percent.

4. In order to find what percent one number **x** is of another number **y**, you can write $\frac{x}{y}$ and find a percent for the fraction.

   EXAMPLE: 3 is what percent of 8?

   SOLUTION: $\frac{3}{8} = 0.37\frac{1}{2} = 37\frac{1}{2}\%$

   3 is $37\frac{1}{2}\%$ of 8.

   Use your answers to Exercise 2 to complete each statement.

   A 9 is ▢% of 20. B 6 is ▢% of 15. C 7 is ▢% of 16.

1. Find the number for each ▒.

   A 4 is ▒% of 20
   B 3 is ▒% of 10
   C 8 is ▒% of 16
   D 4 is ▒% of 5
   E 1 is ▒% of 100
   F 54 is ▒% of 200
   G 27 is ▒% of 300
   H 19 is ▒% of 76
   I 12 is ▒% of 160

Solve each problem.

2. If 1 of every 2 persons has at least one cold a year, what percent of all people have at least one yearly cold?

3. If 4 of every 10 trees in an orchard are apple trees, what percent of all trees are apple trees?

4. If 1 litre of fruit juice is used in making 5 litres of punch, what percent of the punch is fruit juice?

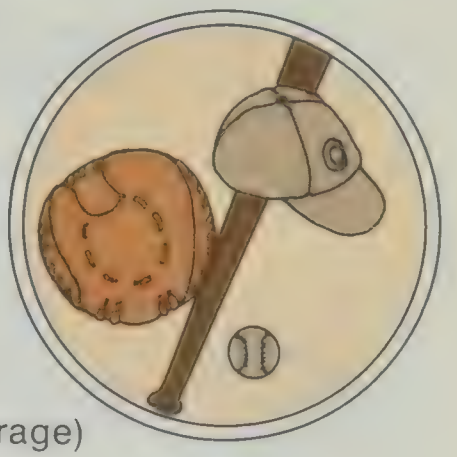
5. The areas of two pieces of property are approximately in the ratio 3:5. Express the area of lot A as a percent of the area of lot B.

6. In a group of 500 people, 35 are more than seventy years old. What percent of the group is more than seventy years old?

7. A coat was priced at $60 but was later sold for $48. The sale price was what percent of the original price?

8. Find what percent the first number is of the second. Round your answers to the nearest tenth of a percent.

   A 11, 46
   ANSWER: 23.9%
   B 28, 55
   C 34, 175
   D 87, 266
   E 28, 21
   F 3.2, 12.4
   G 136,541
   H 725, 8088



9. In baseball, a "300 hitter" is a person who makes a hit in at least 0.300 or 30% of his times at bat. In 1973, Pete Rose made 230 hits in 680 times at bat.

   A Was Pete Rose a "300 hitter" that year?
   B Express the percent of times he got hits (batting average) as a percent to the nearest tenth of a percent.

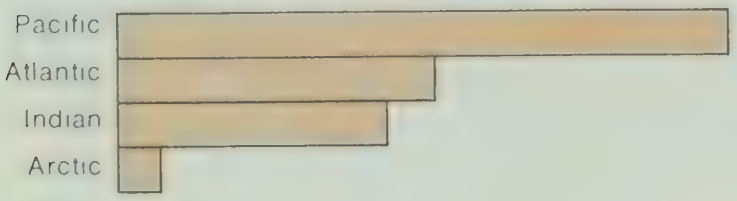
10. Don Kessinger has been at bat 311 times and made 90 hits.

   A What is his average?
   B How many consecutive hits does he need in order to be a "300 hitter"?
   C What will his average be if he is hitless for 7 times at bat?

More practice, page S-24, Set 41

Investigating the Ideas

The bar graph compares areas of the oceans of the world. The area of the Pacific Ocean is about 166 000 000 square kilometres.



Can you estimate the area of each of the other oceans?	Use a reference book to check your estimates.

Discussing the Ideas

1. How did you make each of your estimates?

2. The area of the Atlantic Ocean is about what percent of the area of the Pacific Ocean?

3. Is the area of the Arctic Ocean more or less than 10% of the area of the Pacific Ocean?

4. How did you make your estimate of the area of the Indian Ocean?

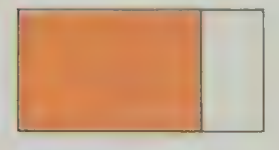
5. To estimate 31.7% of 496.4, you can round the numbers and think "30% of 500 is 150." Therefore, 31.7% of 496 is about 150. Explain how you would make estimates for these products.

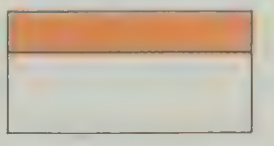
   A 48% of 81.42 **B** $6\frac{1}{2}$% of 7295 **C** 19.8% of 698.2

6. If the Arctic Ocean is about 12.1% as large as the Atlantic Ocean and the Atlantic Ocean is 86 550 000 square kilometres, what would you estimate the area of the Arctic Ocean to be?

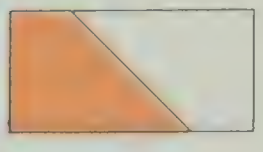
7. What is your estimate for the percent of the rectangle that is shaded?

1. Estimate what percent of each rectangular region is shaded.

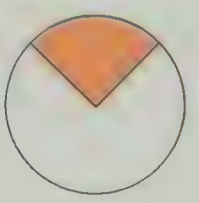
A 

B 

C

D 

2. Estimate what percent of each circular region is shaded.

A 

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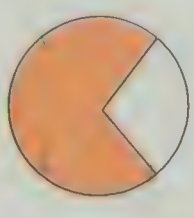
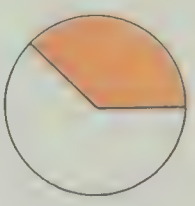
C 

D 

3. The bar graph compares the areas of the seven continents.
   - A Which continent is about 50% as large as Asia?
   - B Which continent is about 50% as large as South America?
   - C Antarctica is about what percent of the size of Asia?

4. Mr. Freed took his family to a restaurant for dinner. The check for the dinner totalled $23.85. Mr. Freed figured approximately 15% of this amount for a tip. Which amount is the best approximation of the tip?

A $3.00 B $3.50 C $3.75 D $4.00

5. Estimate the sales tax, to the nearest dollar. on a used car which is priced at $2499 if the provincial sales tax rate is

A 5% B 6% C 7% D 8% E 9%

6. Choose the best estimate for each exercise.

A	3% of 369	(100, 10, 1000, .1, 0.01)
B	27% of 46	(12, 1.2, 120, 0.12, 0.012)
C	98% of 3.7	(0.37, 0.037, 37, 3.7, 370)
D	16% of 284	(0.45, 0.045, 450, 4.5, 45)
E	49% of 68.4	(34, 3.4, 0.34, 340, 0.034)
F	33% of 200	(660, 6.6, 66, 0.66, 0.066)
G	.9% of 167	(167, 1.67, 16.7, .167, 1670)

think

Six similar bricks balance 3 large blocks that are also similar. Each block weighs 2 kg more than a brick. What is the mass of 1 block?

REVIEWING THE IDEAS

1. Express each ratio in simplest terms.

A 8:12 B 20:50 C 35:25 D 16:4

2. In the proportion $3:8 = 15:40$,

A which numbers are the means?

B which numbers are the extremes?

3. Solve the proportions for n.

A $\frac{2}{5} = \frac{n}{20}$ C $\frac{3}{n} = \frac{4}{12}$

B $4:15 = 12:n$ D $n:6 = 20:8$

4. Express each ratio in simplest terms.

A 300 cm to 5 m C 240 mm to 12 cm

B 40 dm to 3 m D 600 mm to 1 m

5. The scale for a drawing of a house is 1:100. The living room is pictured on the drawing as a 6.2 by 9.5-cm rectangle. What is the actual length and width of the living room?

6. Express each percent as a decimal.

A 23% C 250% E 0.01%

B 6% D 1.5% F 0.003%

7. Express each decimal as a percent.

A 0.35 C 1.25 E 0.7

B 0.04 D 5 F 0.009

8. Express each percent as a lowest-terms fraction.

A 50% C 75% E 2%

B 30% D 80% F $\frac{1}{2}$%

9. Express each fraction as a percent.

A $\frac{1}{4}$ C $\frac{7}{10}$ E $\frac{3}{8}$

B $\frac{3}{5}$ D $\frac{3}{10}$ F $\frac{1}{500}$

10. Find the percent of each number.

A 42% of 87 C 60% of \$72.95

B 9% of 283 D $37\frac{1}{2}$% of \$800

11. A sweater costs \$14. The provincial sales tax rate is 7%.

A How much tax on the sweater?

B What is the total cost of the sweater?

12. Find the interest on each amount.

A \$200 at 7% per year for 1 year

B \$450 at 6% per year for 3 years

C \$225 at 8% per year for $1\frac{1}{2}$ years

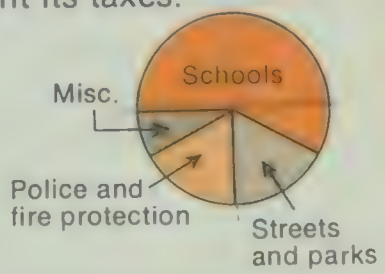
D \$600 at 5% per year for 4 months

13. What is the yearly interest rate if the monthly rate is $1\frac{1}{4}$%?

14. A baseball team won 9 out of 21 games. What percent of the games, to the nearest tenth, were won?

15. A television set priced at \$300 was reduced in price by \$24. What percent was it reduced?

16. The circle graph pictures how a certain city spent its taxes. Estimate the percent of all tax money spent for each category.



17. Choose the number closest to 2% of 79 534.12.

A 160 000 C 1600 E 16

B 16 000 D 160 F 1.6

★ **18.** The French franc is worth 20% as much as the dollar.

A What is the value in dollars of 500 French francs?

B How many francs could you get in exchange for \$50?

TEST YOURSELF

1. What is the simplest terms ratio for 16:20?

2. Solve the proportion: $\frac{n}{8} = \frac{12}{32}$

3. The ratio of 18 mm to 3 cm is

   A $\frac{18}{3}$ **B** $\frac{3}{5}$ **C** $\frac{3}{18}$ **D** $\frac{5}{3}$

4. A scale on a map is 1 cm = 48 km. What is the actual distance between two cities if the distance on the map is 3.5 cm?

5. Express as a decimal.

   A 17% **B** 6.2% **C** 0.4%

6. Express as a percent.

   A $\frac{4}{5}$ **B** $\frac{1}{8}$ **C** $0.08\frac{3}{4}$ **D** 0.237

7. What is 38% of 420?

8. **A** How much sales tax must be paid on a purchase of $26.95 if the sales tax rate is 5%?

   B How much interest on $500 at a rate of 8% per year for $2\frac{1}{2}$ years?

9. In a class of 20 children, 4 of them wear glasses. What percent of the children wear glasses?

10. Which is the best estimate for $8\frac{3}{4}$% of 319?

    A 27 **B** 2.7 **C** 0.27 **D** 270

RESEARCH PROJECTS

A Make a scale drawing of the floor plan of your home or of some building. Choose a suitable scale to show the dimensions of the building as well as some details such as windows and doors.

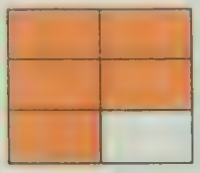
B Select some newspaper advertisements that show different percents of discounts for articles that are on sale. Make a chart that shows the regular price, the percent of reduction (discount), the sale price, and the amount saved by purchasing the article during a sale.

C Obtain a copy of a sales tax table for your province. Show how it is used by retail sales people. Local business firms or provincial tax offices can provide tax tables.

D Visit a local bank to find out about different interest rates that are charged for borrowing money or for depositing money in savings accounts. Try to determine how long it would take for $100 of savings to double in value if it were left in a savings account.

CUMULATIVE REVIEW

1. What fractional part of the figure is shaded?

A 

B

2. Which fraction is equivalent to $\frac{6}{16}$?

A $\frac{3}{4}$ B $\frac{7}{8}$ C $\frac{1}{3}$ D $\frac{3}{8}$

3. Give three fractions that are equivalent to:

A $\frac{1}{3}$ B $\frac{3}{10}$ C $\frac{2}{9}$

4. In each set, which fraction is not equivalent to any of the other fractions in the set?

A $\{\frac{1}{3}, \frac{3}{12}, \frac{5}{15}, \frac{8}{24}, \frac{6}{18}\}$

B $\{\frac{6}{8}, \frac{15}{20}, \frac{3}{4}, \frac{9}{16}, \frac{18}{24}\}$

5. Give the lowest-terms fraction for each fraction.

A $\frac{15}{25}$ B $\frac{16}{40}$ C $\frac{14}{35}$ D $\frac{25}{150}$

6. Give a fractional number for each lettered point on the number line.

7. Give the correct symbol ($<$, $=$, or $>$) for each .

A $\frac{6}{7}$ $\frac{5}{6}$

B $\frac{5}{8}$ $\frac{7}{12}$

C $\frac{1}{2}$ $\frac{5}{9}$

D $\frac{11}{5}$ $\frac{13}{6}$

8. Give an improper fraction for each mixed numeral.

A $1\frac{5}{8}$ B $3\frac{2}{5}$ C $4\frac{9}{10}$

9. Give a mixed numeral for each improper fraction.

A $\frac{11}{7}$ B $\frac{43}{10}$ C $\frac{42}{8}$

10. What is the least common denominator for $\frac{3}{5}$ and $\frac{1}{4}$?

11. Find the sums.

A $\frac{5}{12} + \frac{3}{4}$ C $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

B $1\frac{2}{3} + \frac{5}{6}$ D $2\frac{3}{4} + 4\frac{3}{5}$

12. Find the differences.

A $\frac{11}{12} - \frac{3}{4}$ C $7 - 3\frac{2}{5}$

B $5\frac{1}{4} - \frac{7}{8}$ D $2\frac{2}{3} - 1\frac{1}{4}$

13. Give the missing numbers in each function table.

A

Function Rule $n + \frac{1}{6}$	
n	$f(n)$
2	$2\frac{1}{6}$
$\frac{1}{2}$	
0	
$\frac{2}{5}$	
	1

E

Function Rule $n - \frac{3}{4}$	
n	$f(n)$
2	$1\frac{1}{4}$
$1\frac{1}{2}$	
$4\frac{1}{8}$	
$3\frac{5}{6}$	
10	

14. Round each number to the nearest whole number.

A $3\frac{7}{8}$ B $\frac{19}{3}$ C $\frac{56}{8}$

15. It took $46\frac{9}{10}$ litres of gasoline to fill Mr. Wagner's car. The next time he filled his car, it took $43\frac{1}{2}$ litres. How many litres did it take for both fill ups?

16. Solve the equations.

A $\frac{1}{3} \cdot \frac{1}{2} = n$ C $3 \cdot \frac{1}{4} = n$

B $\frac{1}{4} \cdot \frac{1}{6} = n$ D $\frac{1}{8} \cdot 5 = n$

17. Find the products.

A $\frac{1}{4}$ of 12 C $\frac{1}{3}$ of 36

B $\frac{1}{8}$ of 72 D $\frac{1}{10}$ of 250

18. Find the products.

A $\frac{3}{5} \cdot \frac{1}{3}$ C $1\frac{2}{3} \cdot \frac{3}{4}$

B $\frac{9}{5} \cdot \frac{1}{3}$ D $2\frac{1}{2} \cdot 7\frac{3}{5}$

19. Give the reciprocals for each.

A $\frac{2}{7}$ B 5 C $2\frac{1}{3}$

20. Find the quotients.

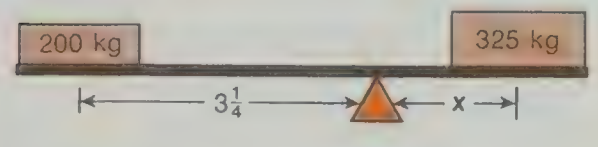
A $\frac{3}{4} \div \frac{1}{2}$ C $1 \div \frac{9}{10}$

B $\frac{3}{5} \div \frac{2}{3}$ D $4\frac{1}{4} \div 2\frac{1}{8}$

21. Simplify each complex fraction.

A $\dfrac{\frac{3}{4}}{\frac{5}{6}}$ B $\dfrac{\frac{6}{7}}{\frac{9}{14}}$

22. If the system is in equilibrium, what is the distance x?



23. Write a fraction for each decimal.

A 1.7 B 0.318 C 0.04

24. Write a decimal for each fraction.

A $\frac{37}{100}$ B $1\frac{356}{1000}$ C $6\frac{9}{100}$

25. Find the sums.

A $2.8 + 3.47 + 19.068$

B $167.3 + 84.9 + 72.006$

26. Find the differences.

A $12.47 - 9.6$

B $83.49 - 17.864$

27. Find the products.

A 6.2×0.12 B 43.09×7.6

28. Find the quotients.

A $1.9\overline{)7.22}$ B $0.008\overline{)31.2}$

29. Find the repeating decimal for $\frac{4}{9}$.

30. Express $\frac{5}{6}$ as a mixed decimal numeral.

31. Solve the proportion: $\frac{n}{9} = \frac{12}{36}$

32. Express the ratio "300 cm to 5 m" in simplest terms.

33. Express each percent as a decimal.

A 27% B 115% C 8%

34. Express each decimal as a percent.

A 0.89 B 0.9 C 0.03

35. Express each percent as a lowest-terms fraction.

A 20% B 35% C 4%

36. Express each fraction as percent.

A $\frac{2}{5}$ B $\frac{3}{4}$ C $\frac{5}{8}$

37. What is 15% of 642?

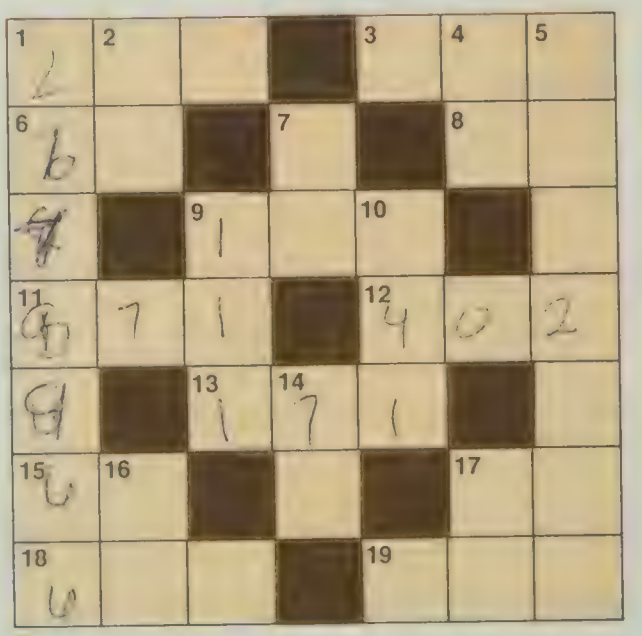
38. A pair of shoes cost $18.95 plus 5% sales tax. What was the total cost of the shoes?

39. How much interest on $600 at a rate of $5\frac{1}{2}$% per year for 3 years?

40. In a school survey, 12 students out of 40 preferred a shorter lunch hour. What percent of the students preferred a shorter lunch hour?

A Cross Number Puzzle

Make a copy of the cross number puzzle grid on graph paper. Then find the numbers for the squares.



ACROSS

1. 2^8
3. The number of a large jet airplane.
6. XLVII
8. 3^4
9. The number of centimetres in 1.78 metres.
11. Base two numeral for 5.
12. 2381 − 1979
13. 19 · 9
15. $15\frac{3}{4} \times 4$
17. $0.002\overline{)0.07}$
18. The square of a number greater than 15 that ends with a five.
19. CMVII

DOWN

1. 2154 × 1129
2. 19.38 ÷ 0.34
4. $\frac{12}{25} =$ ▮▮▮%
5. The digits in the first seven decimal places for $\frac{5}{7}$.
7. The largest prime number less than 100.
9. $[(3 \cdot 3 \cdot 2 \cdot 2) + 1] \cdot 3$
10. 29^2
14. $2^2 \cdot 3^2$
16. Two to the fifth power.
17. A number whose prime factors are 2, 3, and 5.

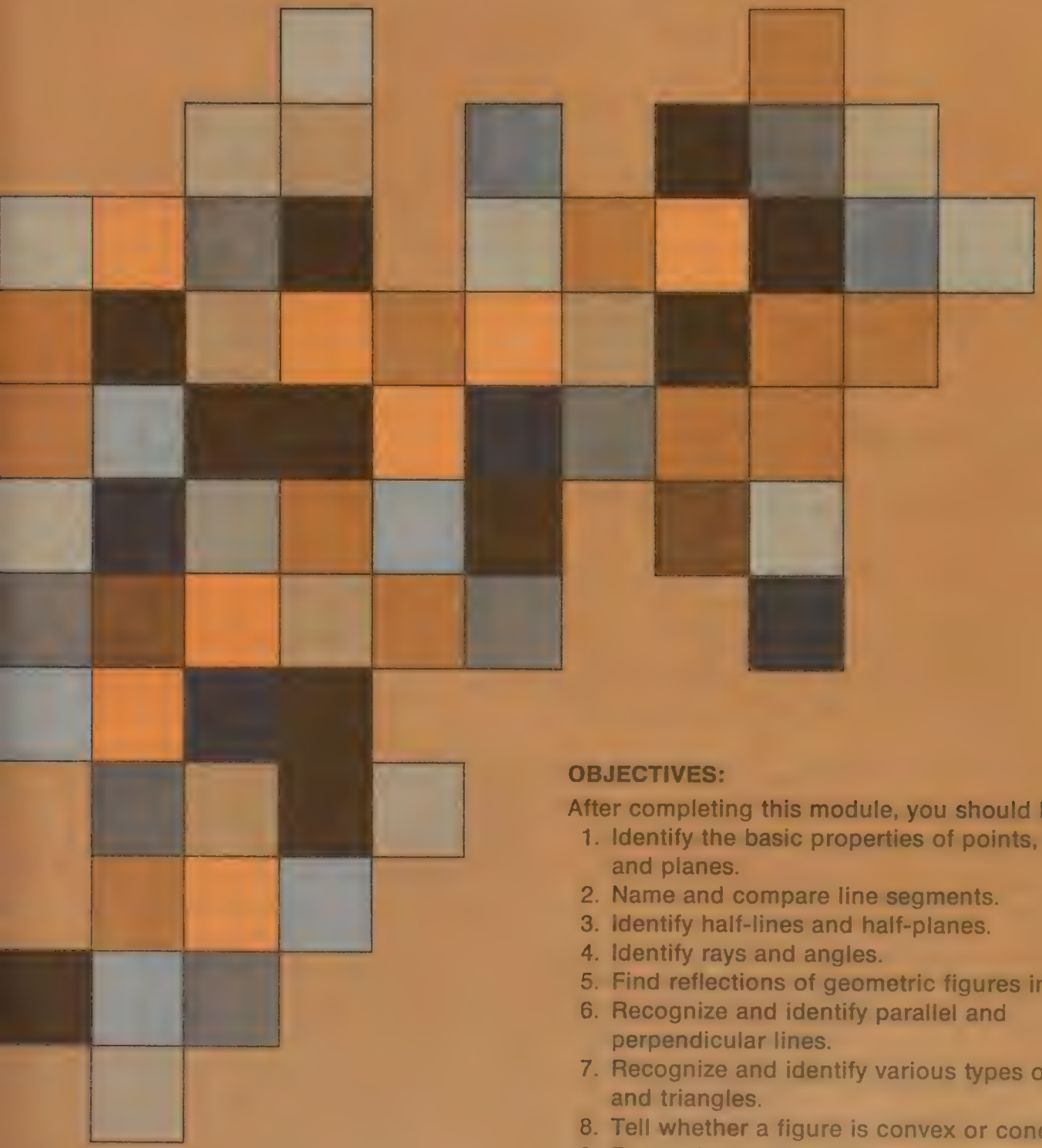
RECREATIONS MATHEMATICAL

TEST YOURSELF Answers

1. 4:5 2. $n = 3$ 3. B 4. 168 km 5. A 0.17
B 0.062 C 0.004 6. A 80% B 12½% C 8⅓% D 23.7%
7. 159.6 8. A \$1.08 ; B \$100 9. 20% 10. A

UNIT C: *Geometry and Measurement*

MODULE 1: *Basic Concepts of Geometry*



OBJECTIVES:

After completing this module, you should be able to:

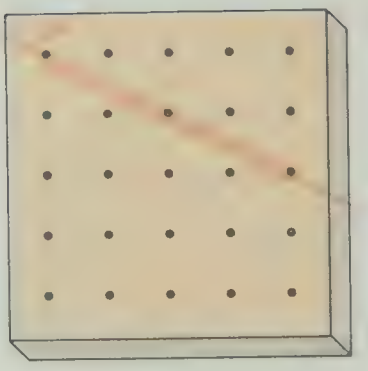
1. Identify the basic properties of points, lines, and planes.
2. Name and compare line segments.
3. Identify half-lines and half-planes.
4. Identify rays and angles.
5. Find reflections of geometric figures in a line.
6. Recognize and identify parallel and perpendicular lines.
7. Recognize and identify various types of angles and triangles.
8. Tell whether a figure is convex or concave.
9. Recognize and identify special quadrilaterals.

Investigating the Ideas

There is exactly one line that contains any two points.

"line AB"

SYMBOL: $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$



How many different "lines" can you find on your geoboard that contain point P?	Record your findings on dot paper or graph paper.

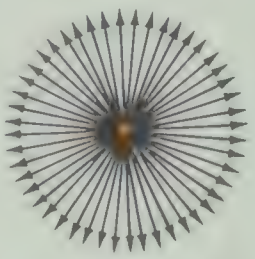
Discussing the Ideas

1. How many lines did you find that contain point P?

2. What are some objects that remind you of the idea of a point?

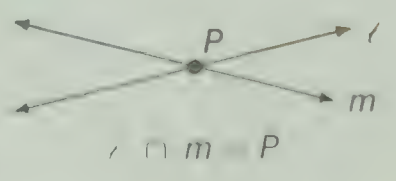
3. Lines, in geometry, never end. However, we often see physical objects which remind us of lines. What are some objects that suggest lines?

4. When we say "draw a line," we mean **draw a picture to represent a line**. What does the picture at the right suggest about the number of lines that can contain one point?



5. A line can have many names. Three names for this line are $\overleftrightarrow{RS}$, $\overleftrightarrow{TR}$, and ℓ. What are some other names for this same line?

6. The figure shows lines ℓ and m intersecting at point P. Can two lines intersect in more then one point?



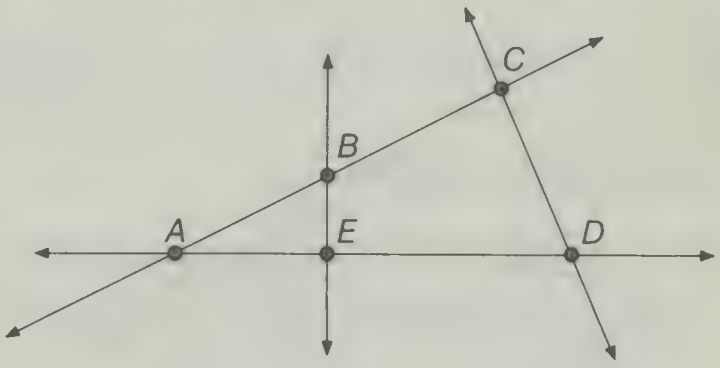
$\ell \cap m = P$

1. Mark two points on your paper and label them P and Q.
Use your ruler to draw the line through P and Q.
Mark a point A not on $\overleftrightarrow{PQ}$. Draw $\overleftrightarrow{AP}$ and $\overleftrightarrow{AQ}$.

2. If three points are chosen not on the same line, how many lines can be drawn that contain pairs of these points?

3. Mark four points on your paper and label them A, B, C, and D.
Make sure no three of them are on the same line.
Draw all the lines that contain pairs of these points.
Give names for the lines such as $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, and so on.
You should have six lines in all.

4. A How many lines are shown in the figure at the right?
   B Give two other names for $\overleftrightarrow{AE}$.
   C Give two other names for $\overleftrightarrow{AC}$.
   D What is $\overleftrightarrow{AB} \cap \overleftrightarrow{CD}$?
   E What is $\overleftrightarrow{BC} \cap \overleftrightarrow{ED}$?



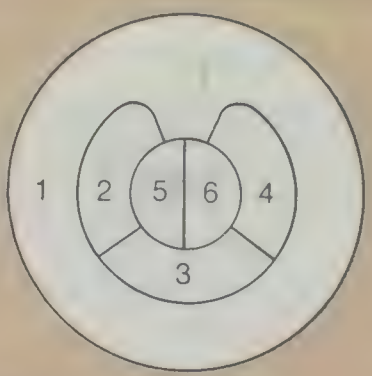
5. Draw four lines and label them m, n, o, and p.
Make sure that no three of them go through the same point and that each one intersects the other three.
How many points are determined by the intersections of pairs of these lines?

★ 6. How many lines are determined by five points if no three of them were on the same line?

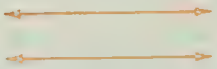
★ 7. How many lines are determined by six points if no three of them were on the same line?

think

Copy the figure on your paper.
If you had six different crayons, you could easily color the six regions of this "map" so that no two bordering regions had the same color.
Try to do it using only five colors.
Then try to color the map using only four colors.
Can you color the regions using only three colors?
Remember, bordering regions may not have the same color.



Investigating the Ideas

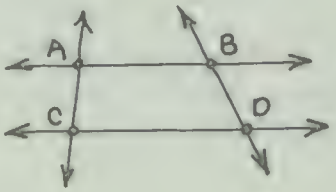
Two lines in a plane can have  **no point of intersection** or **exactly one point of intersection.**

Three lines in a plane can have **zero** **one** **two** or **three points of intersection.**

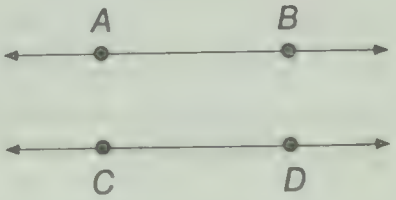
How many different points of intersection can you find for four lines?

Discussing the Ideas

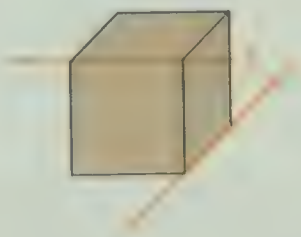
1. Andrea drew this picture to illustrate four lines intersecting in four points. Was she correct? Explain your answer.



2. Two lines that are in the same plane and do not intersect are called **parallel lines**. To show that $\overleftrightarrow{AB}$ **is parallel to** $\overleftrightarrow{CD}$, we write $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. What are some physical objects that remind you of parallel lines?



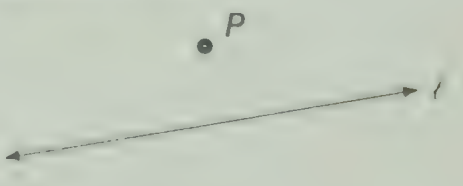
3. **Skew lines** are two lines that are not in the same plane. In the figure lines s and t illustrate skew lines. They do not intersect, yet they are not parallel lines.



Use two pencils to demonstrate

A intersecting lines **B** parallel lines **C** skew lines

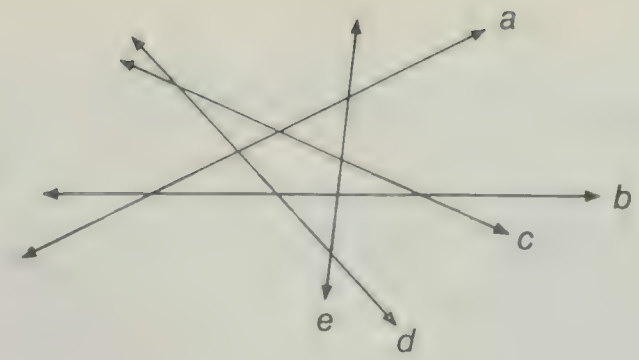
4. Suppose point P is not on a line ℓ.



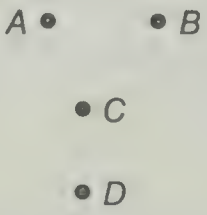
A Is there a line in the plane of P and ℓ that contains point P and is parallel to line ℓ?

B Is there more than one line through P that is parallel to line ℓ?

1. How many points of intersection are shown in the figure for five lines?



2. A Mark four points on your paper in the positions of points A, B, C, and D.

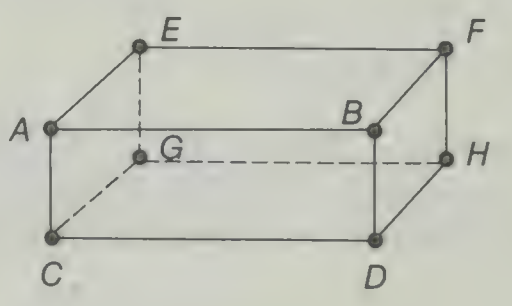


B Draw all the lines that you can through pairs of points.

C How many new points of intersection, other than A, B, C, and D, do the lines form?

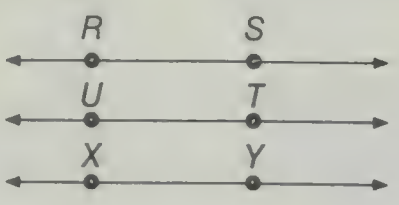
3. Use the points at the corners of the box in the figure to name some parallel lines. How many pairs can you find?

EXAMPLE: $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$



4. Name some skew lines using the points at the corners of the box in Exercise 3.

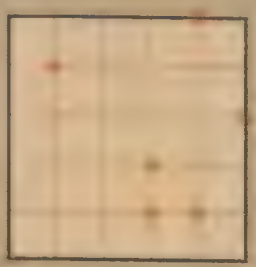
5. Suppose $\overleftrightarrow{RS} \parallel \overleftrightarrow{UT}$ and $\overleftrightarrow{UT} \parallel \overleftrightarrow{XY}$. Is $\overleftrightarrow{RS} \parallel \overleftrightarrow{XY}$?



6. Draw a picture to illustrate the following:
Five lines a, b, c, d, and e in a plane such that $a \parallel b$ and $c \parallel d$, a is not parallel to c, e intersects the other four lines, and the five lines intersect in four points.

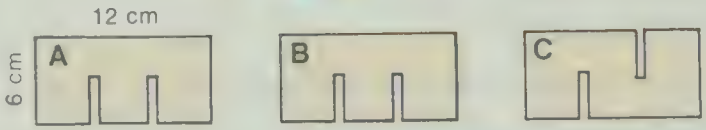
think

Trace the square and the seven points. Draw three straight line segments joining the sides of the square so that each of the seven points will be separated from each other.



Investigating the Ideas

A **plane** is a flat surface that extends indefinitely. You can make some models of planes using pieces of cardboard or tag board. Cut out three pieces of cardboard like A, B, and C. Then cut the slots so that they go at least halfway through each piece.



Can you place the three pieces together to make a model of three intersecting planes?

Discussing the Ideas

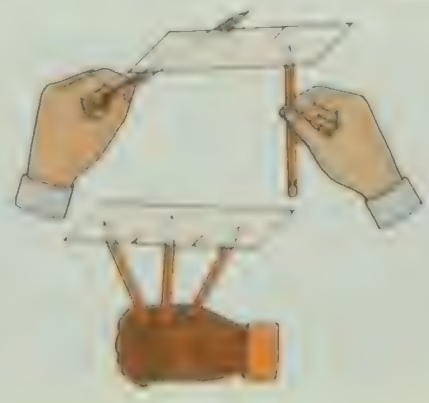
1. The intersection of two planes is the set of all points that are common to both planes. Look at your model. What geometric figure is formed by the intersection of two planes?

2. How many lines of intersection are formed by the planes in your model?

3. Imagine that your model will just fit into a rectangular box. Into how many separate compartments would the model partition the box?

4. Can you arrange the pieces of your model so that it would partition a box into six compartments?

5. There is only one plane that will contain a line and a point not on the line. Hold two pencils to balance one of your planes in order to illustrate this.

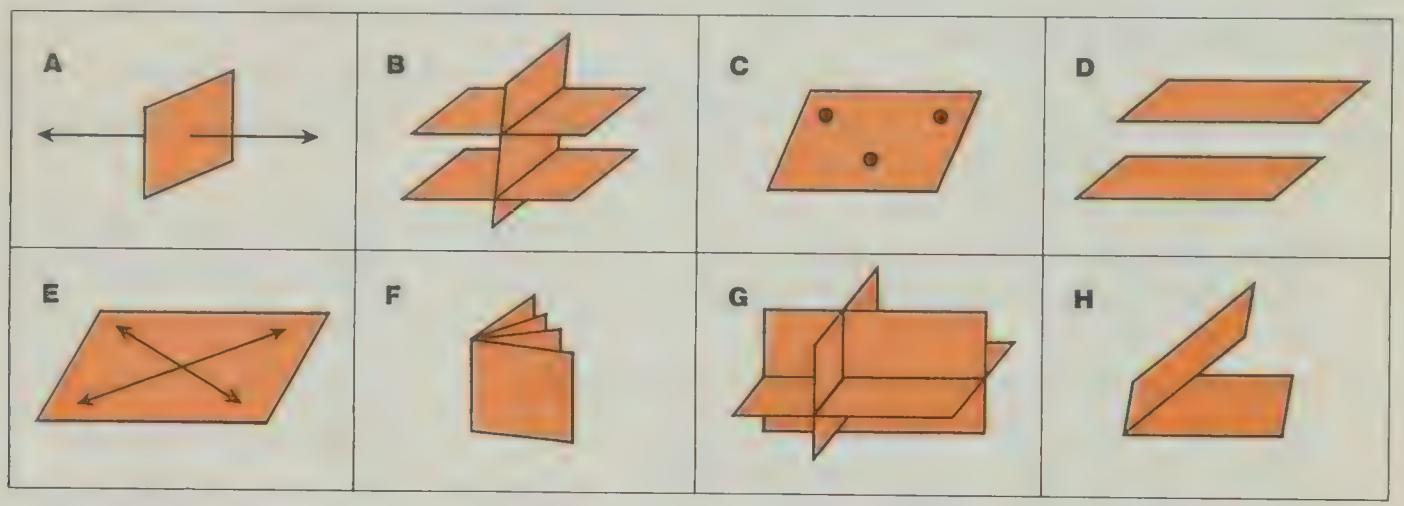


6. If three points are not in a line, then there is just one plane that contains all three points. Try to balance your model of a plane on three pencil points.

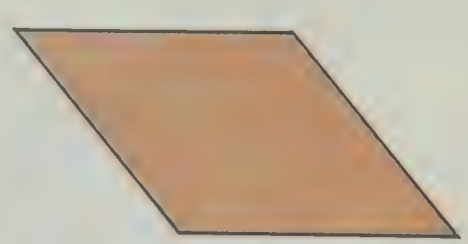
1. Mark three points on your paper so that only one plane contains these points.

2. Mark three points on your paper so that each of many planes contains these points.

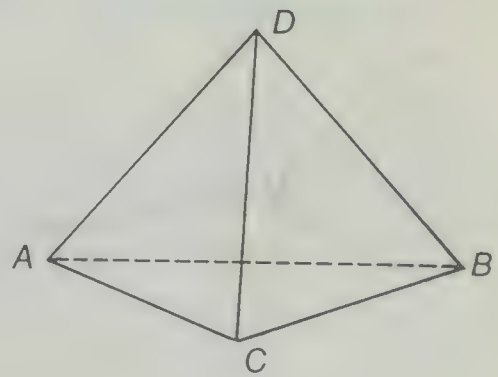
3. Name the figure suggested by the phrase.
   - **A** Two planes that do not intersect.
   - **B** Four planes intersecting in one line.
   - **C** A line intersecting a plane in a point.
   - **D** Two intersecting lines in a plane.
   - **E** Three planes intersecting.
   - **F** Two planes intersecting in a line.
   - **G** Three points in a plane.
   - **H** Three planes intersecting in two lines.



4. **A** Draw a picture of a plane.
   - **B** Mark two points X and Y in the plane.
   - **C** Draw $\overleftrightarrow{XY}$.
   - **D** Mark a point P on $\overleftrightarrow{XY}$ between X and Y.
   - **E** Mark a point Q in the plane but not on $\overleftrightarrow{XY}$.
   - **F** Draw $\overleftrightarrow{PQ}$.



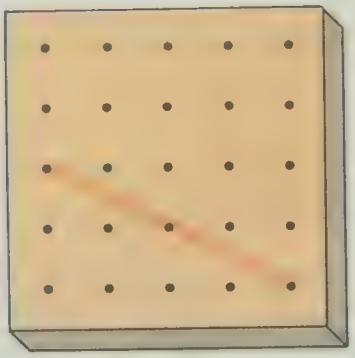
5. In the figure there are four points so that no three lie on one line and no plane contains all of them. Points A, C, and D determine one plane called plane ACD.
   Name three other planes determined by other sets of three points in the figure.



6. How many planes would be determined by five points? (See Exercise 5.)

Investigating the Ideas

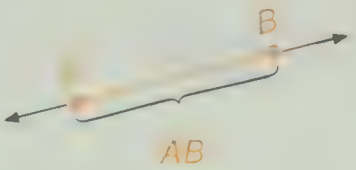
You can represent a **segment** on your geoboard by looping a rubber band around any two nails on the board.



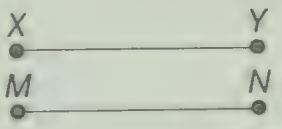
How many segments can you find on your geoboard that are just as long as the one shown above?	Record your findings on dot paper.

Discussing the Ideas

1. The segment AB with endpoints A and B is the set made up of points A and B and all points between A and B on $\overleftrightarrow{AB}$.
The symbol for segment AB is AB.
In what way is a segment different from a line?

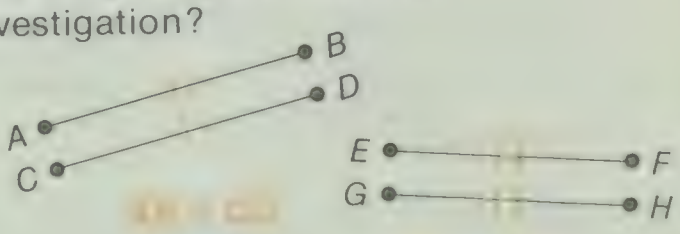


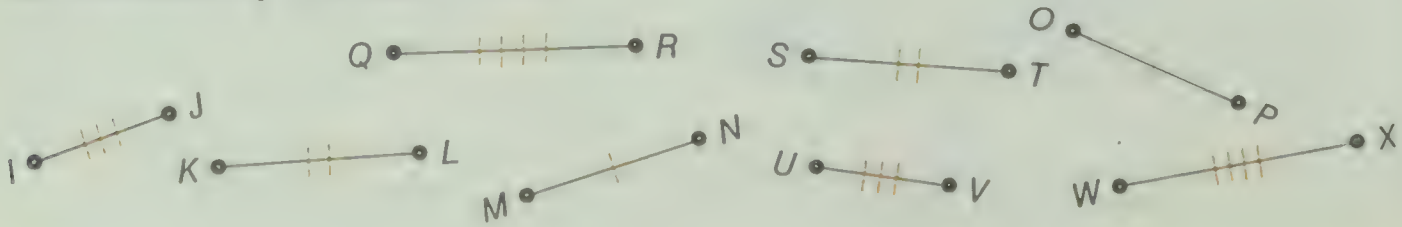
2. If two segments have their endpoints equally far apart, they are congruent.
In the figure, $\overline{XY}$ is congruent to $\overline{MN}$ and we write $\overline{XY} \cong \overline{MN}$.
Draw and name two segments that you think are congruent.



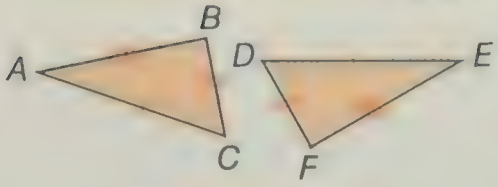
3. How many different segments did you find on your geoboard that were congruent to the segment in the Investigation?

4. We often use slash marks to indicate that two segments are congruent. The same number of slashes indicates that the two segments are congruent.
Name the pairs of congruent segments as indicated by the slash marks.



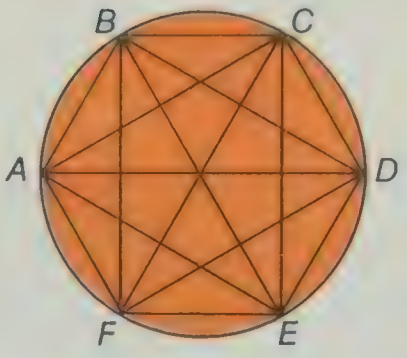


1. List the pairs of congruent segments shown in the two triangles.



2. The six points A, B, C, D, E, and F are equally spaced around the circle. List all the segments in the figure that are congruent to

A $\overline{AB}$ **B** $\overline{AC}$ **C** $\overline{AD}$



3. How many segments are shown in the figure for Exercise 2?

4. Complete the sentence: If $\overline{JK} \cong \overline{DE}$ and $\overline{DE} \cong \overline{HG}$, then $\overline{JK} \cong$? .

5. Mark two points X and Y on your paper. Then mark a third point Z so that $\overline{XY} \cong \overline{XZ}$ and $\overline{XY} \cong \overline{ZY}$.

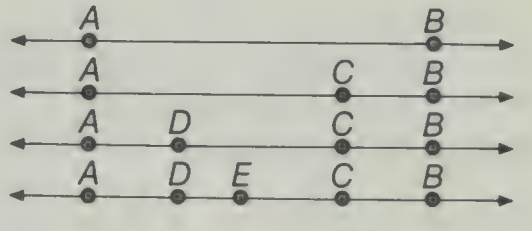
6. How many segments can you name using

A two points on a line?

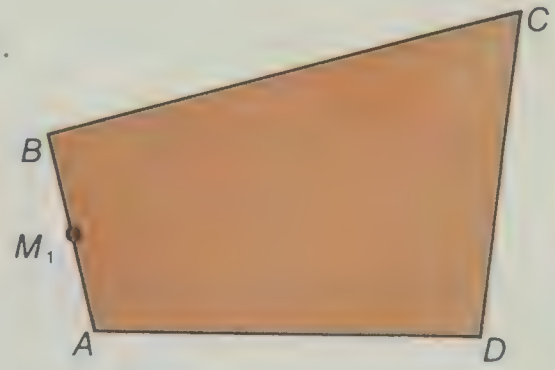
B three points on a line?

C four points on a line?

D five points on line?

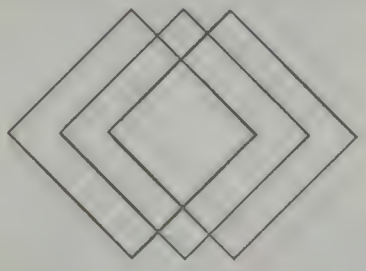


7. The **midpoint** of a segment divides a segment into two congruent segments. Draw a 4-sided figure such as $ABCD$. Label the midpoints of $\overline{AB}$ as M_1, the midpoint of $\overline{BC}$ as M_2, the midpoint of $\overline{CD}$ as M_3, and the midpoint of $\overline{AD}$ as M_4. Draw the figure $M_1M_2M_3M_4$. Can you find some congruent segments in your drawing?



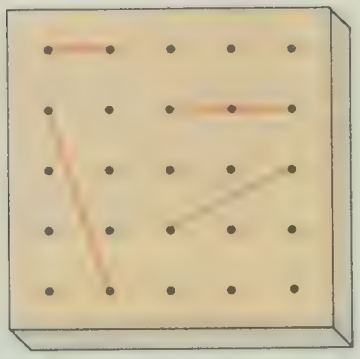
Think

Starting at any point, draw the figure without crossing any lines, retracing any lines, or lifting your pencil from the paper.



Investigating the Ideas

Use your geoboard for this Investigation.



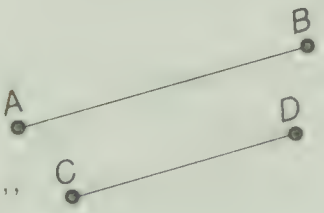
How many different segments can you find on your geoboard so that no two are congruent?

Record your findings on dot paper.

Discussing the Ideas

1. Are these two geoboard segments congruent? How can you tell?

2. The endpoints of $\overline{AB}$ are farther apart than the endpoints of $\overline{CD}$. We say that "AB **is greater than** CD" or "CD **is less than** AB." We write this as $AB > CD$ or $CD < AB$.



Compare the pairs of segments below.

A

B

C

3. Label four points A, C, E, and K in that order on a line. Name all the different segments. You should have six in all.

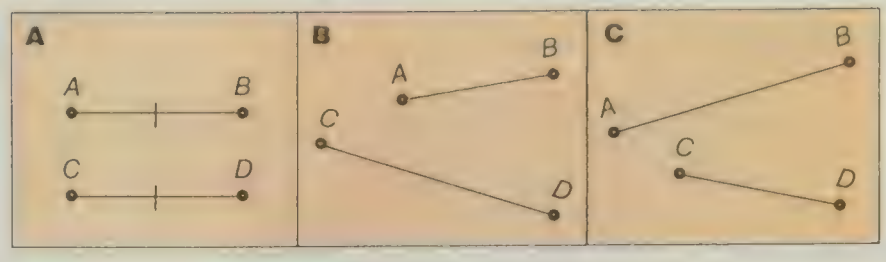
4. Find and name three inequalities about the six segments in Exercise 3. EXAMPLE: $AC < AE$

5. In the figure, point P is the midpoint of AB.

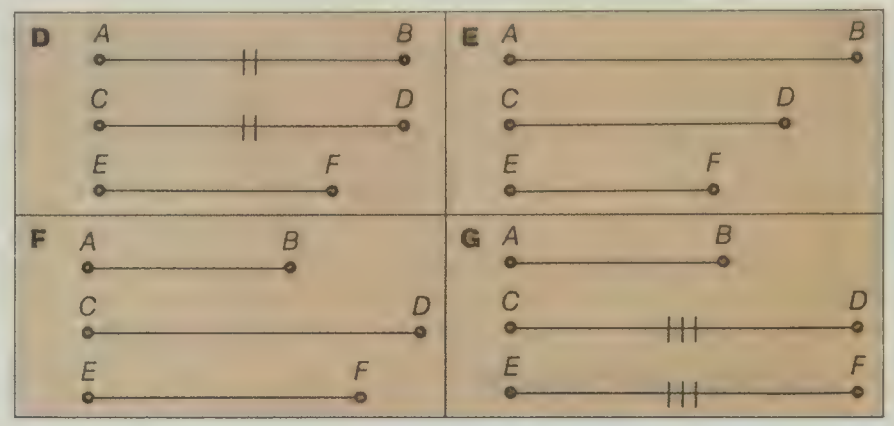
What is the correct symbol for each ▒?

A AB ▒ BP B AP ▒ PB C PB ▒ AB

1. Select the correct figure.
   - A $\overline{AB} > \overline{CD}$
   - B $\overline{AB} \cong \overline{CD}$
   - C $\overline{CD} < \overline{AB}$
   - D $\overline{CD} > \overline{AB}$
   - E $\overline{AB} < \overline{CD}$

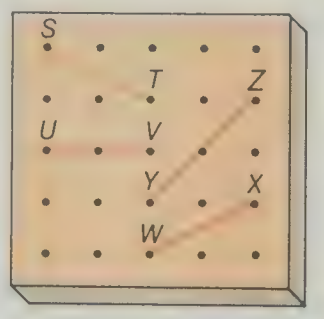


2. Select the correct figure.
   - A $\overline{AB} < \overline{CD}$ and $\overline{EF} < \overline{CD}$
   - B $\overline{AB} > \overline{CD}$ and $\overline{CD} > \overline{EF}$
   - C $\overline{AB} < \overline{CD}$ and $\overline{CD} \cong \overline{EF}$
   - D $\overline{CD} > \overline{EF}$ and $\overline{EF} > \overline{AB}$
   - E $\overline{AB} \cong \overline{CD}$ and $\overline{AB} > \overline{EF}$
   - F $\overline{AB} > \overline{EF}$ and $\overline{CD} < \overline{AB}$

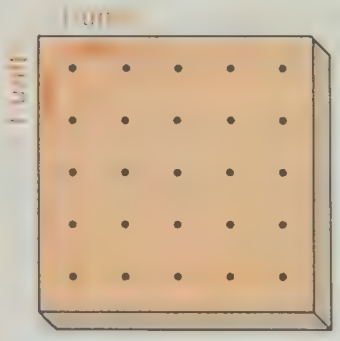


3. Select the statement that you think completes the sentence correctly. Be prepared to explain your selection.
   - A If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, we know that $\begin{cases} \overline{AB} \cong \overline{EF}. \\ \overline{AB} < \overline{EF}. \end{cases}$
   - B If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} > \overline{EF}$, we know that $\begin{cases} \overline{AB} \cong \overline{EF}. \\ \overline{AB} > \overline{EF}. \end{cases}$
   - C If $\overline{AB} < \overline{CD}$ and $\overline{CD} < \overline{EF}$, we know that $\begin{cases} \overline{AB} < \overline{EF}. \\ \overline{AB} \cong \overline{EF}. \end{cases}$

★ 4. Write as many statements as you can that will compare different pairs of segments shown on the geoboard.



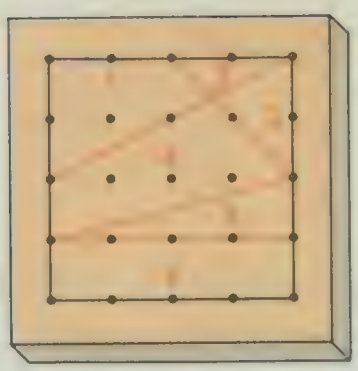
★ 5. Suppose that the distance between two adjacent nails on a geoboard either across or down is 1 unit.
   - A How many segments are 1 unit long?
   - B How many segments are 2 units long?
   - C How many segments are 3 units long?
   - D How many segments are 4 units long?



Half-lines and Half-planes

Investigating the Ideas

The four colored segments divide the square region on the geoboard into six smaller regions.



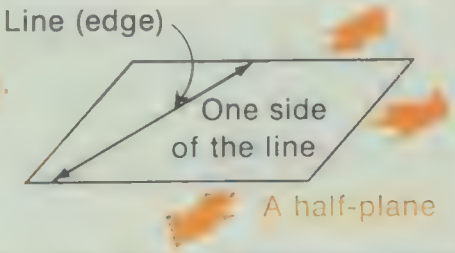
Can you find the fewest and the greatest number of regions into which a square can be divided by four segments whose endpoints are on two different sides of the square?

Record your findings on dot paper.

Discussing the Ideas

The region on each of the two sides of a line in a plane is called a **half-plane**.

We say: A line in a plane separates the plane into three sets, the line and two half-planes.

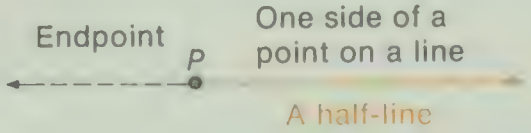


1. The red line on a hockey rink suggests a line that separates a plane. What are some other examples?

2. Do you think that a half-plane includes the separating line or edge?

Each of the two sides of a point on a line is called a **half-line**.

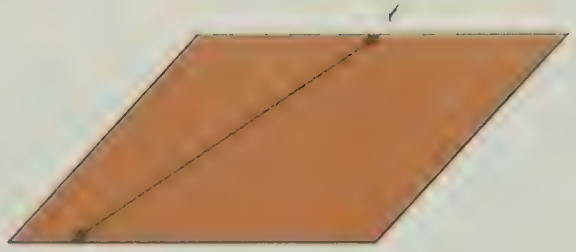
We say: A point on a line separates the line into three sets, the point and two half-lines.



3. What are some physical objects that suggest that a point separates a line?

4. Does a half-line include the endpoint?

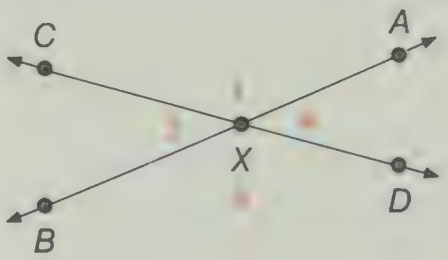
1. **A** Draw a picture of a plane with line ℓ separating the plane into two half-planes.



B Mark a point P in one of the half-planes and a point Q in the other half-plane. Draw segment PQ. Does segment PQ necessarily intersect line ℓ?

C Mark another point R in the same half-plane as point Q. Does RQ intersect line ℓ? Does PR intersect line ℓ?

2. **A** Draw a figure like the one at the right.



B Shade (▨) the half-plane that is the C-side of $\overleftrightarrow{AB}$.

C Shade (▤) the half-plane that is the B-side of $\overleftrightarrow{CD}$.

D Which region has been shaded twice, 1, 2, 3, or 4?

3. Refer to $\overleftrightarrow{WX}$ to give the correct letter for each ▨.

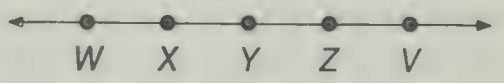
A X and Z are on opposite sides of ▨.

B Z and ▨ are on the same side of Y.

C W and Y are on opposite sides of ▨.

D W, Y and Z are on the same side of ▨.

E V and X are on the same side of ▨.

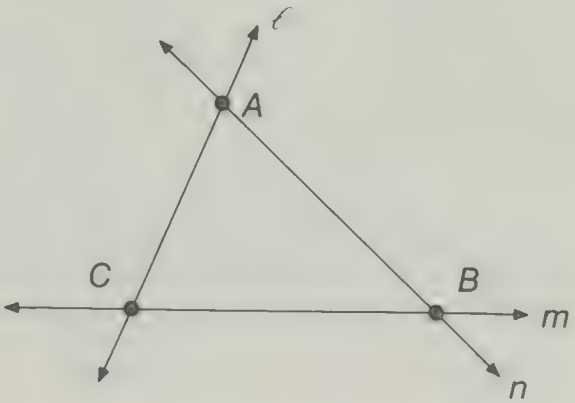


4. **A** Shade (▩) the set of points that are on the B-side of ℓ.

B Shade (▧) the set of points that are on the C-side of n.

C Shade (▨) the set of points that are on the A-side of m.

D Describe the set of points that are in all three half-planes.

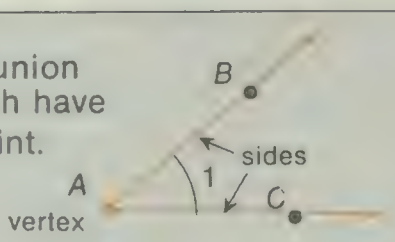


Investigating the Ideas

A ray is the union of a half-line and its endpoint.

NAME: Ray AB SYMBOL: $\overrightarrow{AB}$

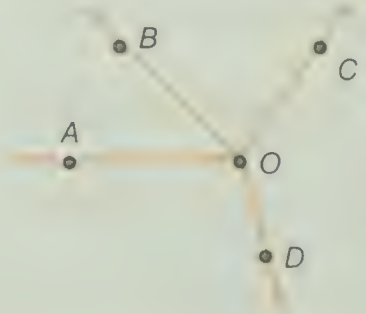
An angle is the union of two rays which have the same endpoint.



NAMES: Angle A, Angle BAC, Angle 1

SYMBOLS: $\angle A$, $\angle BAC$, $\angle 1$

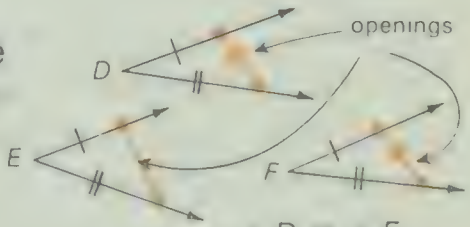
How many rays and angles can you find and name in the figure at the right?



Discussing the Ideas

1. In naming an angle with three letters, such as $\angle BAC$, which letter denotes the vertex of the angle?

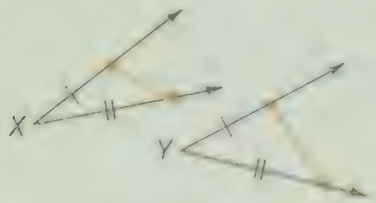
2. If the openings of two angles are the same, then the two angles are **congruent** ($\cong$). If the two angles are not congruent, then one angle is **greater than** or **less than** the other. In the following pairs of figures, the same number of slash marks on the segments tells you which pairs of segments are congruent. How do each pair of angles below compare in size?



$\angle D \cong \angle F$

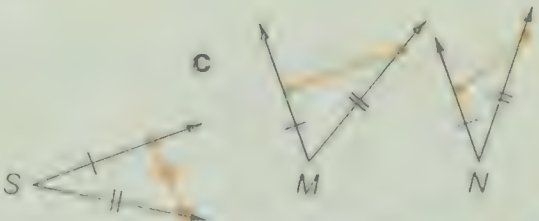
$\angle E > \angle F$, $\angle D < \angle E$

A



B

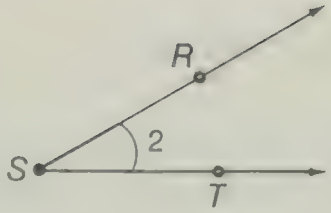
C



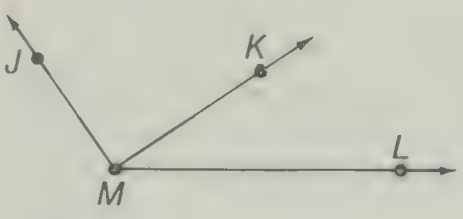
3. A The **interior** of $\angle BAC$ is the intersection of two half-planes as shown in the figure. Describe the two half-planes.

   B What is meant by the **exterior** of the angle?

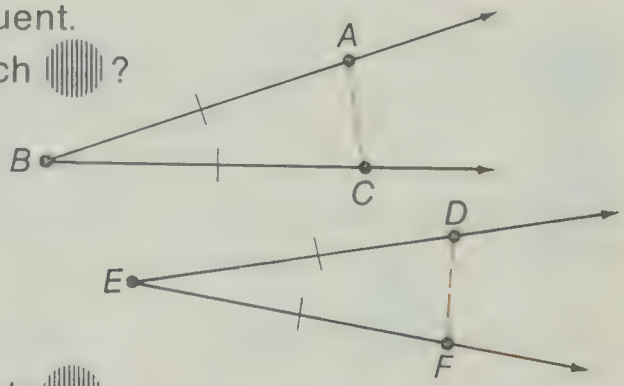
1. **A** Name the rays that form the angle.
   B Which point is the vertex of the angle?
   C Use three different symbols to name the angle.



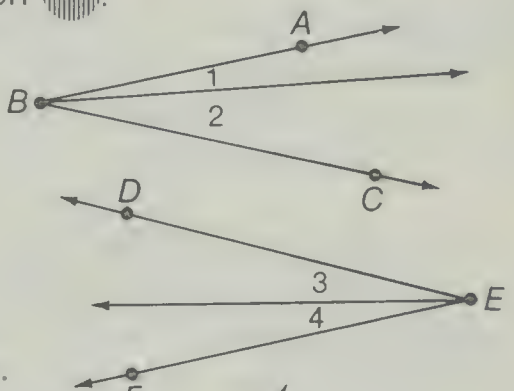
2. How many different angles are shown in the picture? Use three letters to name each angle.



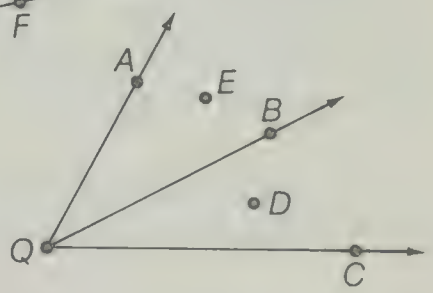
3. Suppose that $\overline{BA}$, $\overline{BC}$, $\overline{ED}$, and $\overline{EF}$ are congruent.
   Which symbol ($<$, $>$, or $\cong$) should go in each ▨?
   A If $\overline{AC} < \overline{DF}$, then $\angle B$ ▨ $\angle E$.
   B If $\overline{AC} > \overline{DF}$, then $\angle B$ ▨ $\angle E$.
   C If $\overline{AC} \cong \overline{DF}$, then $\angle B$ ▨ $\angle E$.



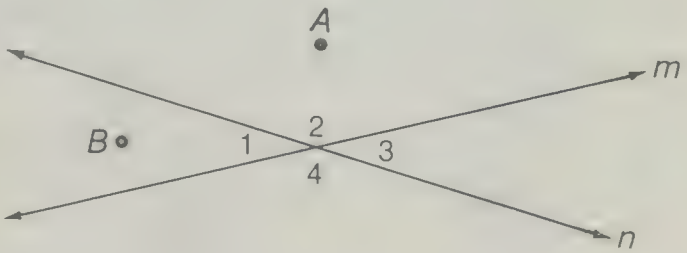
4. Give the correct symbol ($<$, $>$, or $\cong$) for each ▨.
   A If $\left\{\begin{array}{l}\angle 1 \cong \angle 4 \\ \angle 2 \approx \angle 3\end{array}\right\}$ then $\angle ABC$ ▨ $\angle DEF$.
   B If $\left\{\begin{array}{l}\angle 1 < \angle 4 \\ \angle 2 \cong \angle 3\end{array}\right\}$ then $\angle ABC$ ▨ $\angle DEF$.
   C If $\left\{\begin{array}{l}\angle ABC > \angle DEF \\ \angle 1 \cong \angle 4\end{array}\right\}$ then $\angle 2$ ▨ $\angle 3$.



5. Complete each sentence by naming an angle.
   A Point E is in the interior of $\angle$?.
   B Point D is in the exterior of $\angle$?.
   C Point B is in the interior of $\angle$?.
   D Point A is in the exterior of $\angle$?.

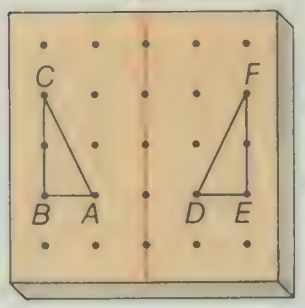


6. Draw two intersecting lines m and n and choose points A and B as shown.
   Shade (▨) the half-plane that is on the B-side of m.
   Shade (▨) the half-plane that is on the A-side of n.
   A What is the intersection of the shaded regions?
   B What is their union?



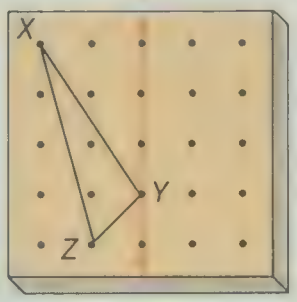
Investigating the Ideas

If line ℓ on the geoboard represented an upright mirror, you could see the reflection of $\triangle ABC$ in the mirror. The reflection is $\triangle DEF$. You can visualize this by thinking of folding the geoboard at the line. Then the two triangles would match.



Can you show how the reflection of of $\triangle XYZ$ would look on the geoboard?

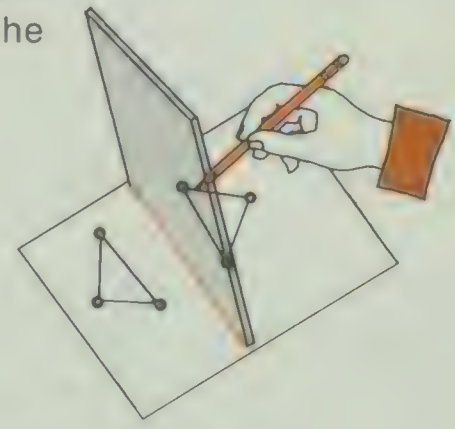
Record the reflection on dot paper.



Discussing the Ideas

1. The triangle that you found for the question above is the **reflection image** of triangle XYZ in the line ℓ. Does the reflection image that you found have the same size and shape as the original triangle?

2. You can use a rectangular piece of clear plastic to help you draw reflection images of figures on paper. Hold the plastic upright to the paper. Look through the plastic to see the reflection image. Try drawing the reflections of various figures.



3. Do you think that the reflection image of a geometric figure in a line is always the same size and shape as the original figure?

4. The region shown is a **symmetric region** with ℓ as a **line of symmetry**. Each half of the region is a reflection image of the other.

How can you use a plastic mirror to see if these regions are symmetric?

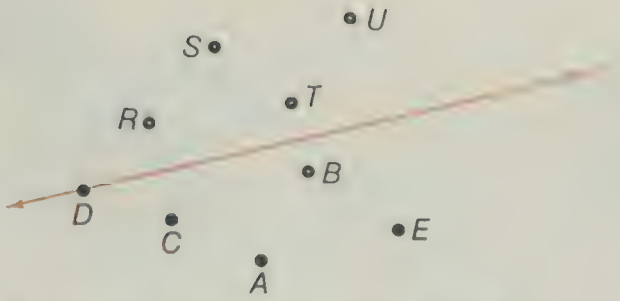
A

B

C

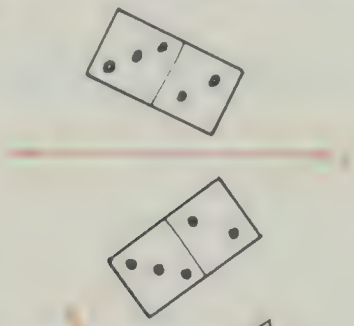
D

1. Which point is the reflection image of each of the points *A*, *B*, *C*, *D*, and *E* in line ℓ?

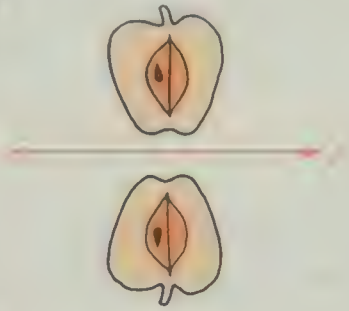


2. For each picture, tell whether the two objects are reflection images of each other in the line.

A

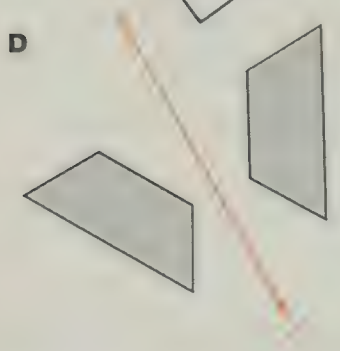


B



C

D



E

CHOICE

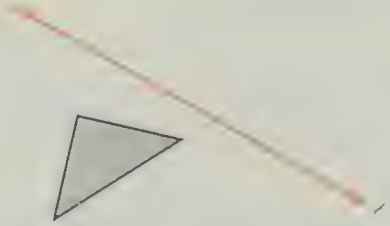
CHOICE

F

3. Copy each figure below. Then draw the reflection image of each figure in the line.

A

B



C

4. Which regions are symmetric?

A

B

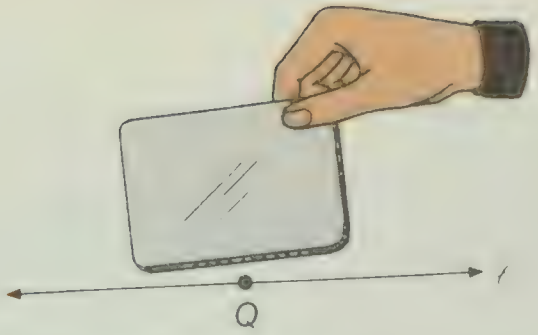
C

D

5. Draw four segments on your paper. Use your plastic mirror and reflections to find the **midpoint** (middle point) of each segment.

Investigating the Ideas

Draw a line ℓ and mark a point Q on the line.



Can you find a line m through Q so that ℓ is the reflection image of itself when m is the reflecting line?

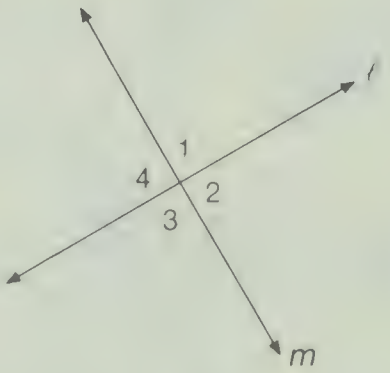
Discussing the Ideas

1. The lines ℓ and m are called **perpendicular lines**. You can write this as $\ell \perp m$.

   Is each line the reflection image of itself in the other line?

2. What are some physical objects that remind you of perpendicular lines?

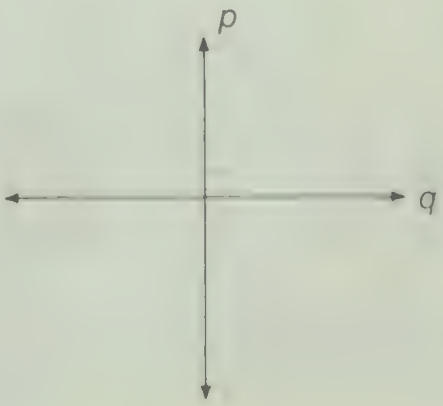
3. If $\ell \perp m$, the angles shown are congruent to each other. Explain how you can use a plastic mirror to check whether or not lines ℓ and m are perpendicular.



4. When the two rays of an angle lie on a line and are opposite in direction, the angle is called a **straight angle**. Give some names for the straight angle at the right with point S as the vertex.

5. The four non-straight angles formed by two perpendicular lines are called **right angles**. The symbol ⌝ is used on drawings to indicate a right angle.

   What are some objects in your classroom that suggest right angles?



6. Can you explain how to use your plastic mirror to draw a right angle.

1. Which pairs of lines are perpendicular?

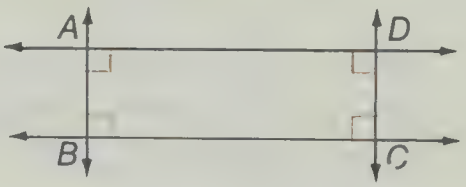
2. A Draw a line t and then mark a point *P* on line *t*. Use your mirror to draw a line perpendicular to line *t* through point *P*.

   B Mark another point *Q* on line *t* and then draw a perpendicular line to line *t* through point *Q*.

   C What do you think is the relationship between the two lines perpendicular to line *t*?

3. Use your plastic mirror to draw a right angle.

4. Use your plastic mirror to draw a figure like this one.



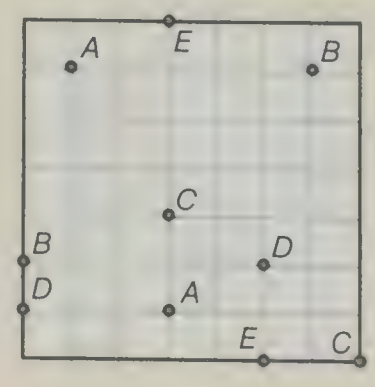
5. Use your mirror to draw a line through point *Q* that is perpendicular to line ℓ.

6. Construct a square with the aid of your plastic mirror.

think

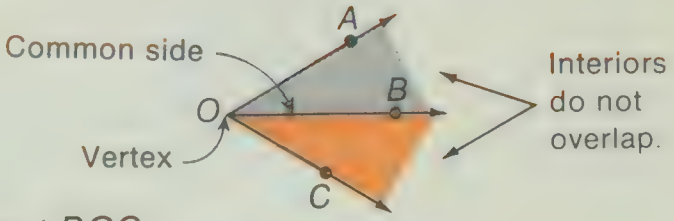
Two points are labelled *A*, two points are labelled *B*, two *C*, two *D*, and two *E*.

Join *A* to *A*, *B* to *B*, *C* to *C*, *D* to *D*, and *E* to *E* with paths along the lines of the grid so that no two paths cross or touch at any point.



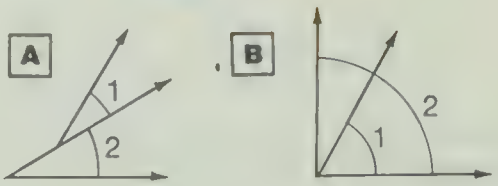
Discussing the Ideas

1. The figure shows that $\angle AOB$ and $\angle BOC$ are **adjacent angles**.

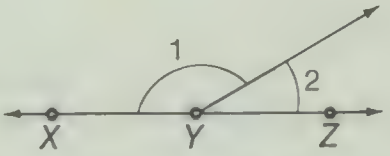


We say: $\angle AOB$ is **adjacent to** $\angle BOC$.

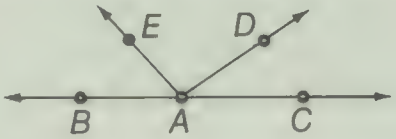
In **A** and **B**, explain why $\angle 1$ and $\angle 2$ are not adjacent angles.



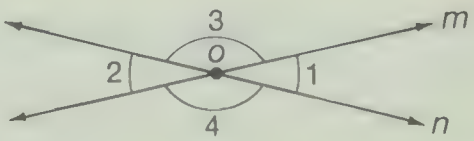
2. If $\angle XYZ$ is a straight angle, then $\angle 1$ and $\angle 2$ are **supplementary angles**.



Which two pairs of angles are supplementary if $\angle BAC$ is a straight angle?



3. Lines m and n intersect at point O.



We say: $\angle 1$ and $\angle 2$ are **vertical angles**.
$\angle 3$ and $\angle 4$ are **vertical angles**.

A Do you think pairs of vertical angles are the same size?

B Can you find a way to use reflections to compare two vertical angles?

4. An angle smaller than a right angle is an **acute angle**.

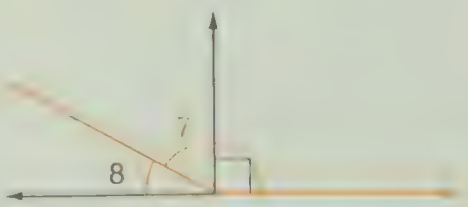
Is $\angle 6$ an acute angle?

$\angle 5$ is an acute angle

5. An angle greater than a right angle but less than a straight angle is called an **obtuse angle**.

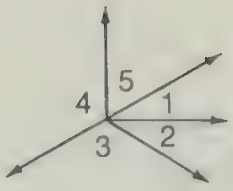
A Is $\angle 8$ obtuse or acute?

B What name is given to the pair $\angle 7$ and $\angle 8$?

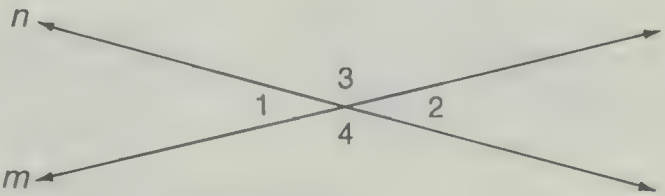


$\angle 7$ is an obtuse angle

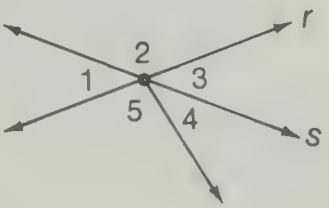
1. Using the figure at the right, name
   - **A** five pairs of adjacent angles.
   - **B** two pairs of angles that are not adjacent.



2. Complete each sentence with the correct word (**vertical**, **supplementary**).
   - **A** Angles 1 and 3 are ? .
   - **B** Angles 3 and 4 are ? .
   - **C** Angles 2 and 4 are ? .
   - **D** Angles 1 and 2 are ? .
   - **E** Angles 2 and 3 are ? .



3. In the figure, r and s are lines. Indicate which pairs of angles are
   - **A** vertical angles.
   - **B** supplementary angles.

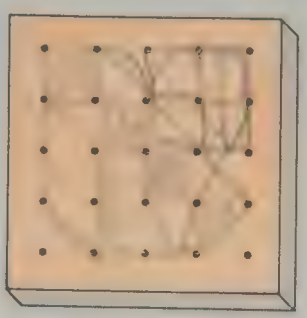


4. Tell whether each of the numbered angles in the picture at the right is
   - **A** right.
   - **B** acute.
   - **C** obtuse.
   - **D** straight.

5. Use the words **acute** or **obtuse** to complete each sentence.
   - **A** An angle that is congruent to an obtuse angle is an ? angle.
   - **B** An angle that is supplementary to an acute angle is an ? angle.
   - **C** If $\angle A$ and $\angle B$ are vertical angles with $\angle A$ acute, then $\angle B$ is ? .
   - **D** The supplement of an obtuse angle is an ? angle.
   - **E** An angle that is less than a right angle is an ? angle.
   - **F** A right angle is less than an ? angle.

think

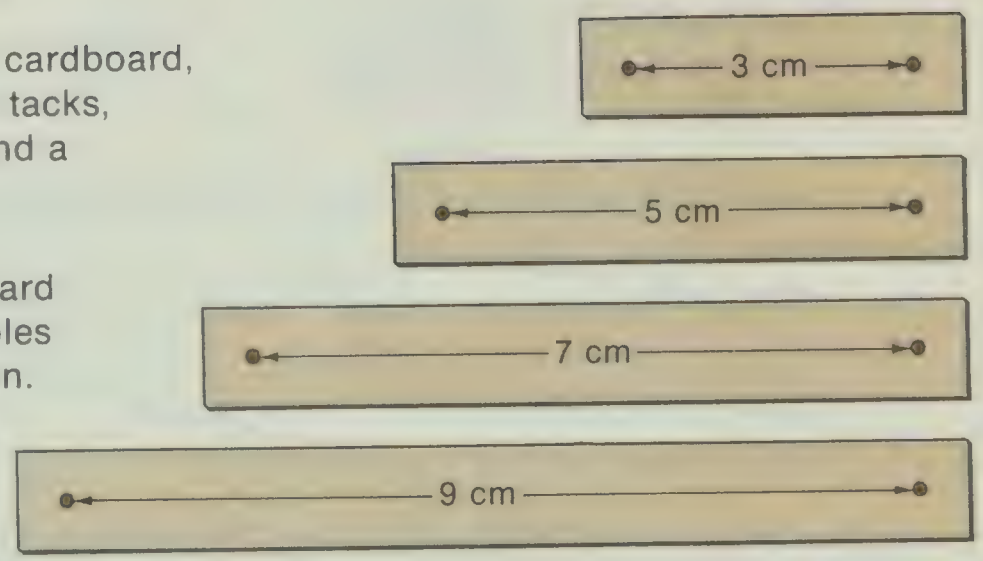
Form an outline of a cross with a rubber band on a geoboard so that five nails are inside the cross and eight nails are outside the cross and all sides of the cross are congruent segments.



Investigating the Ideas

You will need some cardboard, three pins or thumb tacks, a pair of scissors, and a centimetre ruler.

Cut out four cardboard strips and punch holes in the ends as shown.



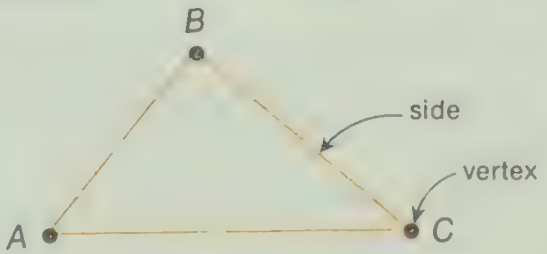
How many different shaped triangles can you make by choosing any three of the strips and putting pins through the holes?

Discussing the Ideas

1. Did you find three of the strips that would not form a triangle? Which three?

2. Can you make triangles of different shape using the same three strips?

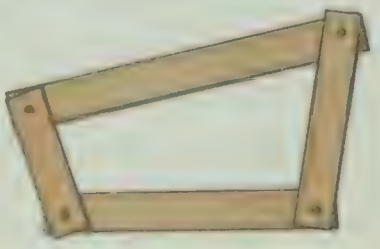
3. A **triangle** in geometry, consists of three points, not all on one line, and all the points on the segments determined by the three points.



A triangle is said to be a **rigid** geometric figure. Can you use the strips to demonstrate why a triangle is a rigid figure?

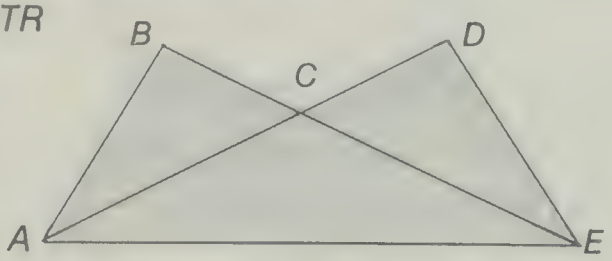
4. A How many **sides** does every triangle have?
   B How many **vertices**? (The plural of vertex is *vertices*.)

5. Join the four strips together to form a 4-sided figure. Is it a rigid figure like a triangle?



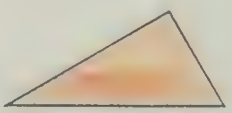
1. **A** Mark three points not in a line on your paper and label them *R*, *S*, and *T*.
   B Join the points with segments.
   C You have drawn triangle *RST* (SYMBOL: $\triangle RST$). What are some other symbols that could name the triangle? EXAMPLE: $\triangle STR$

2. Name as many triangles as you can in this figure.

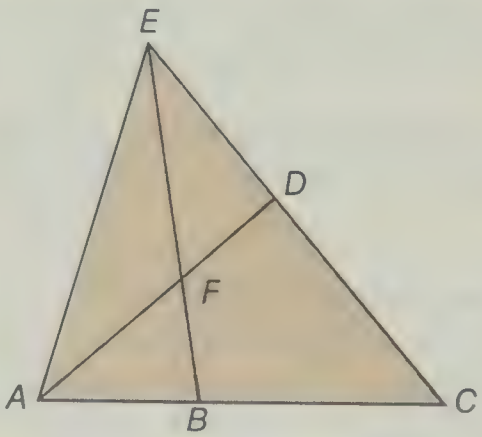


3. Draw a figure that illustrates each situation.
   A A line intersecting a triangle in two points.
   B A line intersecting a triangle in only one point.
   C A line that contains many points of a triangle.

4. A point is in the **interior of a triangle** if it is in the interior of each angle of the triangle. If a point is not in the interior or on the triangle, it is in the **exterior of the triangle.**



Draw a figure and label it like the one shown. Then mark each point described below.



   A Point *X* is in the interior of $\triangle ABE$ and in the exterior of $\triangle AFB$.
   B Point *Y* is in the interior of $\triangle ADE$ and in the exterior of $\triangle AFE$.
   C Point *Z* is in the interior of $\triangle ADC$ and in the interior of $\triangle BCE$.

Think

Choose a 2-digit number such as 48.

Multiply the two digits. $4 \cdot 8 = 32$

Multiply again until you get a 1-digit number. $3 \cdot 2 = 6$

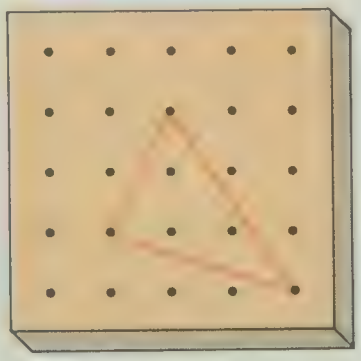
This gives a sequence of three numbers. 48, 32, 6

What 2-digit number will give the longest sequence?

Investigating the Ideas

How many of the triangles described below can you find on your geoboard?	Record your findings on dot paper.

- **A** A triangle with no two sides congruent.
- **B** A triangle with a pair of congruent sides.
- **C** A triangle with all sides congruent.
- **D** A triangle that has a right angle.
- **E** A triangle with the greatest possible number of nails on its sides.
- **F** A triangle with the greatest possible number of nails in its interior.



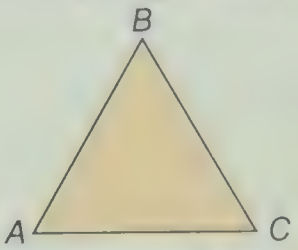
Discussing the Ideas

1. One of the triangles above is impossible to form on the geoboard. Which one do think it is?

2. A **scalene triangle** has no two of its sides congruent. Draw a scalene triangle. Does it have any lines of symmetry?

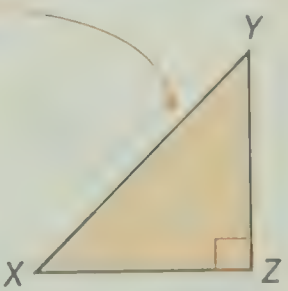
3. An **isoceles triangle** has at least two sides congruent. How many lines of symmetry does an isoceles triangle have?

4. An **equilateral triangle** has all sides congruent. How many lines of symmetry does an equilateral triangle have?



Equilateral $\triangle ABC$

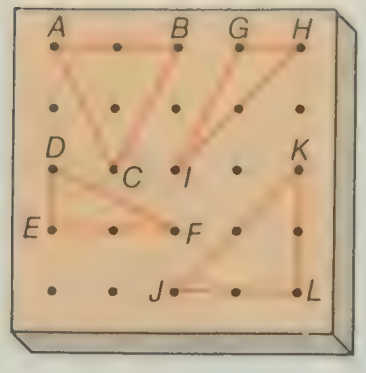
5. A **right triangle** has a right angle.
   - **A** Show a right triangle on the geoboard that is also an isoceles triangle.
   - **B** The longest side of a right triangle is called the **hypotenuse**. Where is the hypotenuse of a right triangle located with respect to the right angle of the triangle?



Right $\triangle XYZ$

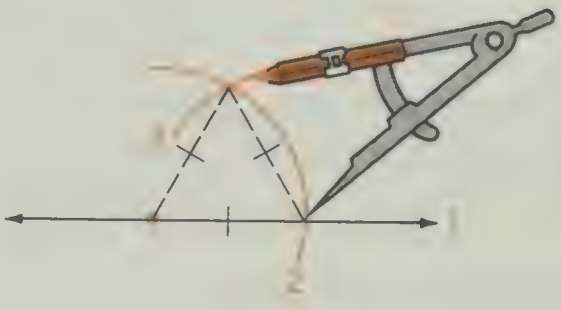
6. Can a triangle have two right angles? Explain.

1. Each triangle on the geoboard has a special name, **scalene**, **isoceles**, **equilateral**, or **right**.
   Give the name of each triangle.
   - **A** $\triangle ABC$
   - **B** $\triangle DEF$
   - **C** $\triangle GHI$
   - **D** $\triangle JKL$



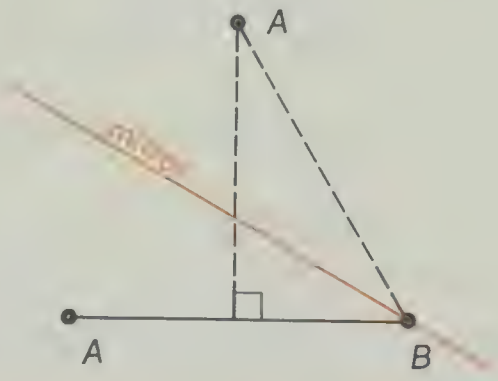
2. What properties would an **isoceles right triangle** have?

3. The drawing shows how a compass may be used to draw an equilateral triangle. Draw an equilateral triangle with sides 6 centimetres long.



4. Draw an isoceles triangle that has a pair of sides 5 centimetres long.

5. Use your plastic mirror and your knowledge about reflections to find each figure.
   - **A** Isoceles triangle
   - **B** Right triangle
   - **C** Equilateral triangle
     (HINT: See the figure at the right.)

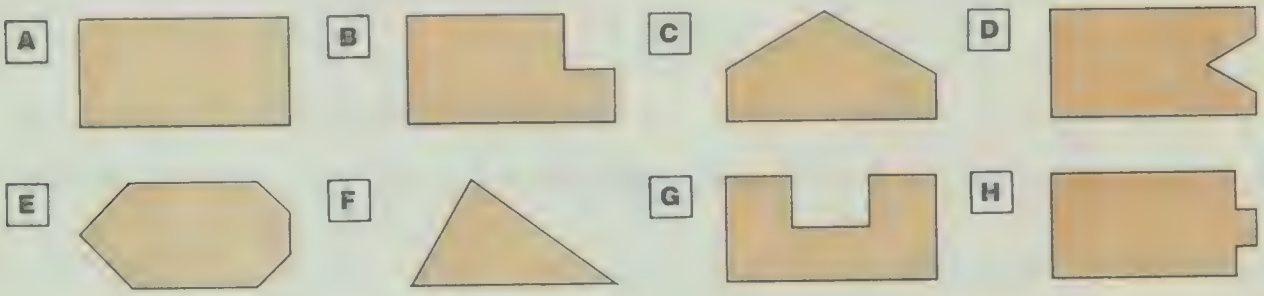


think

There are 47 different triangles in the figure. How many can you name?

Investigating the Ideas

Suppose each drawing below represents a floor plan of a room and you are standing inside the room.



In which rooms could you see all parts of the room no matter at what point you stood in the room?

Discussing the Ideas

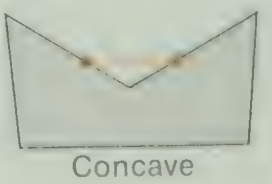
1. The rooms in which you can see all parts of the room from any inside point are **convex** shaped rooms. The others are **non-convex** shaped rooms. Is your classroom convex or non-convex in shape?

2. Can you draw a non-convex triangle?

3. Is the statement true or false?
   All 4-sided geometric figures are convex.
   Draw some pictures that would illustrate your answer.

4. Another way in which you can think about convex and non-convex figures is illustrated at the right.

Convex

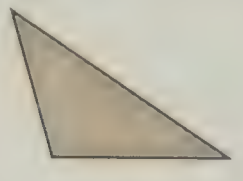

Concave

   A figure is **convex** if every segment joining any two points on the boundary of the figure contains only points in the interior of the figure or on the boundary. Non-convex figures are **concave** figures.

   A Draw a 6-sided figure that is convex.

   B Draw a 6-sided figure and show that it is concave.

1. Tell whether each figure is convex or concave.

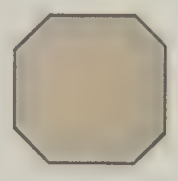
A 

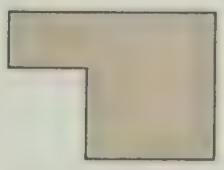
B

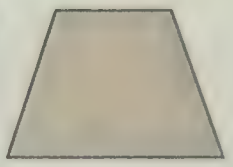
C

D

E

F 

G 

H 

2. Make a copy of each figure below. Then draw a segment that will show that each figure is concave.

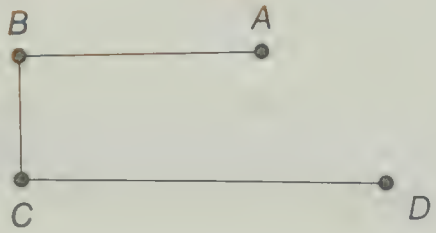
A

B

C

D

3. Draw a figure like the one shown at the right.



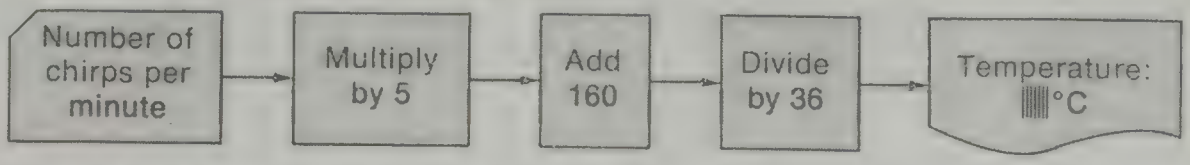
A If *A* and *D* are connected by a segment, is the figure convex or concave?

B Complete the figure with two segments connecting *D* with *A* to form a convex figure.

C Complete the figure with two segments connecting *D* with *A* to form a concave figure.

Think

It is said that the approximate temperature can be found by counting the number of chirps per minute of a cricket. The flow chart below gives the method.



What is the approximate temperature when the number of chirps per minute is

A 40 B 112 C 184 D 4 E 256

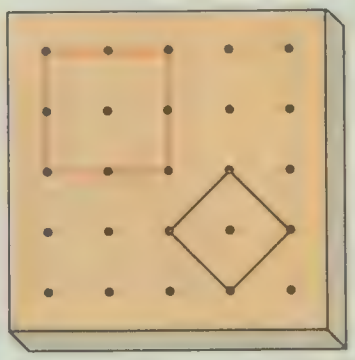
At what temperature would a cricket stop chirping?

Investigating the Ideas

Two squares of different size are shown on the geoboard.

How many squares of different size can you find on your geoboard?

Record your findings on dot paper or graph paper.



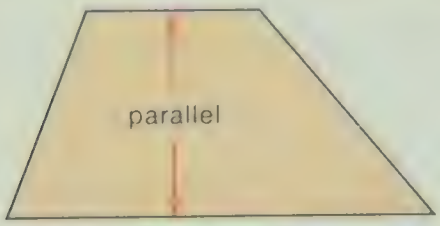
Discussing the Ideas

1. Squares are a special kind of **quadrilateral**. Do you think that a square is a convex or a concave quadrilateral? Give your reasons.

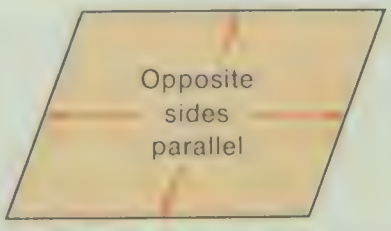
convex quadrilateral

concave quadrilateral

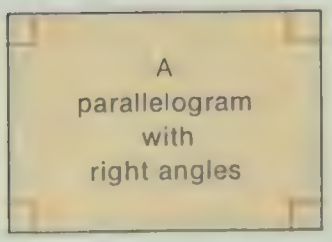
2. Study the special quadrilaterals named below. What other properties do you think each quadrilateral might have besides those that are listed?



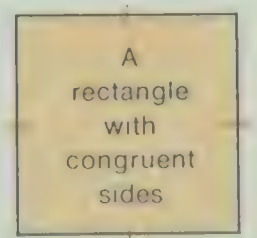
Trapezoid



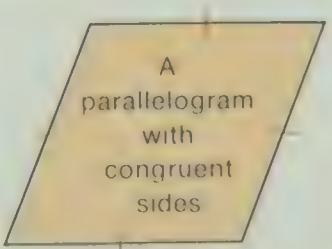
Parallelogram



Rectangle



Square



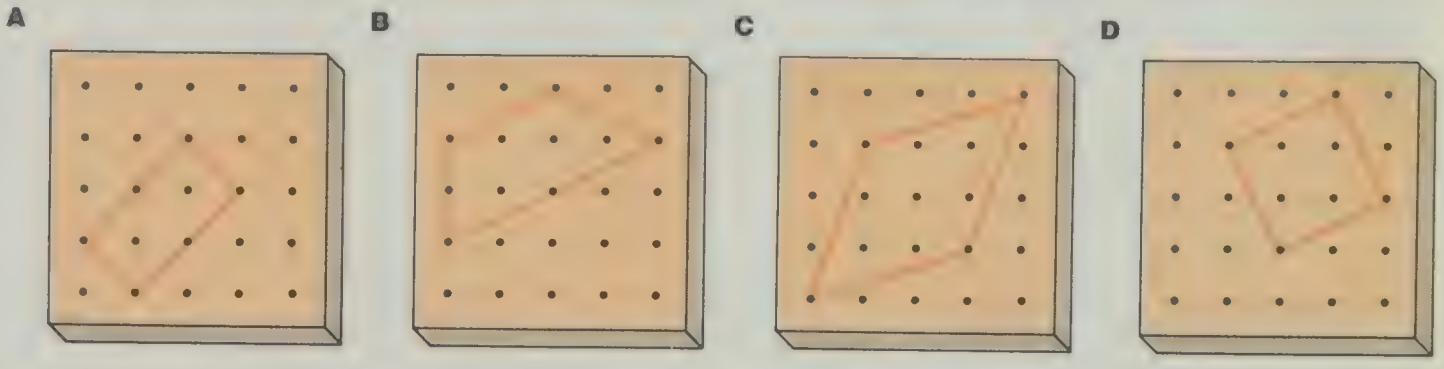
Rhombus

3. Can a quadrilateral have two right angles and not be a rectangle? Illustrate this with your geoboard or by drawing a picture.

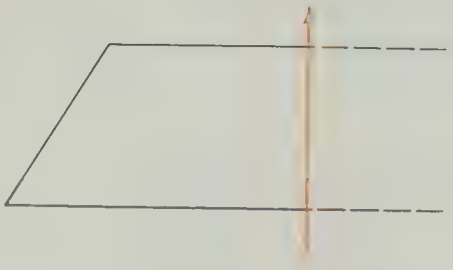
4. Can a quadrilateral have two right angles and not be a trapezoid? Illustrate your answer by drawing a picture.

1. Of the five special kinds of quadrilaterals shown in Exercise 2 on the preceding page, tell which ones have
   - **A** at least one pair of parallel sides.
   - **B** two pairs of parallel sides.
   - **C** all right angles.
   - **D** all sides congruent.

2. Give the name of each special quadrilateral shown on the geoboards.



3. Draw a trapezoid that has a line of symmetry.



You have drawn an **isoceles trapezoid**; that is, a trapezoid with a pair of congruent sides.

4. How many lines of symmetry does a rectangle have? Draw a picture to show the lines of symmetry.

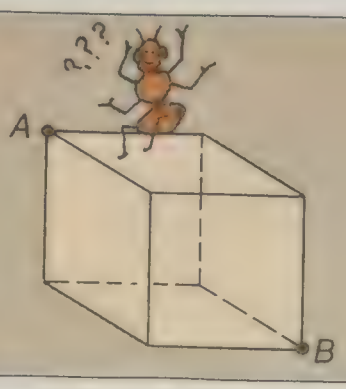
5. Draw a rhombus. Show any lines of symmetry.

6. Draw a square. Show all its lines of symmetry.

7. Construct a rhombus by means of reflections.

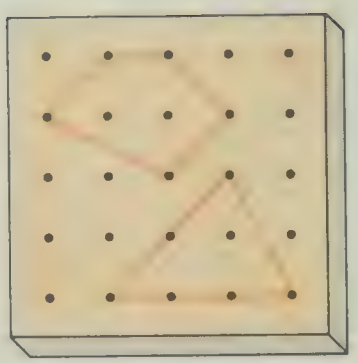
think

Suppose a bug is at point *A*. What is the shortest path he can crawl in order to get to point *B*? Draw this path.



Investigating the Ideas

A triangle is a closed geometric figure with the fewest number of sides.

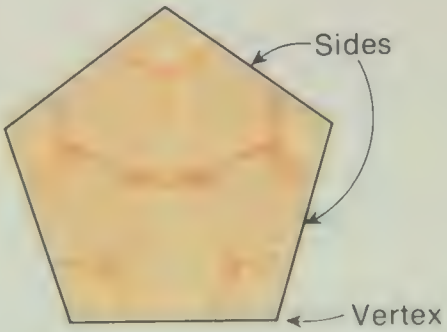


What is the greatest number of sides of any convex figure that you can show on a 5 by 5 nail geoboard?

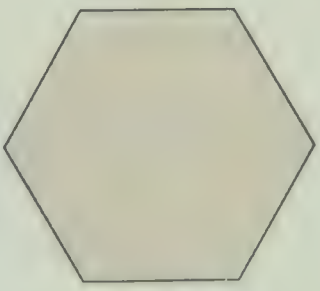
Discussing the Ideas

1. A **polygon** is a closed geometric figure whose boundary is composed of segments. The word polygon comes from
   poly meaning "many" and
   gonia meaning "angle."

   Does every convex polygon have the same number of angles as it has sides?

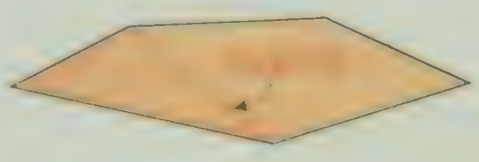


2. A convex polygon is a **regular polygon** if all of its sides are congruent segments and all of its angles are congruent.
   - A What is the name of a regular polygon that has three sides?
   - B What is the name of a regular polygon that has four sides?



3. A Draw a polygon that has four congruent sides but which is not a regular polygon.
   B Draw a polygon that has four angles, all congruent to each other, but which is not a regular polygon.

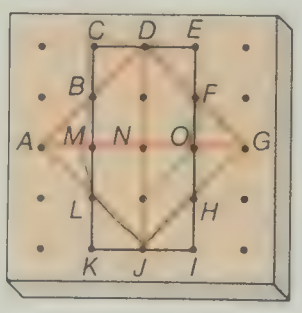
4. A **diagonal** of a polygon is a segment connecting two nonconsecutive vertices of a polygon.
   - A Does a triangle have any diagonals?
   - B How many diagonals does a quadrilateral have?



1. Complete the table.

POLYGON	NAME	NUMBER OF SIDES	NUMBER OF DIAGONALS
	pentagon		
	hexagon		
	heptagon		
	octagon		
	nonagon		
	decagon		

2. Give the name of the polygon (**triangle**, **square**, and so on) shown on the geoboard whose vertices are named below.



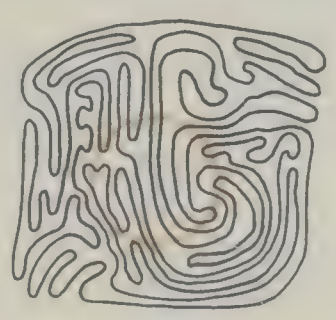
A *ADGJ*
B *FGH*
C *CEIK*
D *MBDFO*
E *DFHJ*
F *BDFHJK*

3. Using the points on the geoboard in Exercise 2, name an octagon. Draw a picture of the octagon.

4. How many regular polygons are shown on the geoboard? Name them.

think

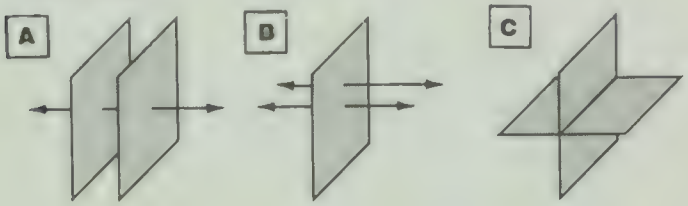
Use tracing paper to copy this figure. It is a **simple closed curve**. (Think of a loop of string that does not cross itself.) Within the colored circle, shade the interior of this simple closed curve.



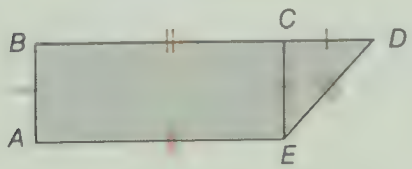
REVIEWING THE IDEAS

1. Give three names for this line.

2. Choose the correct figure suggested by the phrase "two planes intersect in a line."



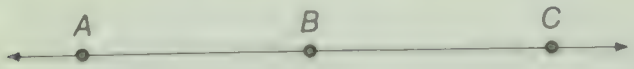
3. Name the pairs of congruent segments in this figure.



4. Complete each sentence.
   - A If $\overline{XY} \cong \overline{RS}$ and $\overline{RS} \cong \overline{MN}$, then $\overline{XY} \cong$? .
   - B If $\overline{CD} < \overline{JK}$ and $\overline{JK} \cong \overline{PQ}$, then ? .

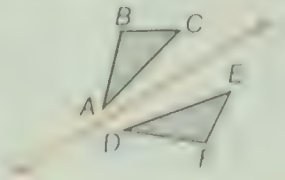
5. A Each of the two sides of a point on a line is called a ? .
   - B Each of the two sides of a line in a plane is called a ? .

6. Name three different rays.

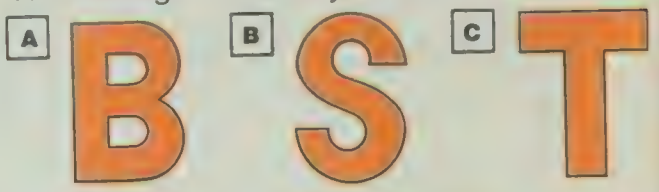


7. Name three angles shown in this figure.

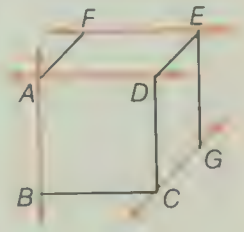
8. Are $\triangle ABC$ and $\triangle DEF$ reflection images of each other?



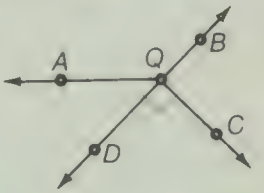
9. Which regions are symmetric?



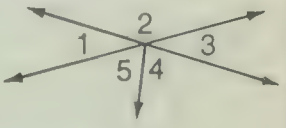
10. Name the kind of lines (**parallel**, **skew**, **perpendicular**) for the pairs shown on the cube.
    - A $\overleftrightarrow{AD}$ and $\overleftrightarrow{FE}$
    - B $\overleftrightarrow{AB}$ and $\overleftrightarrow{CG}$
    - C $\overleftrightarrow{AD}$ and $\overleftrightarrow{AB}$



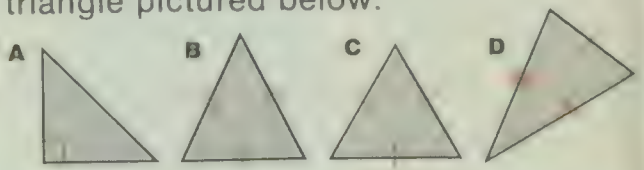
11. Tell which angles are
    - A acute
    - B obtuse
    - C right
    - D straight



12. Tell which pairs of angles are
    - A vertical
    - B supplementary

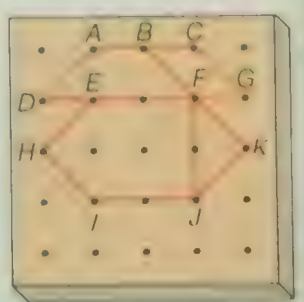


13. Give the name of each kind of triangle pictured below.



14. Which figure is convex?

15. Find and name the following polygons on the geoboard.
    - A trapezoid
    - B parallelogram
    - C pentagon
    - D hexagon



TEST YOURSELF

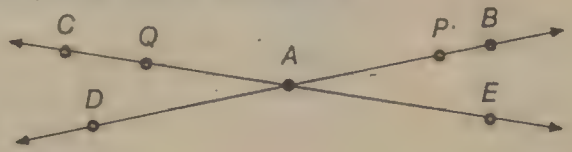
1. Complete the sentence.
   If X and Y are points, there is exactly ? line that contains them.

2. Name all the segments on the line.

3. Use the figure in Exercise 2. Which symbol ($<$, $\cong$, or $>$) goes in the ? $\overline{AB}$? $\overline{AC}$

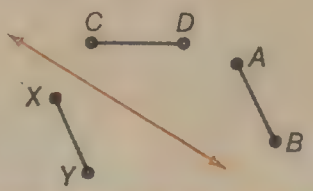
4. All points on one side of a line in a plane is called a ?

5. Refer to the figure below to complete each sentence.



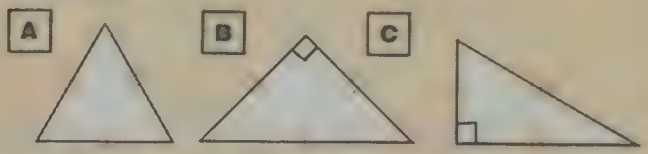
   A The two lines intersect at point ?.
   B The union of $\overrightarrow{AE}$ and $\overrightarrow{AD}$ is $\angle$?.

6. Which segment, $\overline{AB}$ or $\overline{CD}$, is the reflection image of $\overline{XY}$ in the line?

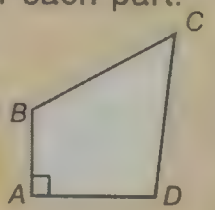


7. Name the kind of lines (**parallel**, **skew**, **perpendicular**) described.
   A Two lines that intersect to form right angles.
   B Two lines in a plane that do not intersect.
   C Two lines that are not in the same plane.

8. Which figure is an isoceles right triangle?



9. Name the type of angle for each part.
   A $\angle BAD$ is a(n) ? angle.
   B $\angle ABC$ is a(n) ? angle.
   C $\angle BCD$ is a(n) ? angle.



10. A Name this special quadrilateral.

   B Is it **convex** or **concave**?

RESEARCH PROJECTS

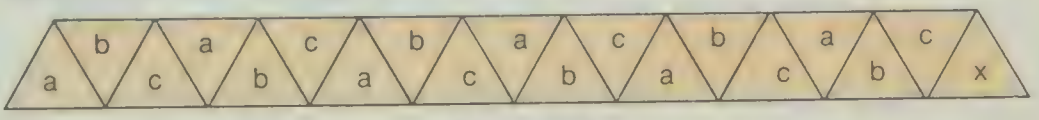
A Straight lines can be used to make some interesting patterns, designs, and figures. Make some designs based on straight lines. (See *Patterns and Puzzles* by Sylvia Horne; Chicago: Lyons and Carnahan, 1968. available from McGraw-Hill Ryerson, Ltd.)

B Make a large poster depicting the Snowflake Curve. This curve has the property that although it has a finite area, it can have infinite perimeter. (See *History of Mathematics* by Carl B. Boyer; New York: 1968, John Wiley and Sons Canada, Ltd., pp. 660–661.)

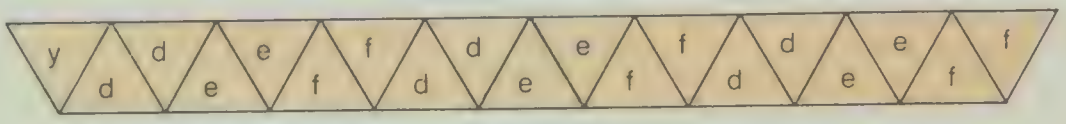
C Do you think that a straight line can be drawn without using a ruler or some other straight edge? Some devices, called **linkages**, have been invented with which to produce straight lines. One such linkage was invented by James Watt. In 1864, a linkage known as Peaucellier's Cell was invented which produced straight line motion. Find out and make some models of these linkages. (See *The Enjoyment of Mathematics* by Hans Rademacher and Otto Toeplitz; Princeton, New Jersey: Princeton University Press, 1957.)

A HEXAFLEXAGON

Use a strip of adding machine paper. Mark off 19 equilateral triangles on the strip. Label the front and back of the strip with the letters as shown in the diagrams below.

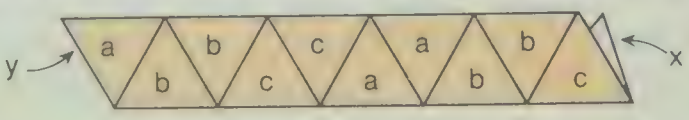


Front side

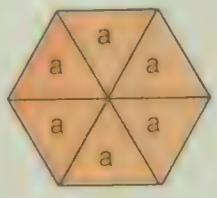


Back side

Fold the strip in a winding motion so that the same letters on the back side face each other; that is, d faces d, e faces e, and f faces f.

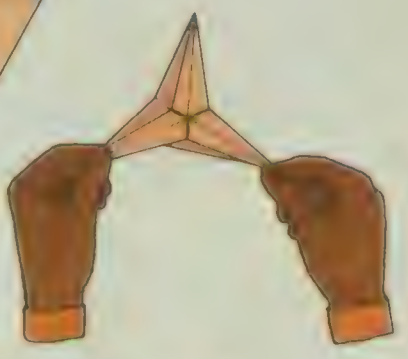


With the strip flat on your desk, fold the strip again with a winding motion so that all the pairs of faces labelled c come face to face. This should cause the strip to fold into a hexagon shaped with all the faces on one side of the hexagon labelled a and all the faces on the other side labelled b.



The two faces labelled x and y are pasted face to face to complete the hexaflexagon.

Now flex your hexagon as shown in the figure by pinching it into a 3-cornered shape and letting it open at the centre. You will expose another face of the hexaflexagon with all of its faces having the same letter on them.



Can you flex your hexaflexagon so that you find all six faces?

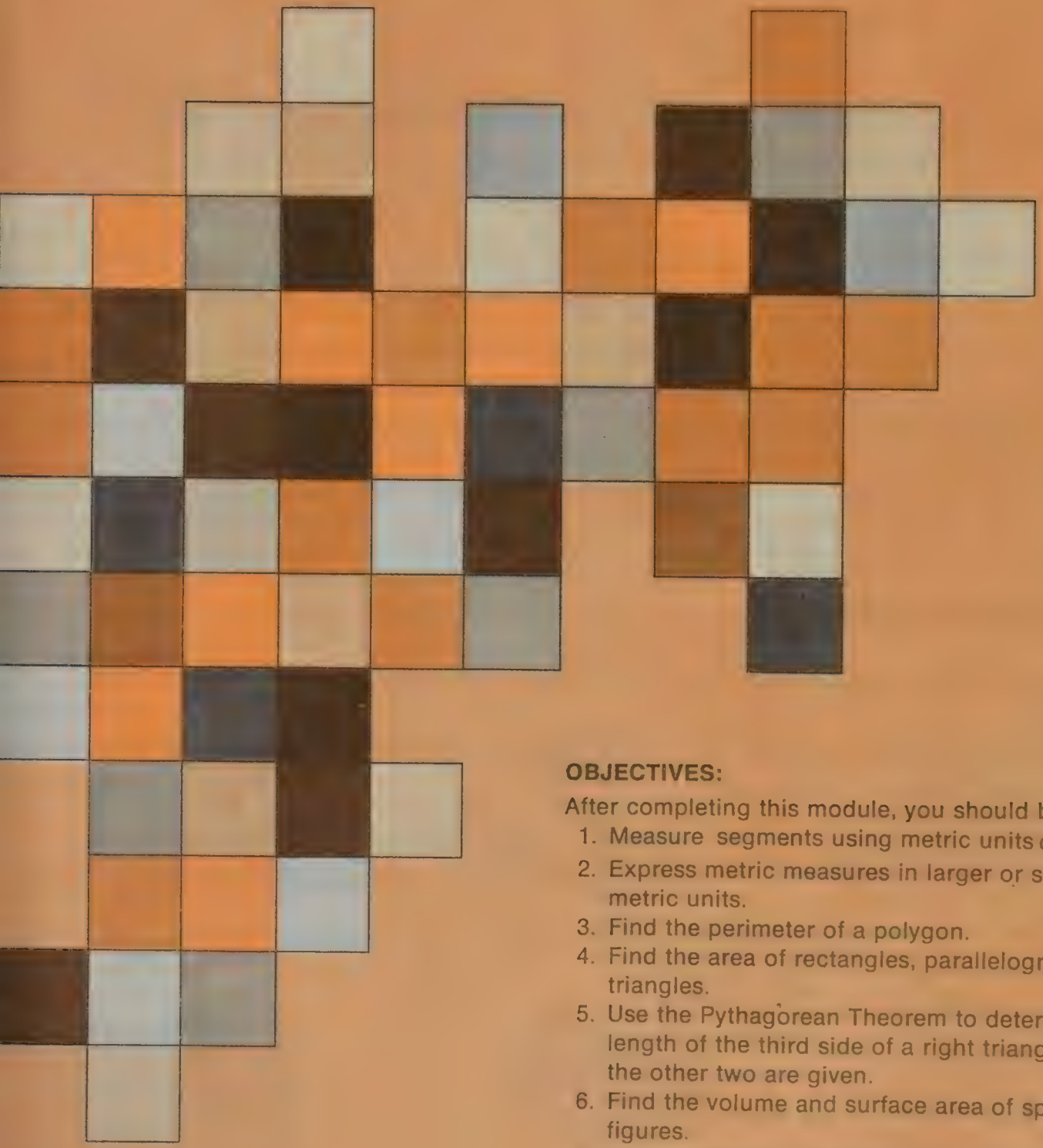
MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

1. one 2. $\overline{AB}$, $\overline{AC}$, $\overline{BC}$ 3. $<$ 4. half-plane 5. A A; B DAE 6. CD 7. A perpendicular; B parallel, C skew 8. B 9. A right; B obtuse; C acute 10. A parallelogram; B convex

UNIT C: *Geometry and Measurement*

MODULE 2: *Measurement*



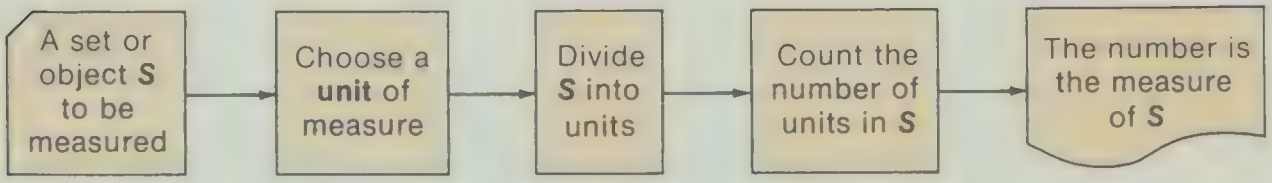
OBJECTIVES:

After completing this module, you should be able to:

1. Measure segments using metric units of length.
2. Express metric measures in larger or smaller metric units.
3. Find the perimeter of a polygon.
4. Find the area of rectangles, parallelograms, and triangles.
5. Use the Pythagorean Theorem to determine the length of the third side of a right triangle when the other two are given.
6. Find the volume and surface area of space figures.
7. Use a protractor to measure angles.
8. Find sums and differences of angle measures given in degrees, minutes, and seconds.
9. Find the sum of the angle measures of triangles and other polygons.

Investigating the Ideas

When you **measure** a set or object, you assign a number to it. The flow chart below shows the measurement process.


A set or object **S** to be measured
Choose a **unit** of measure
Divide **S** into units
Count the number of units in **S**
The number is the measure of **S**

A measure of a segment is called **length**.

A measure of a plane region is called **area**.

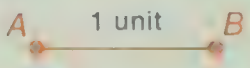
A measure of a space figure is called **volume**.

Can you choose a unit of measure and find the measure of at least one of these objects?	**A** The length of one side of your classroom. **B** The area of a desk top or table top. **C** The volume of a box.

Discussing the Ideas

1. Explain how you measured the object that you chose?

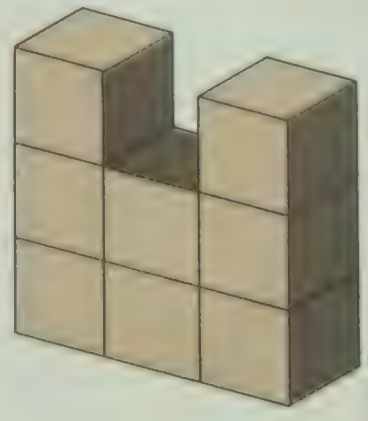
2. To measure **length**, a *segment* must be chosen as the unit.
   - **A** If $\overline{AB}$ is the unit, estimate the length of your desk top.
   - **B** Find a way to check your estimate.



3. To measure the **area** of a plane region, a *square region* is chosen as the unit.
   - **A** Estimate the area of the front cover of your book using the square unit shown.
   - **B** Find a way to check your estimate.

4. To measure the **volume** of a space figure, a *cube* is chosen as the unit.

   Using the cubic unit shown, find the volume of the figure at the right.



1. When measuring each item described, would you be finding length, area, or volume?
   - **A** The height of a flag pole.
   - **B** The space inside of a drawer.
   - **C** The size of a patio.
   - **D** The distance of a bike trail.
   - **E** The floor space in a cabin.
   - **F** The amount of concrete needed for a sidewalk.

2. Tell whether **length**, **area**, or **volume** is being described in each part.
   - **A** Joan claimed that she painted the entire inside of the closet.
   - **B** Joan then said that she had completely filled the closet with boxes of clothes.
   - **C** The closet was wide enough to accomodate three boxes.
   - **D** She also noted that she had used two cans of paint for the job.
   - **E** In putting the cartons into the closet, she figured that she had to walk back and forth across the room 115 times.
   - **F** Joan's father noted that she had forgotten to paint the closet door.

3. If ⊢—⊣ is the unit, how long is each segment.

   A *C* ⊢——⊣ *D* **B** *E* ⊢————⊣ *F*

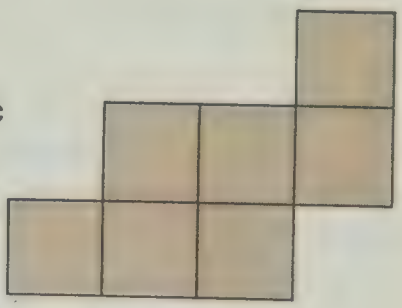
   C *G* ⊢————⊣ *H* **D** *I* ⊢——⊣ *J*

4. If □ is the unit, what is the area of each region?

   A

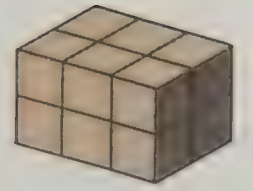
   B

   C

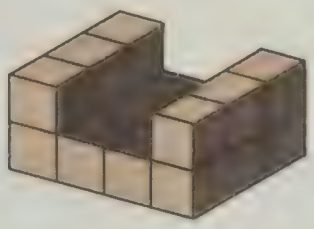
   

5. If is the unit, what is the volume of each space figure?

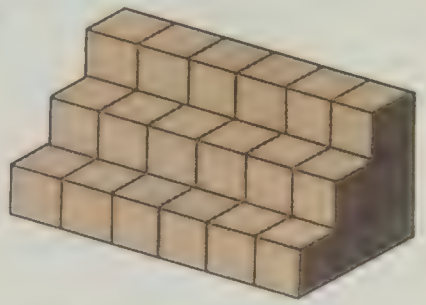
   A

   B

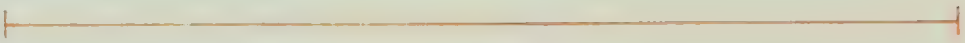
   C

6. Suppose a cube whose edges were only half as long as the cube in Exercise 5 was the unit of measure, what would be the volume of the figure in part **A**?

Investigating the Ideas

This segment has a length of one decimetre.



A Is your hand more or less than one decimetre wide?

B Estimate your height in decimetres.

Can you estimate the length or width of your classroom in decimetres and then find a way to check your estimate?

Discussing the Ideas

1. The standard unit of length in the metric system is a **metre** (m). A **decimetre** is one-tenth as long as a metre.

   Cut a strip of paper or string to be one metre in length. Use the decimetre above to help you.

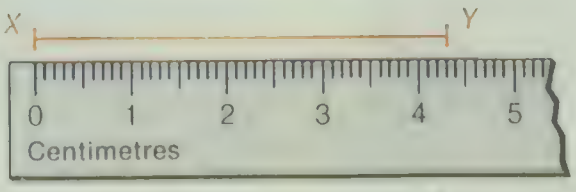
2. **A** About how many decimetres long is one of your arms?
   B What part of a metre is the length of one of your arms?

3. The table of prefixes will help you understand other units in the metric system. Fill in the screens.

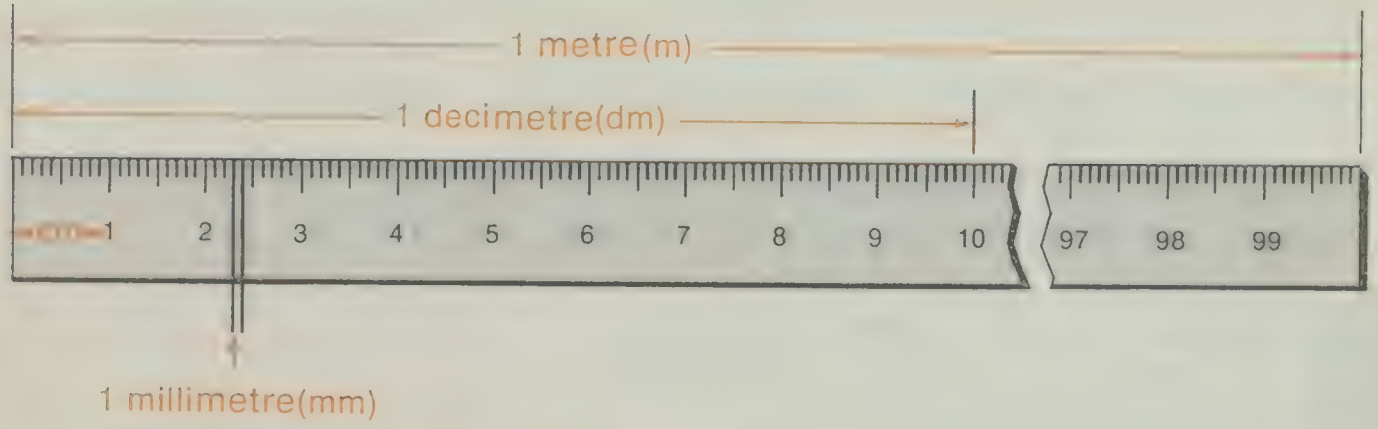
PREFIX	MEANING	PREFIX	MEANING
deca-	ten	**deci-**	one tenth
hecto-	one hundred	**centi-**	one hundredth
kilo-	one thousand	**milli-**	one thousandth

   A 1 kilometre (km) = ▒ m
   B 1 decametre (dam) = ▒ m
   C 1 centimetre (cm) = ▒ m
   D 1 millimetre (mm) = ▒ m
   E 1 decimetre (dm) = ▒ m
   F 1 hectometre (hm) = ▒ m

4. **A** What is the length of $\overline{XY}$ to the nearest whole centimetre?
   B What is the length of XY in millimetres?
   C If the length of XY in centimetres is 4.3 cm, what is its length in decimetres?
   D What is the length of XY in metres?



1. Give the missing number for each part.
The diagram may help you.



A 1 m = ▒▒ cm
B 1 dm = ▒▒ cm
C 1 m = ▒▒ dm
D 1 dm = ▒▒ mm
E 1 m = ▒▒ mm
F 2 cm = ▒▒ mm
G 30 mm = ▒▒ cm
H 500 cm = ▒▒ m
I 80 cm = ▒▒ dm
J 23 cm = ▒▒ mm
K 7 m = ▒▒ cm
L 1000 cm = ▒▒ m

2. Give the length of each segment in millimetres.
Then give the length in centimetres.

A
B
C
D

3. Draw segments having these lengths.

A 9.4 cm
B 7.5 cm
C 60 mm
D 1.2 dm
E 39 mm
F 0.02 m
G 0.5 dm
H 0.083 m

4. Express each measure in metres.

EXAMPLE: 2 m 3 dm 8 cm = 2.38 m

A 5 m 6 dm 2 cm
B 1 m 3 dm 4 cm
C 8 dm 5 cm
D 67 cm
E 23 cm 8 mm
F 2 m 2 dm 2 cm 2 mm
G 1 dam 8 m 7 dm
H 1 km 7 hm 5 dam
I 1 km 5 cm 3 mm

5. A kilometre (km) is 1000 metres. Give the missing numbers for each part.

A 1 km = ▒▒ hm
B 1 km = ▒▒ dam
C 1 km = ▒▒ m
D 1 km = ▒▒ dm
E 1 km = ▒▒ cm
F 1 km = ▒▒ mm

Polygonal Paths and Perimeter

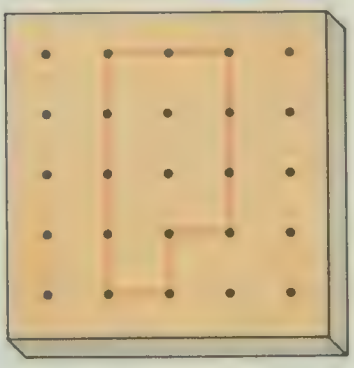
Investigating the Ideas

The **polygonal path** $ABCDEF$ is the union of segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, and $\overline{EF}$.

Its length is: length $\overline{AB}$ + length $\overline{BC}$ + length $\overline{CD}$ + length $\overline{DE}$ + length $\overline{EF}$.

If a region has a boundary which is a polygonal path, then the **perimeter** of the region is the length of the path.

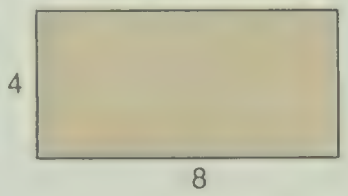
?	Can you show eight different-shaped regions on your geoboard that have a perimeter of 12 units?
	Record your regions on dot paper or graph paper.



Discussing the Ideas

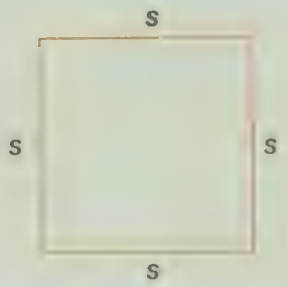
1. What is the length of the polygonal path $ABCDEF$ in the Investigation? Use your centimetre ruler.

2. Explain how to find the perimeter of this rectangular region.



3. If the length of one side of a square is s units, then the perimeter P is given by the formula

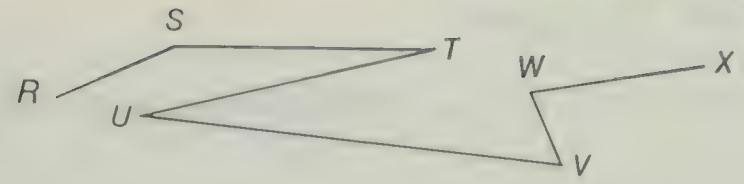
$$P = s + s + s + s = 4 \cdot s$$



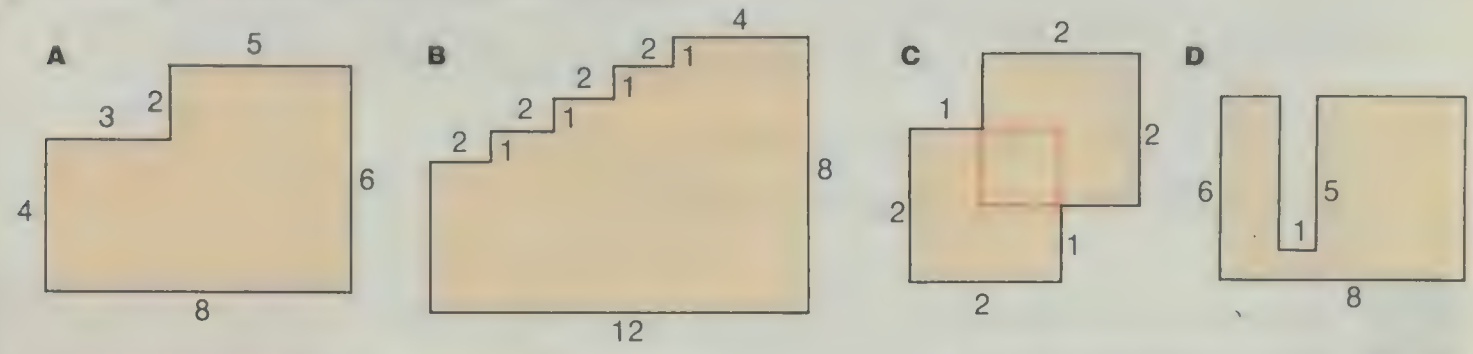
   A What is the perimeter if $s = 7$ cm?
   B What is the perimeter if $s = 2.1$ m?

4. What would be a formula for the perimeter of each of the figures at the right? Each have sides with length s.

1. Find the length of the polygonal path *RSTUVWX*.

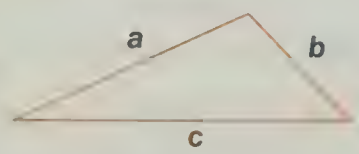


2. What is the perimeter of each region?



3. The formula for the perimeter of a triangle is

$$P = a + b + c$$



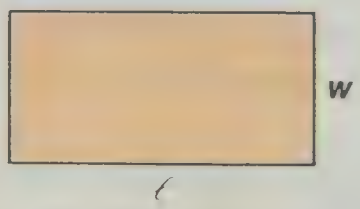
where **a**, **b**, and **c** are the lengths of the sides of the triangle. Use the formula to find the missing part in each row of the table.

	a	b	c	P
A	16	9	24	▒
B	$24\frac{1}{4}$	$19\frac{1}{2}$	▒	67
C	23.7	▒	31.3	78.4

4. A formula for the perimeter of the rectangular region would be

$P = \text{▒} \cdot \ell + \text{▒} \cdot w$ or $P = \text{▒} \cdot (\ell + w)$

Complete the sentence by giving the number for the screens.

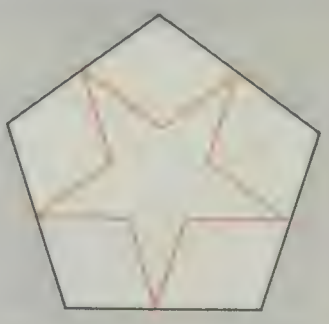


5. Use the formula for the perimeter of a rectangle to find the missing numbers in each part of the table.

	ℓ	w	P
A	12	7	▒
B	4.3	2.9	▒
C	▒	31	200

Think

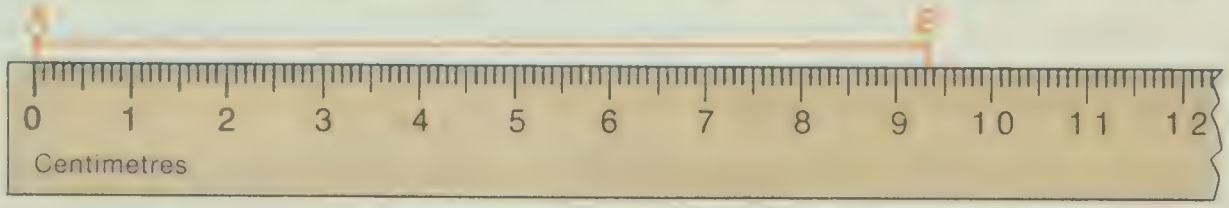
What is the perimeter of the large region? What is the perimeter of the star?



6. A rectangle has a perimeter of 24 centimetres. What could be the length and width of the sides if each side is a whole number of centimetres?

Investigating the Ideas

In measurement, the number that you get depends upon the unit you choose.



The length of $\overline{AB}$ **to the nearest decimetre** is 1.

The length of $\overline{AB}$ **to the nearest centimetre** is 9.

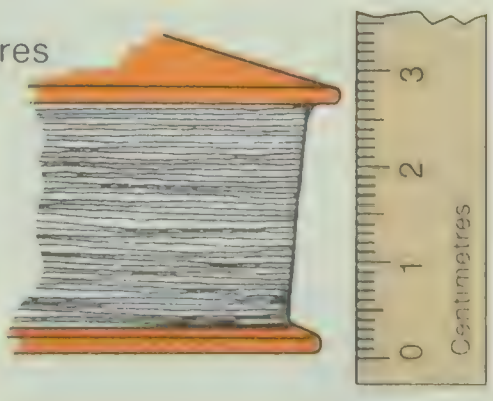
Can you measure one or more of the following objects and record their measures to the nearest unit?

- **A** The length of your desk top in decimetres.
- **B** The length of your room in metres.
- **C** The length of a book in centimetres.

Discussing the Ideas

1. Did any of the measurements that you made come out to exactly a whole number of units?
   How did you decide which number of units to record if the length was between two whole units?

2. Suppose a student said that a book was 3 centimetres thick, **to the nearest centimetre**.
   - **A** Could it be as thin as 2.1 centimetres?
   - **B** Could it be 2.8 centimetres?
   - **C** Could it be as much as 3.7 centimetres thick?



3. When a measurement is reported as 3 centimetres, to the nearest centimetre, the actual length must be between 2.5 centimetres and 3.5 centimetres.
   The **greatest possible error** is 0.5 centimetres.
   The greatest possible error of any measurement is half of the smallest unit of measurement used.

   What is the greatest possible error in each of the measurements you made in the Investigation?

1. For each reported measurement, give the largest and the smallest lengths possible for the actual length.

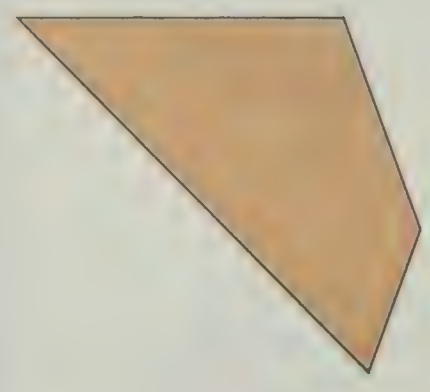
   EXAMPLE: 8 cm

   A 4 km **B** 19 m **C** 37 cm **D** 19 mm **E** 12 cm **F** 154 dm

2. **A** Measure each side of the figure to the nearest whole centimetre. Add the lengths to find the perimeter.

   B Measure each side of the figure to the nearest millimetre and then add these lengths to find the perimeter.

   C What is the difference in the perimeter for **A** and **B** ?



3. The **relative error** of a measurement is found by dividing the greatest possible error by the reported measurement. When you compare two measurements, the one with the smaller relative error is said to be **more precise**.

   EXAMPLE: Reported measure: 10 centimetres
   Greatest possible error: 0.5 centimetre
   Relative error: $0.5 \div 10 = 0.05$ That is, the error may be as much as 5 parts in 100 or 1 part in 20.

   Find the relative error of each measurement.

   A 4 cm **B** 30 m **C** 25 mm **D** 18 km

   Which is the most precise measurement?

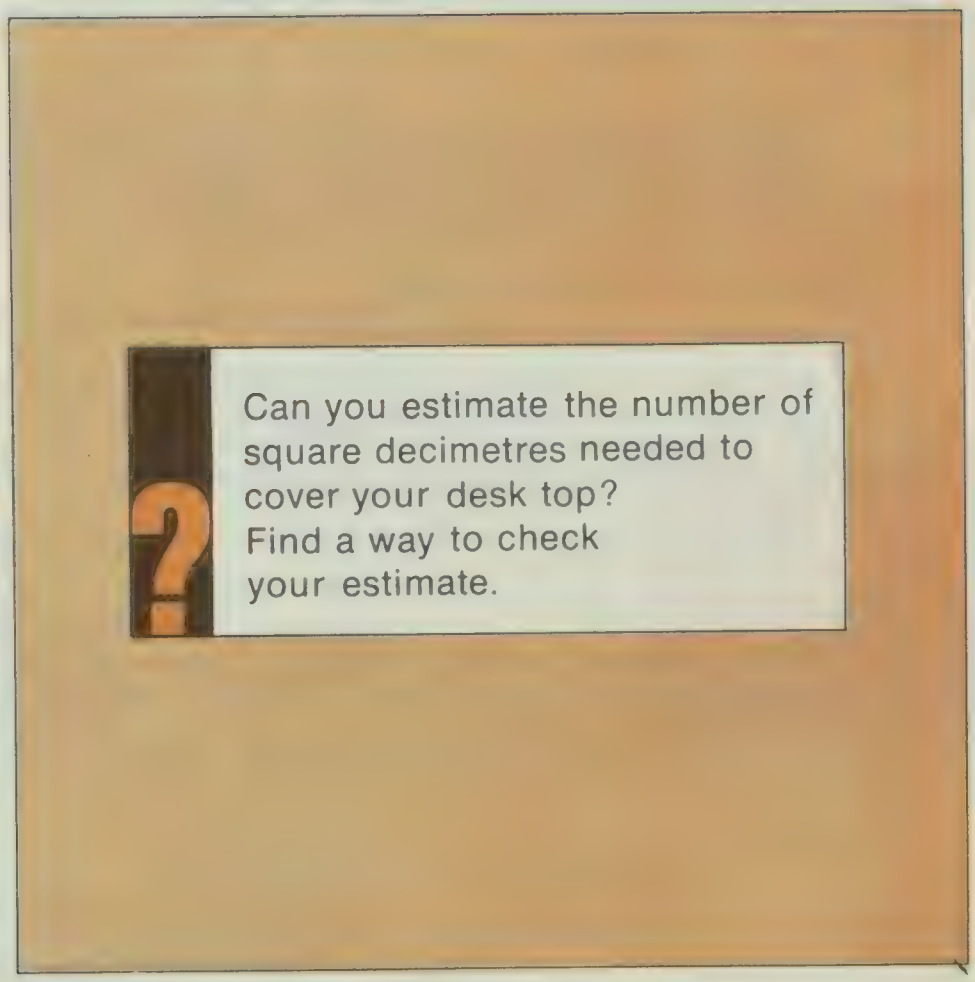
4. Astronomers give the average distance from Jupiter to the sun as 773 million kilometres to the nearest million kilometres. Could the actual distance be as much as 773 700 000 kilometres? Why or why not?

5. An astronomer says that it is about 12 light years from Earth to a certain star. Taking a light year to be 9.4 trillion kilometres, how much could the actual distance (in kilometres) differ from the astronomer's figure?

Investigating the Ideas

This region is one square centimetre.

This region is one square decimetre.

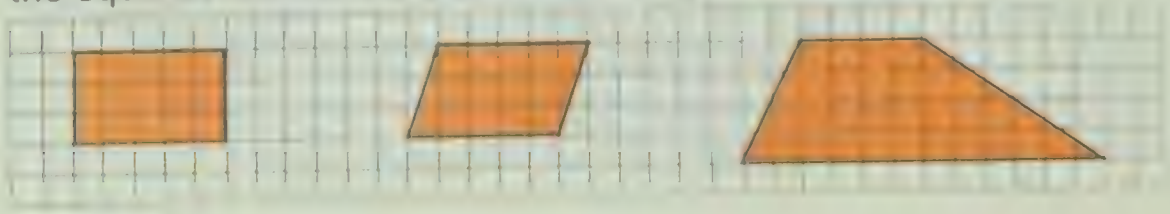


Discussing the Ideas

1. The number of square decimetres, or parts thereof, needed to cover your desk top is the **area** of the desk top. Estimate the area of some other regions using a square decimetre as the unit.

2. Think of a square one metre on each side.
   - A What is the area of the square metre if the square decimetre is the unit?
   - B What is the area if the square centimetre is the unit?

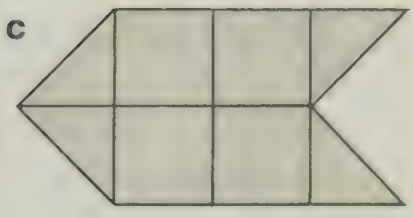
3. Unit squares are indicated on the grid. Count the squares to find the area of each figure.



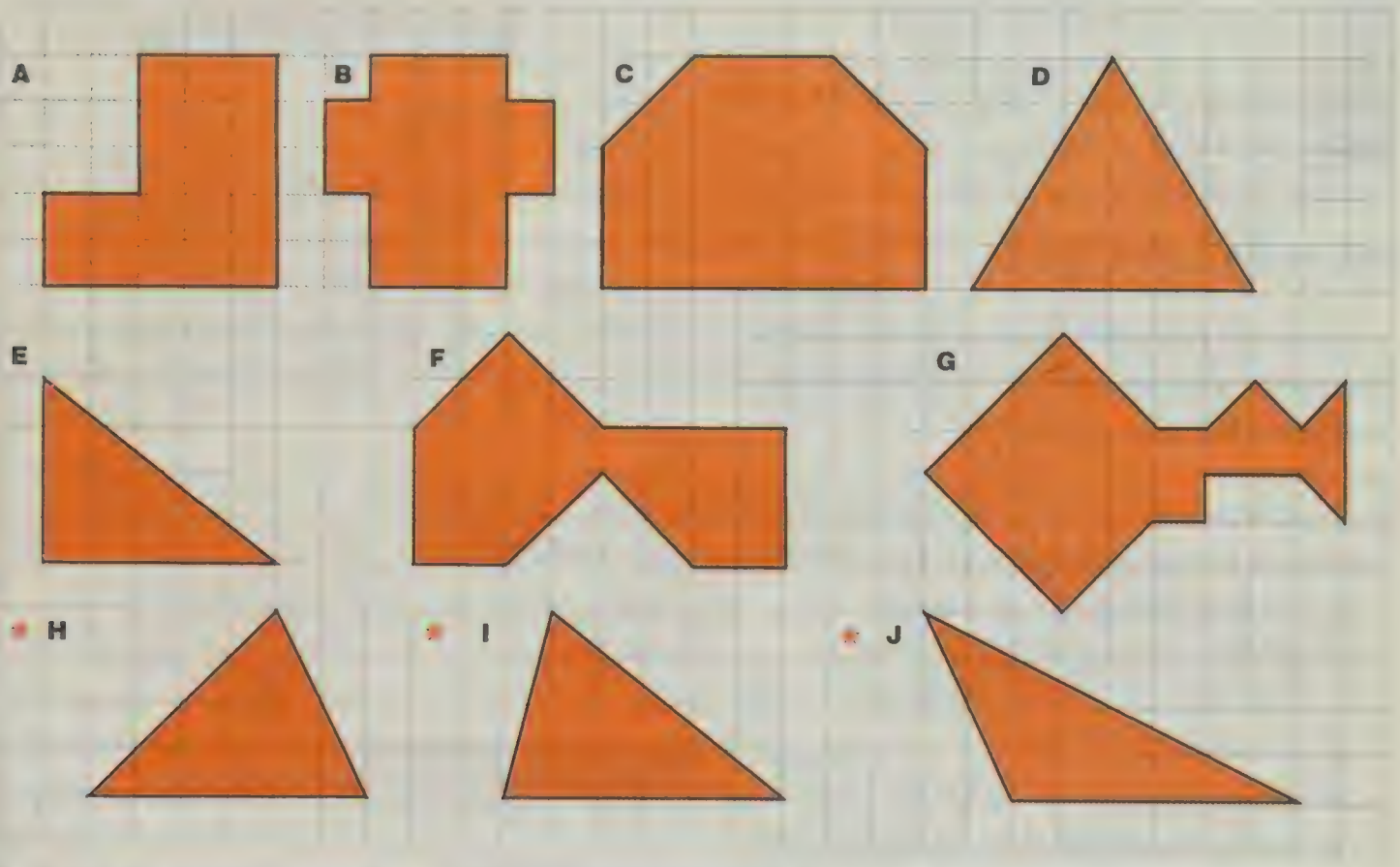
1. Each small square is a square centimetre.
What is the area of each region?

A

B

C 

2. Find the area of each shaded region.
Unit squares are indicated by the grid.



3. An **are** (pronounced "air") is a unit of area. A square ten metres on each side has an area of one are.
How many square metres are there in one **are**?

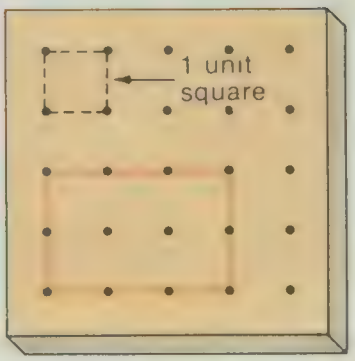
4. A **hectare** is 10 000 square metres. How many ares are there in an hectare?

5. How many hectares are in one square kilometre?

6. Suppose each small square represents an area of 125 000 square kilometres.
Estimate the total area of the country.

Investigating the Ideas

Suppose each of the smallest squares on the geoboard is a unit square
Then the rectangle shown has an area of 6 square units.



Can you find some different-shaped quadrilaterals that also have an area of 6 square units?

Record your regions on dot paper or graph paper.

Discussing the Ideas

1. Would this parallelogram have the same area as the rectangle above? How can you decide?

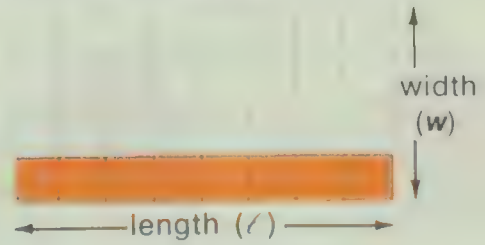
2. Sometimes we say, "the area of a rectangle," when we really mean the **area of the interior of the rectangle**.

   A In the rectangular region at the right, how many unit squares in the bottom row?

   B How many rows?

   C How many unit squares in all?

   D Can you write a formula for the area of any rectangle with length ℓ and width w?

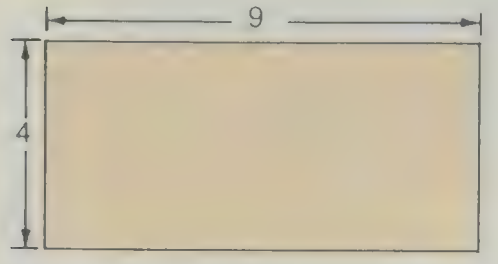


3. Can you find the area of each parallelogram?

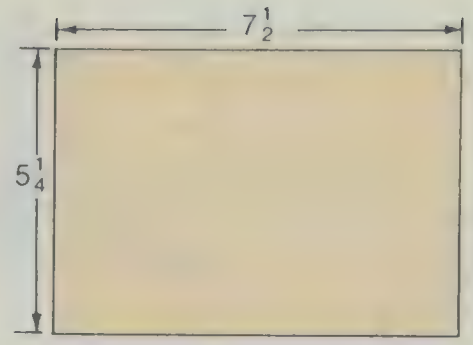
4. In Exercise 3, if b is the length of the **base** and h is the **height** of each of the parallelograms, can you write a formula for the area of a parallelogram?

1. Find the area of each rectangle.

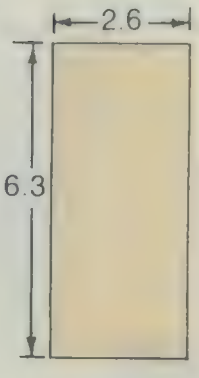
A



B

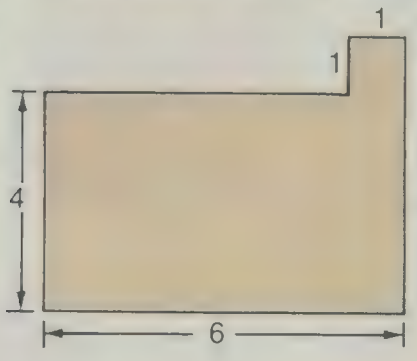


C

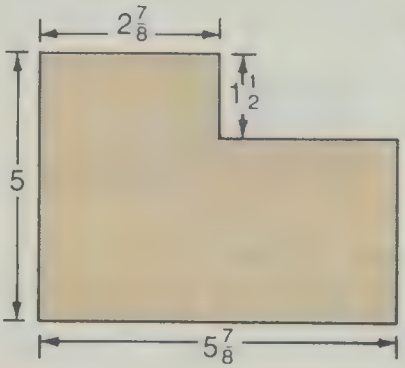


2. Find the area of each region.

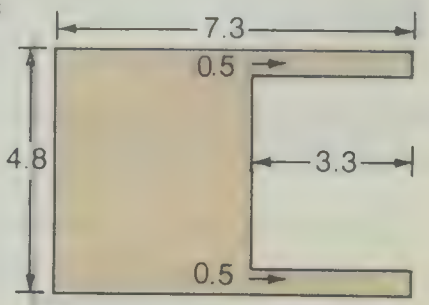
A



B

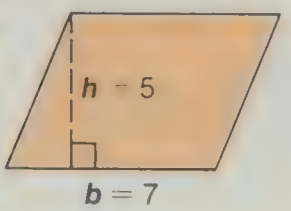


C

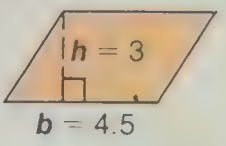


3. Use the formula $A = b \cdot h$ to find the area of each parallelogram.

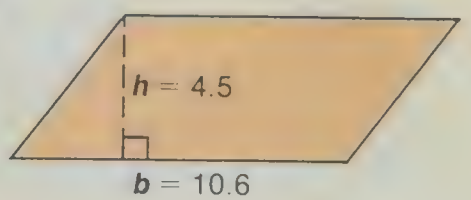
A



B



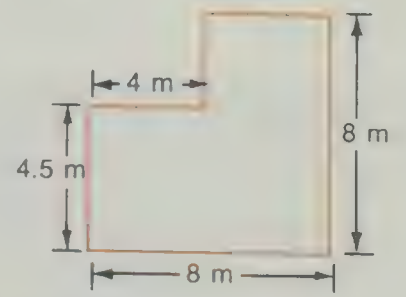
C



4. Find the area of the floor of a rectangular room that is 4.5 metres long and 3.4 metres wide.

5. What would it cost to carpet the room in Exercise 4 if carpeting sells for $14.95 per square metre?

6. A certain rug cleaner charges $1.75 per square metre to clean rugs. How much would be charged to clean the wall-to-wall carpeting for the room shown at the right?

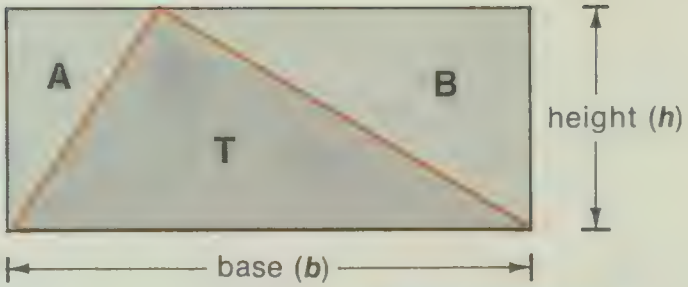


7. A certain brand of tiling is available in squares that are 25 centimetres on each side and cost 29¢ each. About how much would it cost to tile a floor 2.8 metres by 3.4 metres?

More practice, page S-25, Set 42

Investigating the Ideas

Draw a rectangle of any size. Then show a triangle with its base along one side of the rectangle and its height the same as the rectangle. Cut out the rectangle and pieces **A**, **B**, and **T**.

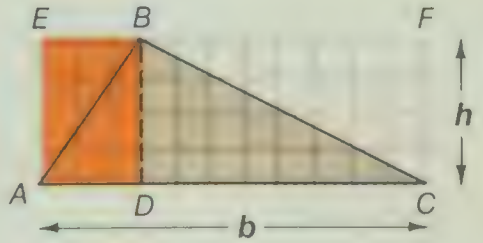


Can you show that pieces **A** and **B** will exactly cover triangle **T**?	Try this with another triangle.	

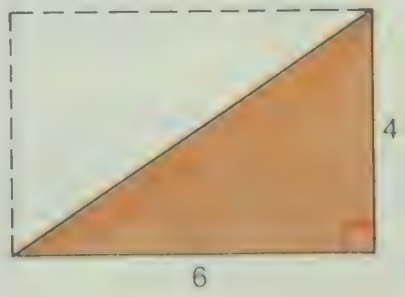
Discussing the Ideas

1. What does the Investigation show about the area of triangle **T** as compared to the area of the rectangle?

2. **A** What is the area of rectangle *ADBE*?
   B What is the area of $\triangle ABD$?
   C What is the area of rectangle *BDCF*?
   D What is the area of $\triangle BDC$?
   E What is the area of $\triangle ABC$?
   F Explain how the figure shows that the formula for the area of a triangle is $A = \frac{1}{2} \cdot b \cdot h$ where b is the base and h is the altitude.

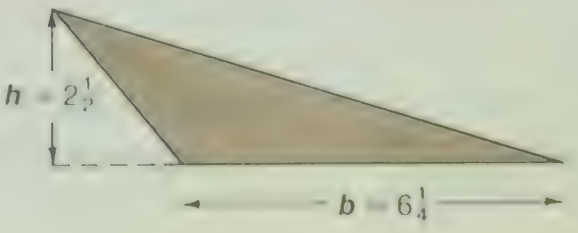


3. **A** What is the height of the right triangle?
   B How can you find its area?
   C What is its area?

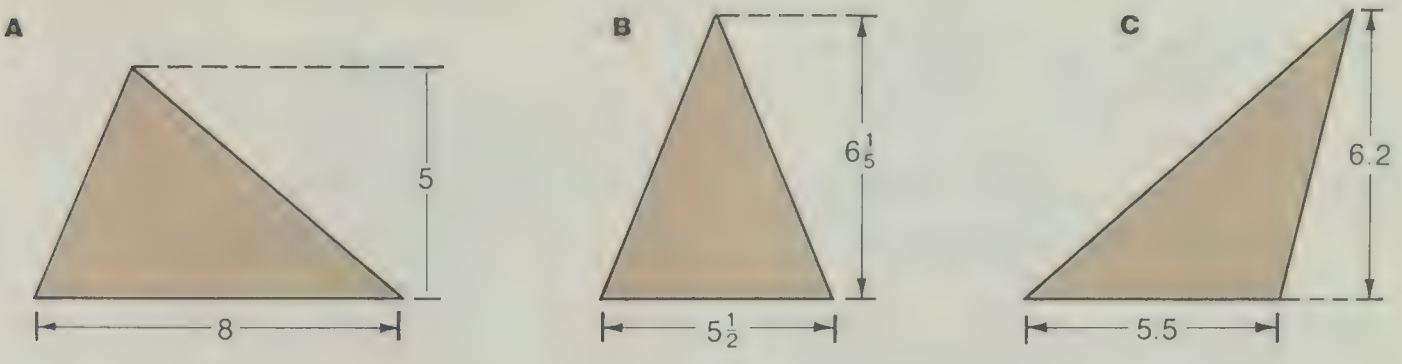


4. Complete the computation to find the area of this triangle.

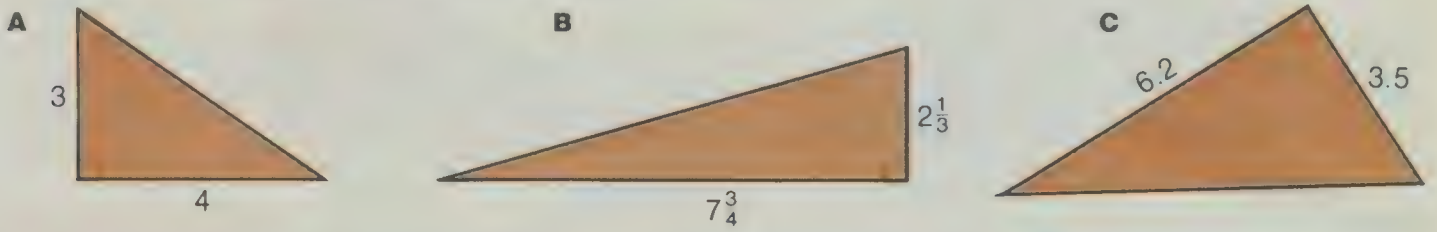
$$A = \frac{1}{2} \cdot b \cdot h$$
$$= \frac{1}{2} \cdot 6\frac{1}{4} \cdot 2\frac{1}{2}$$
$$= \frac{1}{2} \cdot \frac{25}{4} \cdot \frac{5}{2} = \text{▒▒} \text{ square units}$$



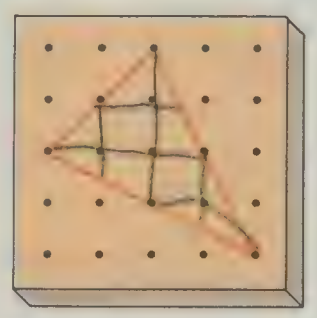
1. Find the area of each triangular region. Use the formula $A = \frac{1}{2} \cdot b \cdot h$.



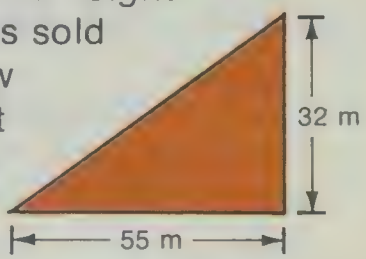
2. Find the area of each right triangular region.



3. What is the area of the largest triangle that can be formed on a 5 by 5 nail geoboard?



4. A triangular lot with a base of 55 metres and a height of 32 metres was sold for $15 960. How much did the lot cost per square metre?



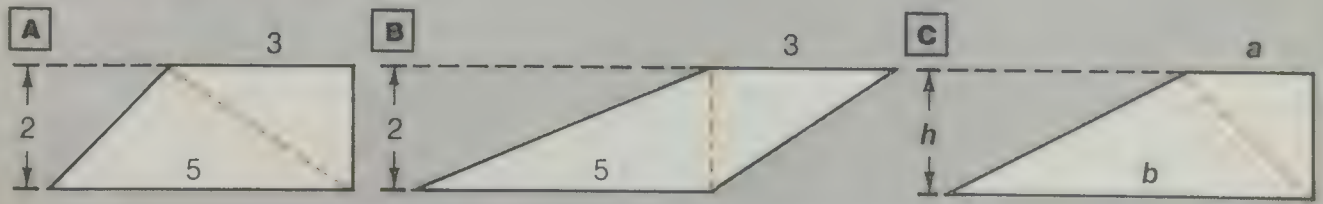
5. Find the areas of the triangles with the following dimensions:

A $b = 240$ mm, $h = 315$ mm
B $b = 3$ m, $h = 4\frac{1}{2}$ m
C $b = 4$ m, $h = 200$ cm
D $b = 6.3$ dm, $h = 28$ cm

6. Design a flow chart for finding the area of a triangle.

think

Find the area of each region.



More practice, page S-25, Set 43

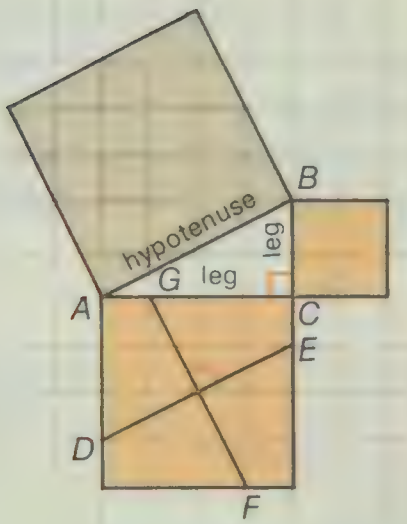
Investigating the Ideas

A Construct right $\triangle ABC$ with a square on each of its sides as shown in the figure.

B Locate the centre of the middle-sized square and call it point O.

C Draw $\overline{DE}$ through point O so that $\overline{DE} \parallel \overline{AB}$. Then draw $\overline{GF} \perp \overline{DE}$. (Four regions are formed in the middle-sized square.)

D Cut out the four regions of the middle-sized square and the small square region.

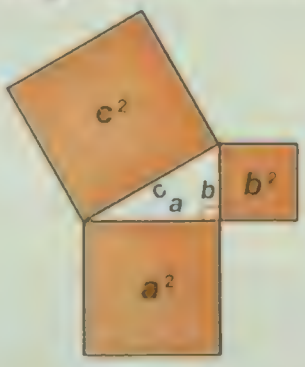


Can you place the five regions of the two smaller squares so that they will exactly cover the large square?

Discussing the Ideas

1. Try the Investigation again using a different-sized triangle for step A. Do the two squares on the legs of the triangle cover the square on the hypotenuse?

2. The Investigation illustrates the **Pythagorean Theorem**.
In any right triangle, the sum of the areas of the squares on the legs is equal to the area of the square on the hypotenuse.
If a = length of a leg
b = length of another leg
c = length of the hypotenuse
then $a^2 + b^2 = c^2$



Can you find three whole numbers a, b, and c, each less than 6, such that $a^2 + b^2 = c^2$?

3. If the area of the square on one leg of a right triangle is 10 square units and the area of the square on the other leg is 6 square units, what is the area of the square on the hypotenuse? How long is the hypotenuse?

1. Copy and complete the table below the figures.

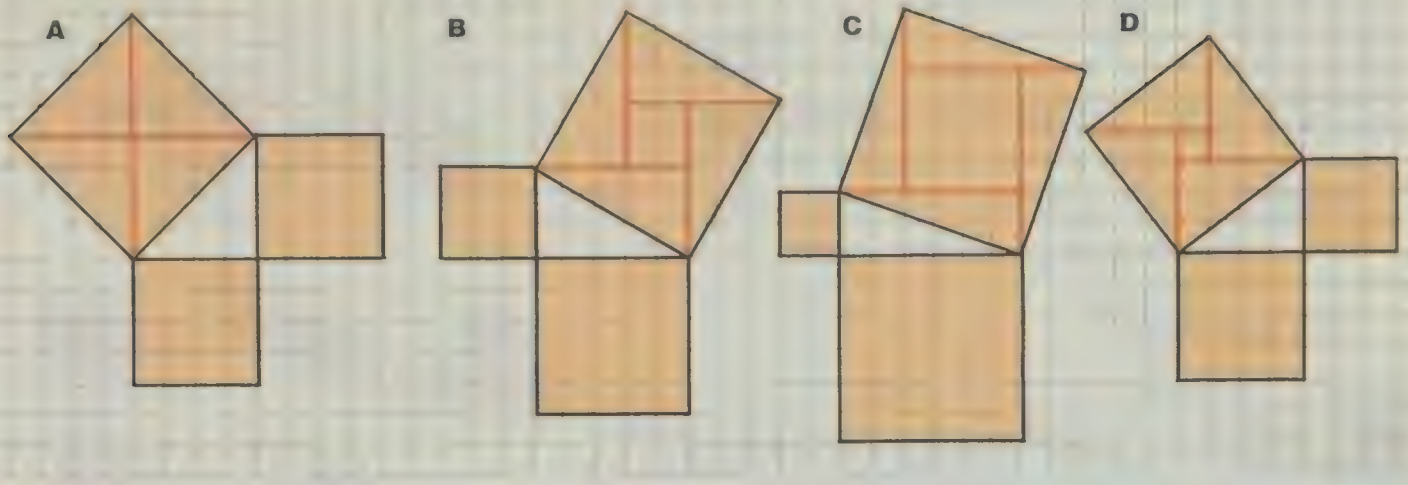


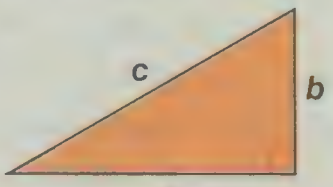
Figure	Area of each of the two small squares		Sum of areas of two small squares	Area of large square
A				
B				
C				
D				

2. Use the formula for the Pythagorean Theorem to complete each part.

   A If $a = 6$ and $b = 8$, then $a^2 + b^2 =$ ▒▒ and $c =$ ▒▒.

   B If $c^2 = 25$ and $b^2 = 16$, then $a^2 =$ ▒▒ and $a =$ ▒▒.

   C If $a = 5$ and $b = 12$, then $c^2 =$ ▒▒ and $c =$ ▒▒.



$a^2 + b^2 = c^2$

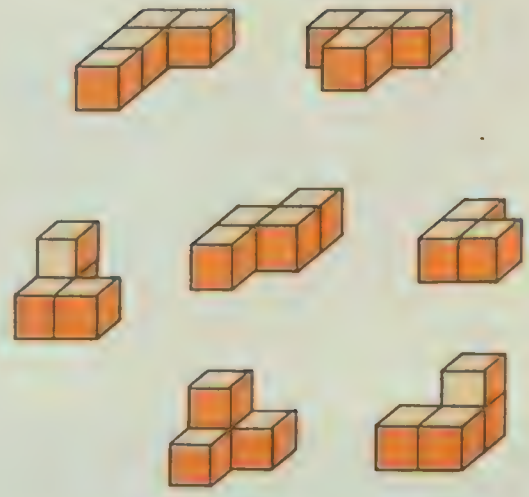
3. Complete the table using the Pythagorean Theorem.

	a	b	a^2	b^2	c^2	c
A	3	4				
B	8	6				
C	12					13
D			64		100	
E	12			81		

Investigating the Ideas

These seven pieces of the **Soma Puzzle** are made from three or four cubes each. When placed together properly, they will make one large Soma Cube.

Can you do it?



Can you find the length of each edge of the Soma Cube?

Discussing the Ideas

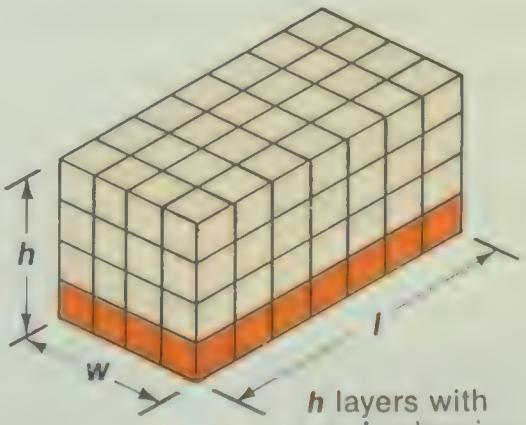
1. The number of unit cubes that it takes to make a space figure is called the **volume** of the figure.

   What is the volume of the Soma Cube if each small cube is a unit of volume?

2. **A** How would you count the number of unit cubes in this rectangular box?
   - **B** How many squares in the base?
   - **C** How many unit cubes in the bottom layer?
   - **D** How many layers?
   - **E** How many unit cubes in all?
   - **F** Explain each formula for the volume of a rectangular box:

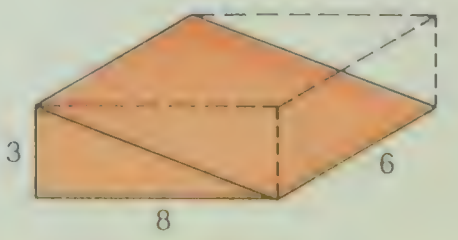
     Volume = area of base times altitude

     and $V = l \cdot w \cdot h$



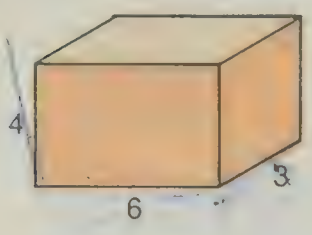
h layers with $w \cdot l$ cubes in each layer

3. **A** What is the volume of the box shown by the dotted segments?
   - **B** How does the volume of the wedge compare to the volume of the box?

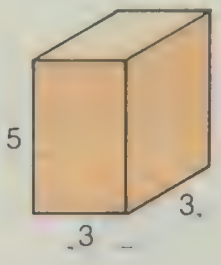


Find the volume of each space figure using the dimensions given.

1.



2.



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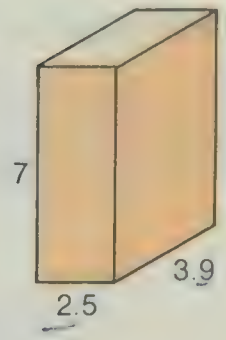

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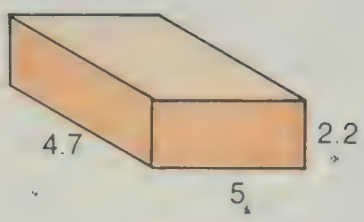

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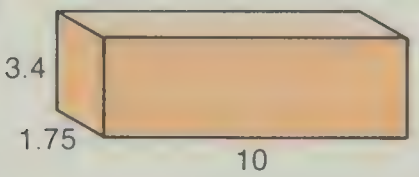

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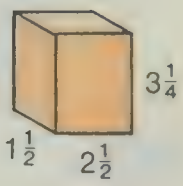

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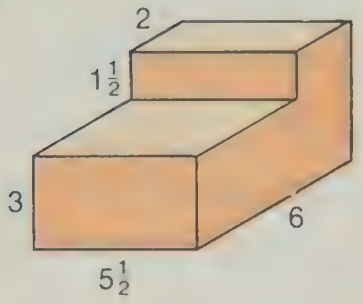

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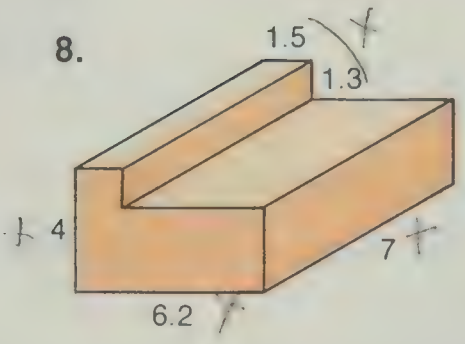

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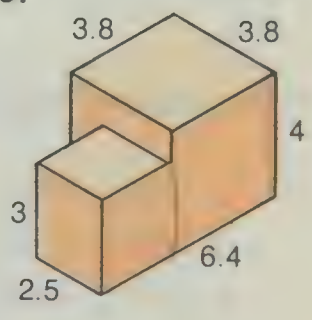

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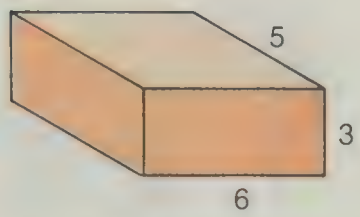

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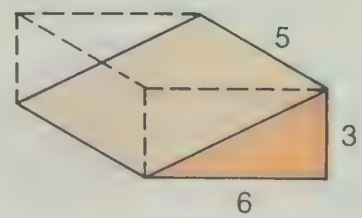

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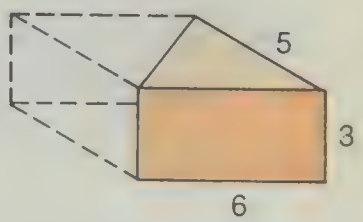
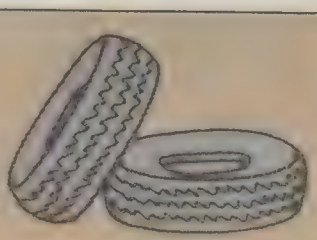


13. A A box is 5 centimetres long, 4 centimetres wide, and 3 centimetres high. What is its volume?

B If each side were twice as long, how many times greater would the volume of the box be?

think

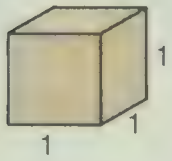
A man bought five new tires for his car. By rotating the tires, he found that at the end of 40 000 kilometres of driving each tire had been used for the same number of kilometres. How many kilometres was this?



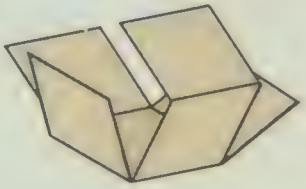
More practice, page S-26, Set 44

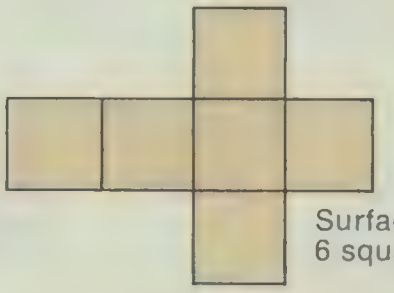
Investigating the Ideas

The diagrams below show that a cube with a volume of one cubic unit has a **surface area** of 6 square units.



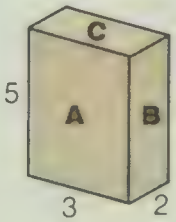
Volume: 1 cubic unit





Surface Area:
6 square units

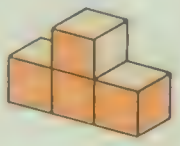
Can you find the volume and the surface area of the space figure at the right?



Discussing the Ideas

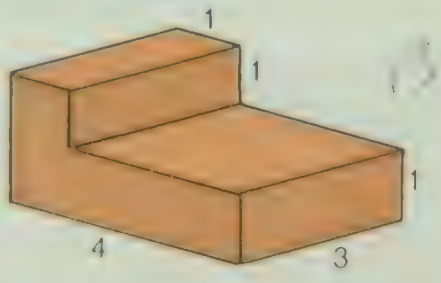
1. A What is the area of face A in the space figure above?
   B What is the area of face B?
   C What is the area of face C?
   D How can you find the area of all six faces (**total surface area**) of the space figure?

2. A What is the volume of this Soma Puzzle piece?
   B What is its surface area?



3. If you wanted to find the total surface area of the walls, the ceiling, and the floor of your mathematics classroom, what measurements would you make?

4. How would you find the volume and surface area of the space figure at the right?



1. Find the volume and surface area of each space figure.

A

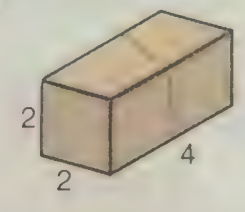
B

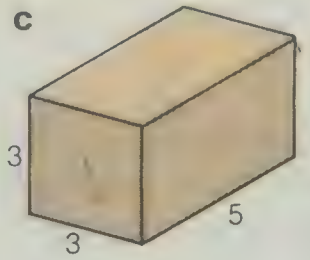
C

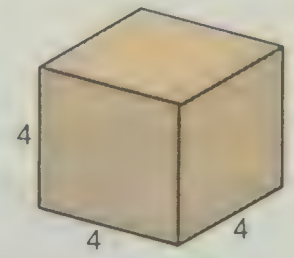
D

2. Find the volume and surface area of each space figure.

A

B 

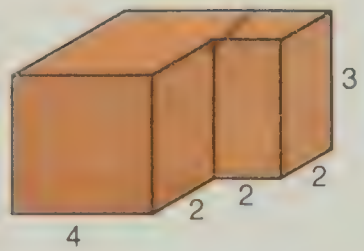
C 

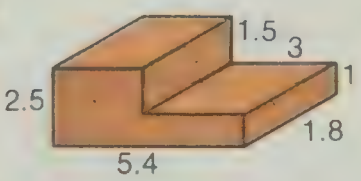
D 

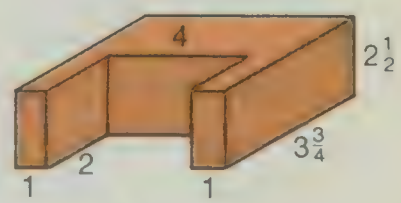
3. A cereal box is 28 centimetres high, 20 centimetres wide, and 9 centimetres thick.
   - A What is the volume of the box?
   - B What is the surface area of the box?

4. A shoe box is 1.7 decimetres by 0.9 decimetres by 3.1 decimetres.
   - A Find the volume of the box.
   - B Find the surface area of the box.

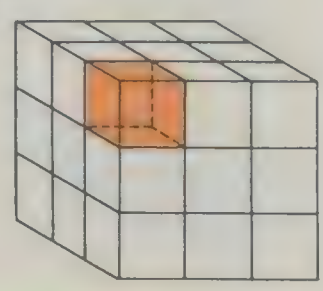
5. Find the volume and surface area of each space figure.

A 

B 

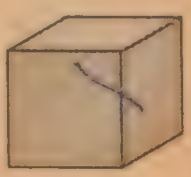
C 

6. A When the colored cube is removed, the volume is decreased by 1 unit but the surface area is unchanged Explain this fact.
   - B What other blocks can you remove from the figure and still keep the same surface area?



think

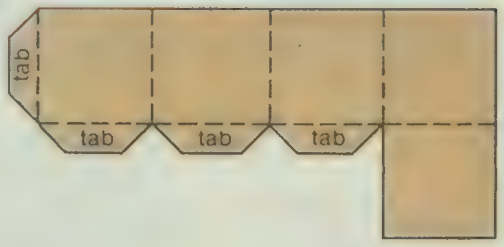
If a cube has the same number of units of volume as it has surface area, how large is the cube?



More practice, page S-26, Set 45

Investigating the Ideas

This is a pattern for an open-top cube.



Can you make a model of an open-top cube using a pattern in which each square is one decimetre on each side?

Discussing the Ideas

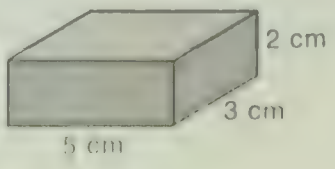
1. What is the volume of your cube in cubic centimetres?

2. If your cube could be filled with water, it would hold one **litre (ℓ)**. A litre is a standard measure of **capacity**.
   - **A** What part of a litre is a millilitre?
   - **B** How large would a cube need to be in order to hold a millilitre?

3. What is the number for each screen?
   - **A** 1 kilolitre (kl) = ▒▒ ℓ
   - **B** 1 decilitre (dl) = ▒▒ ℓ
   - **C** 1 millilitre (ml) = ▒▒ ℓ
   - **D** 1 hectolitre (hl) = ▒▒ ℓ
   - **E** 1 centilitre (cl) = ▒▒ ℓ
   - **F** 1 decalitre (dal) = ▒▒ ℓ

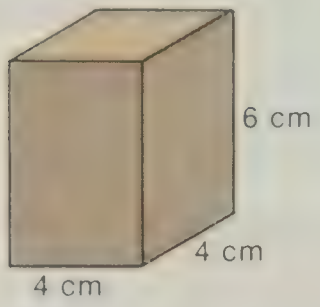
4. Since 1 cubic centimetre (cm^3) has a capacity of 1 ml = 0.001 ℓ you can express units of volume in terms of capacity. Explain how to find the missing numbers.
   - **A** 500 cm^3 is a capacity of ▒▒ ℓ.
   - **B** 2749 cm^3 is a capacity of ▒▒ ℓ.
   - **C** 72 cm^3 is a capacity of ▒▒ ml.
   - **D** ▒▒ cm^3 is a capacity of 3.8 ml.

5. What is the capacity in millilitres of this container?

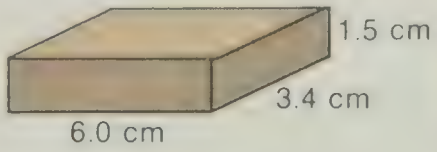


1. Find the capacity of each container in millilitres.
(Use the fact that 1 cm^3 is a capacity of 1 ml.)

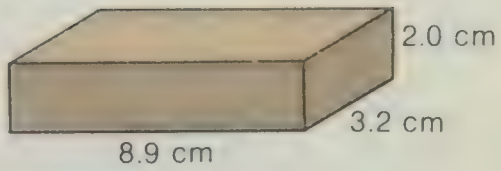
A



B

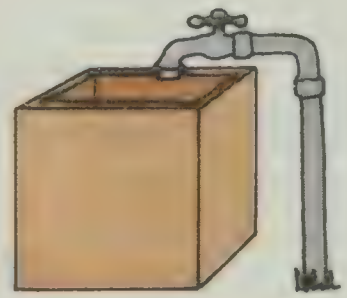


C



2. Express each capacity in Exercise 1 in litres.

3. A tank is in the form of a cube one metre on each edge. How many kilolitres of water will it take to fill the tank?



4. At standard pressure and temperature, **the mass of 1 millilitre of water is 1 gram.**
What is the mass of

A 25 ml of water? **B** 1 ℓ of water? **C** 1 kl of water?

5. A soft drink bottle has a capacity of 236 millilitres.
How many litres in a carton of six bottles?

6. A family uses about three litres of milk per day.
If the cost is 29¢ per litre, how much do they spend for milk each week? each year?

7. Gasoline costs 11.9¢ per litre. How much (to the nearest cent) would it cost to fill a tank whose capacity is 60 litres?

think

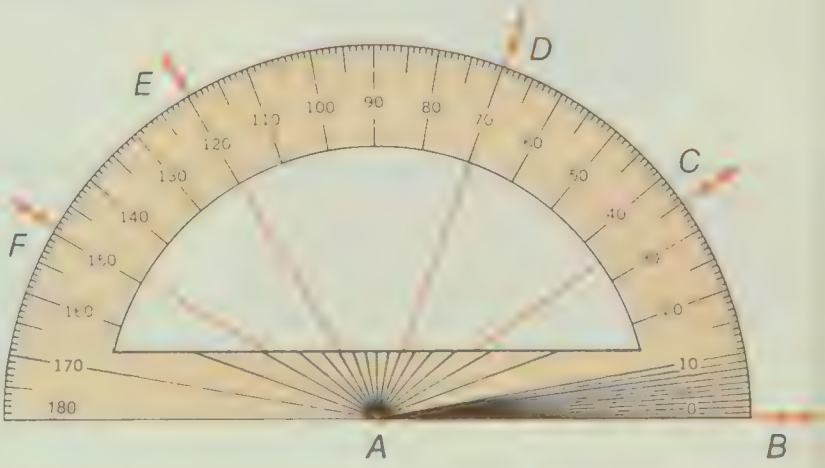
Suppose that you have two containers, one that will hold 4 litres and one that will hold 9 litres. There are no markings on either container to indicate smaller quantities. How can you measure out 6 litres of water using only these two containers?

Investigating the Ideas

A familiar unit angle is called a **degree**. The size of a 1-degree angle is such that the measure of a right angle is 90 degrees.

You can measure an angle by using a **protractor** that has degree units marked around a semicircle.

The protractor shows that the measure of $\angle DAB$ is 70 degrees.
We write: $m\angle DAB = 70°$



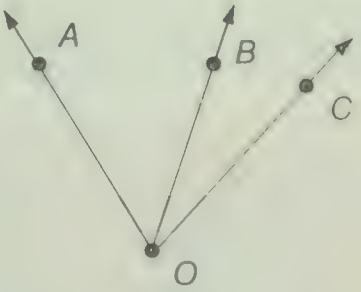
Using the angles shown on the protractor above, how many angles can you name and find the degree measure for?

Discussing the Ideas

1. Draw an angle on your paper or on the chalkboard. Explain how you would use a protractor to find the degree measure of the angle.

2. A degree is what fractional part of a straight angle?

3. Measure each angle.
Does $m\angle AOC = m\angle AOB + m\angle BOC$?



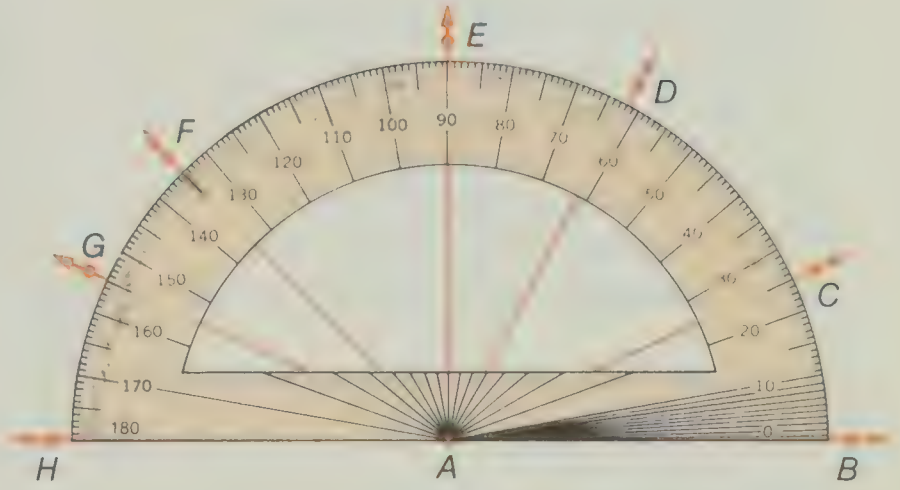
4. Draw an angle and its supplement. Measure each angle, adding their measures. What do you observe?

5. Using your protractor, draw an angle of 45° and an angle of 135°. What can you say about these two angles?

6. Explain how you can use a protractor to draw the **bisector** of an angle.

1. Using the picture of the protractor, give the measure of each angle in degrees.

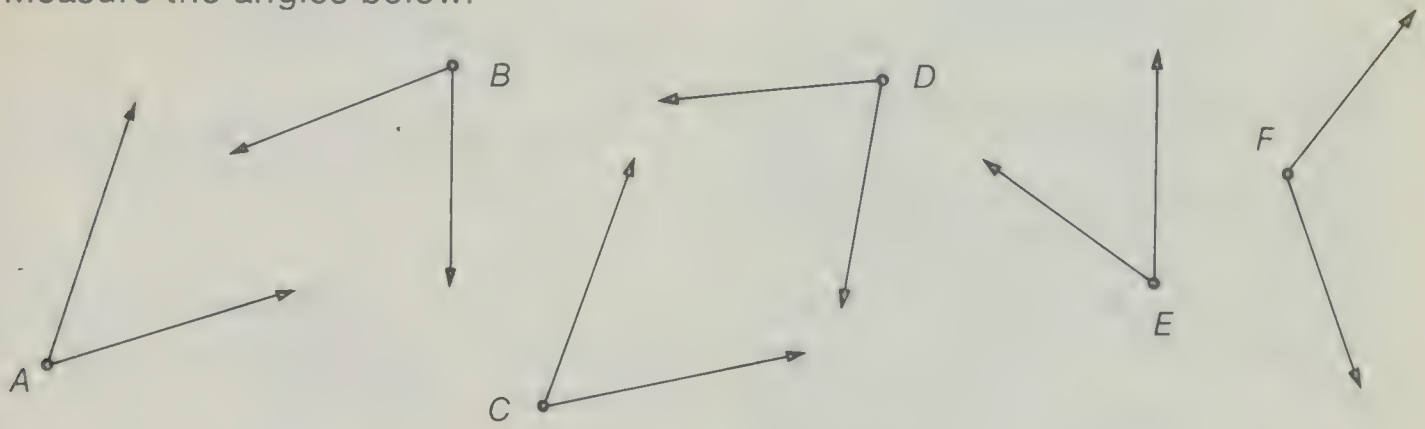
A	$\angle BAC$	**L**	$\angle DAE$
B	$\angle BAD$	**M**	$\angle DAF$
C	$\angle BAE$	**N**	$\angle DAG$
D	$\angle BAF$	**O**	$\angle DAH$
E	$\angle BAG$	**P**	$\angle EAF$
F	$\angle BAH$	**Q**	$\angle EAG$
G	$\angle CAD$	**R**	$\angle EAH$
H	$\angle CAE$	**S**	$\angle FAG$
I	$\angle CAF$	**T**	$\angle FAH$
J	$\angle CAG$	**U**	$\angle GAH$
K	$\angle CAH$	**V**	$\angle HAD$



2. Use a protractor to draw angles with the following measures.

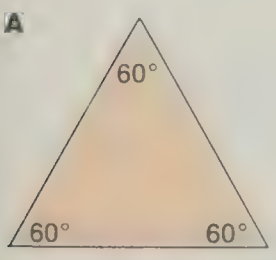
A 37° **B** 65° **C** 108° **D** 135° **E** 29° **F** 100°

3. Measure the angles below.



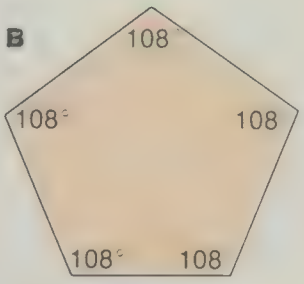
4. Use a protractor to help you draw each of these regular polygons. All the sides of each polygon should have the same length.

A



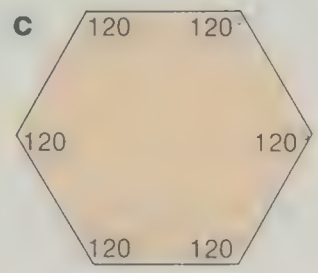
Equilateral triangle

B



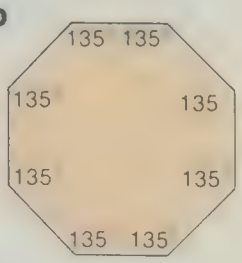
Regular pentagon

C



Regular hexagon

D

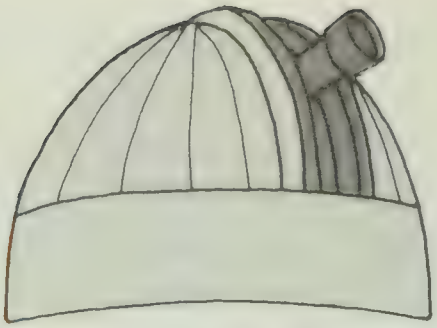


Regular octagon

Degrees, Minutes, and Seconds

Discussing the Ideas

Imagine that an angle of 1° is divided into 60 congruent angles. Each of the 60 small angles is called a **minute** (an angle of 1′). Now think about dividing an angle of 1′ into 60 congruent angles. Each of these smaller angles is called a **second** (an angle of 1″).



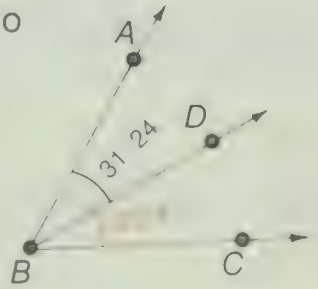
In astronomy, angles are often measured in degrees, minutes, and seconds.

60 minutes (60′) = 1° 60 seconds (60″) = 1′

1. What part of a degree is each of the following?

   A 30′ **B** 15′ **C** 10′ **D** 6′ **E** 45′ **F** 1′ **G** 30″ **H** 15″

2. The example below will help you understand how to add angle measures. The measure of $\angle ABC$ is the sum of the measures of $\angle ABD$ and $\angle DBC$.

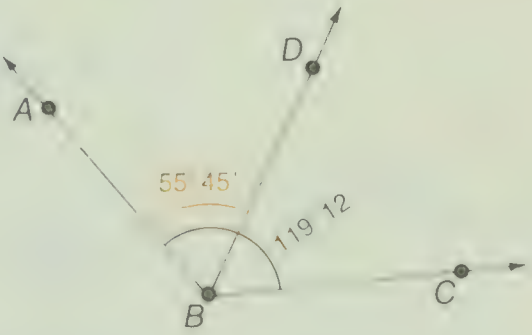


$$\begin{aligned} m\angle ABD &= 31°24' \\ m\angle DBC &= 25°41' \\ \hline m\angle ABC &= 56°65' = 57°5' \end{aligned}$$

   What was done to rewrite 56°65′ as 57°5′?

3. When subtracting angle measures, you may have to regroup some units.

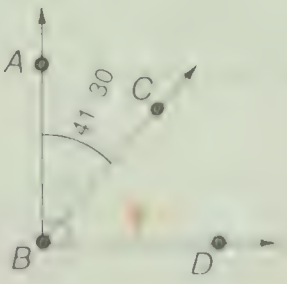
   Find the missing number of minutes and then find $m\angle DBC$.



$$\begin{aligned} m\angle ABC &= 119°12' = 118°\,\square\,' \\ m\angle ABD &= 55°45' = 55°45' \\ \hline m\angle DBC &= \quad ? \quad \square\,° \square\,' \end{aligned}$$

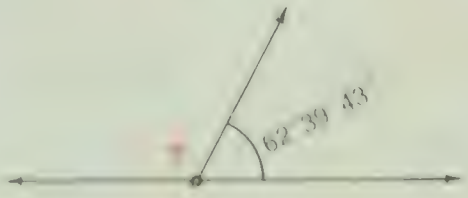
4. Two angles are **complementary** if the sum of their measures is 90°.

   If $m\angle ABC = 41°30'$, what is $m\angle CBD$?



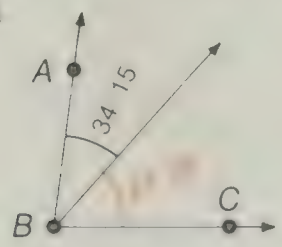
5. Two angles are **supplementary** if the sum of their measures is 180°.

   What is the measure of the supplement of an angle whose measure is 63°39′43″?

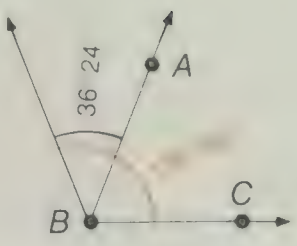


1. Give the measure of $\angle ABC$.

A



B



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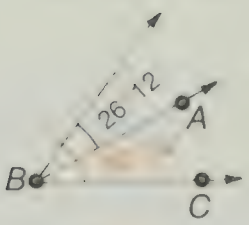
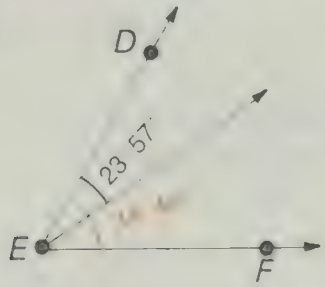


2. Give the measure of $\angle DEF$.

A



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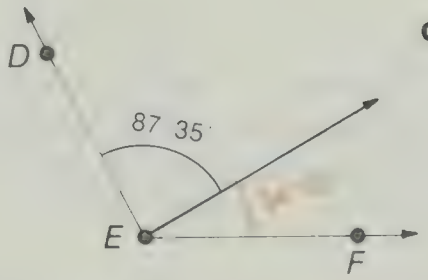
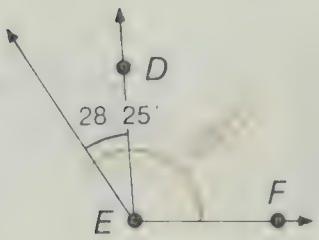


C



3. Find the sums of the angle measures.

EXAMPLE: $37°\ 48'35''$
$\underline{15°\ 57'46''}$
$52°105'81'' = 52°106'21'' = 53°46'21''$

A $25°15'43''$
$\underline{50°36'10''}$

B $86°28'29''$
$\underline{37°21'56''}$

C $34°37'48''$
$\underline{120°46'37''}$

D $67°46'38''$
$\underline{67°13'22''}$

4. Find the **differences** of the angle measures.

EXAMPLE: $37°48'35'' \quad 37°47'95'' \quad 36°107'95''$
$\underline{15°57'46''} \rightarrow \underline{15°57'46''} \rightarrow \underline{15°\ 57'46''}$
$\quad ? \quad\quad ?\ 49'' \quad\quad 21°\ 50'49''$

A $120°56'43''$
$\underline{15°13'28''}$

B $48°53'10''$
$\underline{20°39'26''}$

C $78°43'21''$
$\underline{13°44'52''}$

D $129°34'10''$
$\underline{29°35'59''}$

5. Find the measure of an angle that is complementary to an angle whose measure is

A $38°$ B $27°49'$ C $42°16'38''$ D $58°49''$ E $43'19''$

6. Find the measure of an angle that is supplementary to an angle whose measure is

A $56°$ B $105°20'$ C $84°26'39''$ D $125°58'43''$ E $4°58''$

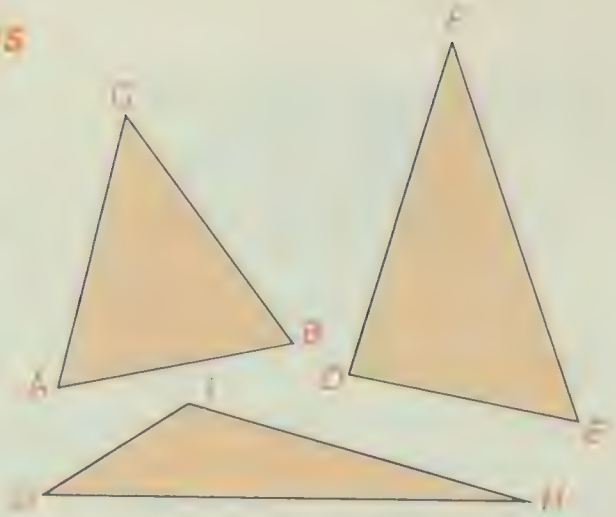
Sums of Angle Measures of Polygons

Investigating the Ideas

A Draw three large differently-shaped triangles on your paper.

B Measure the three angles of each triangle to the nearest degree

C Find the sum of the angle measures of each triangle.



Do you think that the sum of the measures of a triangle depends upon the shape of the triangle?	Compare your sums of the angle measures with those of your classmates.

Discussing the Ideas

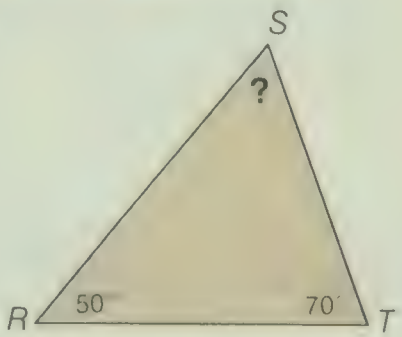
1. Cut out one of the triangles that you drew for the Investigation. Tear off two corners and place them as illustrated in the drawing.

What does this experiment suggest about the sum of the measures of the three angles?

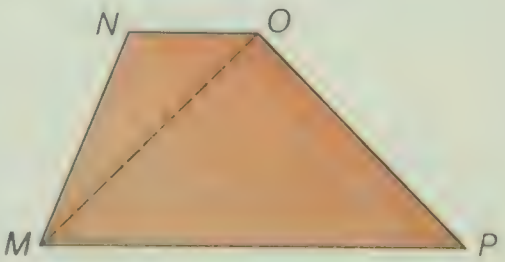
2. You should have convinced yourself of the truth of the following statement:

The sum of the measures of all the angles of any triangle is 180°.

If you know that one angle of a triangle has a measure of 50° and a second angle has a measure of 70°, how could you find the measure of the third angle without using a protractor?

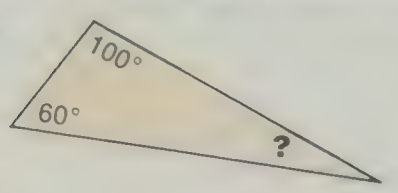


3. Every quadrilateral can be divided into two triangles. Since the sum of the angle measure in each triangle is 180°, what is the sum of the measures of the four angles of quadrilateral *MNOP*?

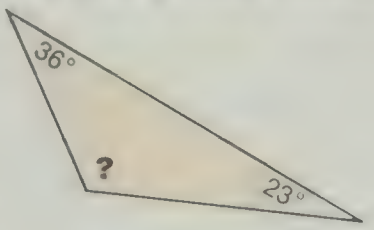


1. Find the degree measure of the angle whose measure is not given.

A



B



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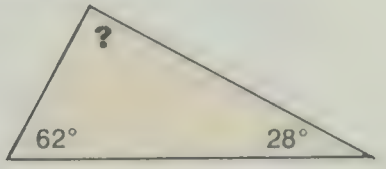

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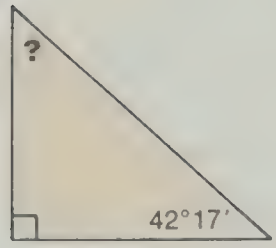

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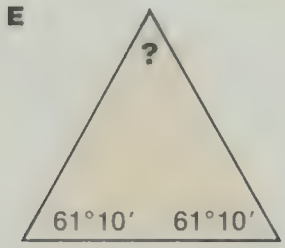

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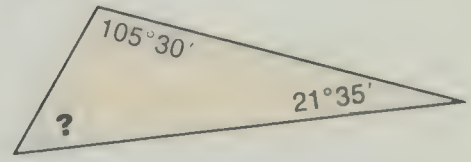
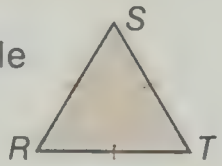
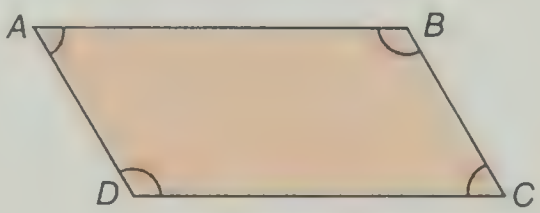


2. What is the measure of each angle of equilateral $\triangle RST$?

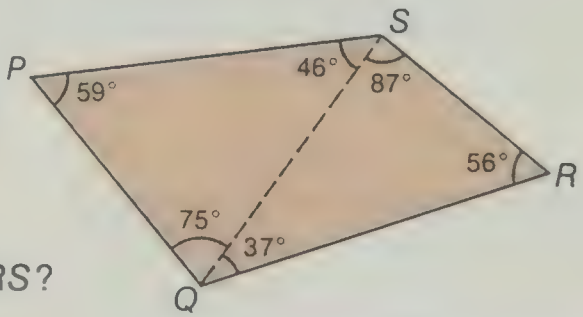


3. In parallelogram $ABCD$ congruent angles are marked by the same number of slashes.

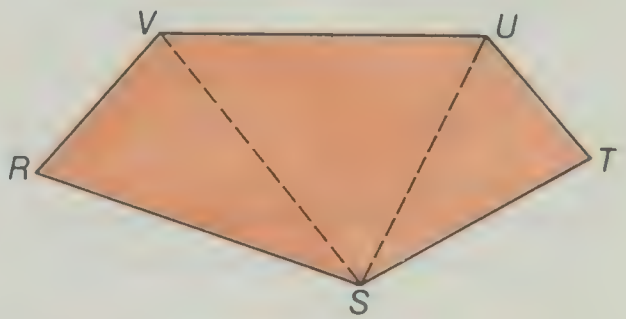


If $m\angle A = 59°10'$ and $\angle D$ is supplementary to $\angle A$, what is the measure of angles B, C, and D?

4. What is the sum of the measures
   - A of the angles of $\triangle PQS$?
   - B of the angles of $\triangle QRS$?
   - C of the angles of quadrilateral $PQRS$?



5. What is the sum of the measures
   - A of the angles of $\triangle RSV$?
   - B of the angles of $\triangle VSU$?
   - C of the angles of $\triangle UST$?
   - D of the angles of pentagon $RSTUV$?

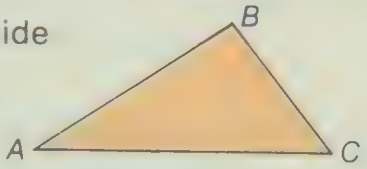


6. What is the sum of the measures of the angles of a hexagon?

7. Can you write a formula for the sums of the angle measures of a polygon having any number n of sides?

REVIEWING THE IDEAS

1. Measure each side of the triangle in millimetres.

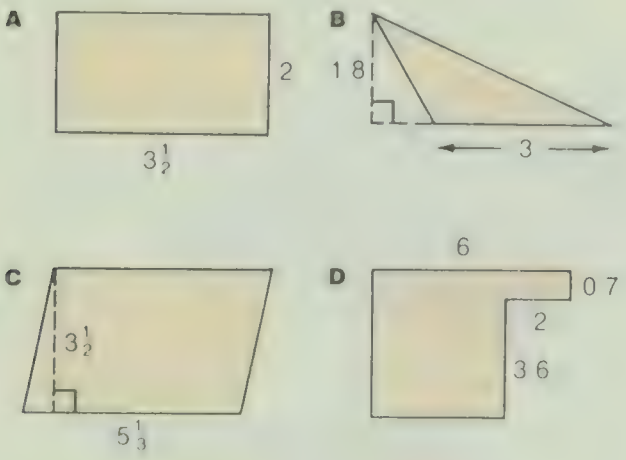


2. What is the perimeter of the triangle in Exercise 1?

3. A 1 metre = ▮▮▮ centimetres
   B 1 centimetre = ▮▮▮ millimetres
   C 1 decimetre = ▮▮▮ metre
   D 23 cm = ▮▮▮ dm
   E 47 dm = ▮▮▮ m
   F ▮▮▮ m = 5 km

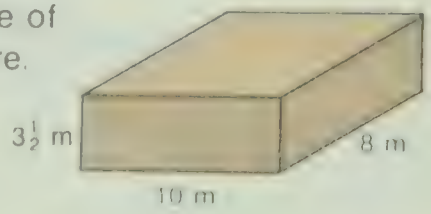
4. When the measure of an object is reported as 8 centimetres (to the nearest centimetre), what is the largest and the smallest measure for its actual length?

5. Find the area of each region.



6. If the lengths of the legs of a right triangle are 6 and 8 metres, what is the length of the hypotenuse?

7. Find the volume of this space figure.



8. Find the surface area of the space figure in Exercise 7.

9. One litre of water has a volume of ▮▮▮ cubic centimetres.

10. What is the capacity in ml of a cube 4 cm on each edge?

11. Give the measure of each angle.

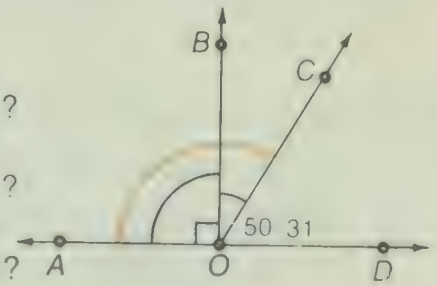
12. Find the **sums** of the angle measures.

A	B
25°15′43″	86°28′29″
50°35′36″	37°49′32″

13. Find the **differences** of the angle measures.

A	B
120°56′23″	98°23′10″
75°13′48″	29°50′26″

14. A What is m∠*AOB*?
    B What is m∠*AOC*?
    C What is m∠*BOC*?

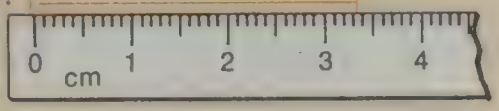


15. What is the measure of an angle which is supplementary to an angle which measures 103°42′?

16. What is the measure of ∠*A*?

TEST YOURSELF

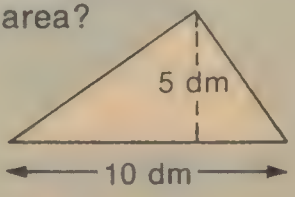
1. How many millimetres long is the segment?



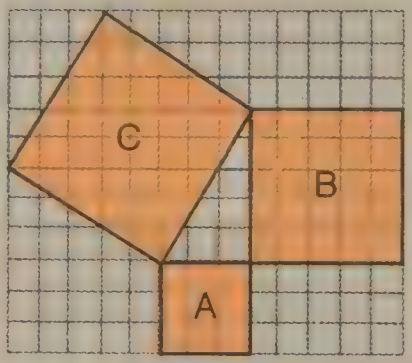
2. 73 centimetres = ▨ millimetres

3. A rectangle is $7\frac{1}{2}$ units long and $2\frac{1}{4}$ units wide.
   - A What is the perimeter?
   - B What is the area?

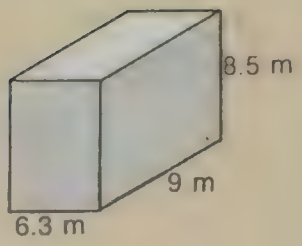
4. What is the area of the triangle?



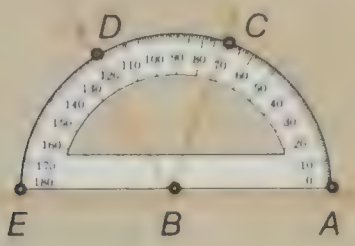
5. A Find the areas of squares A and B.
   B What is the area of square C?



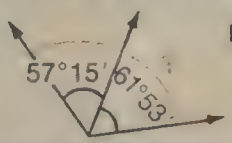
6. A Find the volume of the figure.
   B Find the surface area.



7. What is $m\angle CBD$?



8. Find the sums of the angle measures.
   A



   B $46°34′45″ + 19°28′25″$

9. Find the differences of the angle measures.
   A $158°35′ - 87°48′$
   B $106°48′20″ - 73°35′41″$

10. If two angles of a triangle each have a measure of 54 degrees, what is the measure of the third angle?

RESEARCH PROJECTS

A Can you find a formula for the area of any region on the geoboard when you know the number of nails on the boundary of the figure and the number of nails inside the figure? The formula for the area is called **Pick's Theorem**. (See *Geoboards and Motion Geometry* by John J. Del Grande; Glenview, Illinois: Scott, Foresman, 1972 available from Gage Educational Publishing, Ltd.)

B Another unit of angular measure that is often used in advanced mathematics is called a **radian**. Find out how the radian compareswith the degree, how you can construct an angle whose measure is one radian, and how this unit is used in mathematics.

C There are many different proofs of the **Pythagorean Theorem**. Find an explanation of one or more of them. Make some diagrams or charts that will help explain the proof. (See *The Pythogorean Proposition* by Elisha Scott Loomis; Washington, D.C.: NCTM, 1968.)

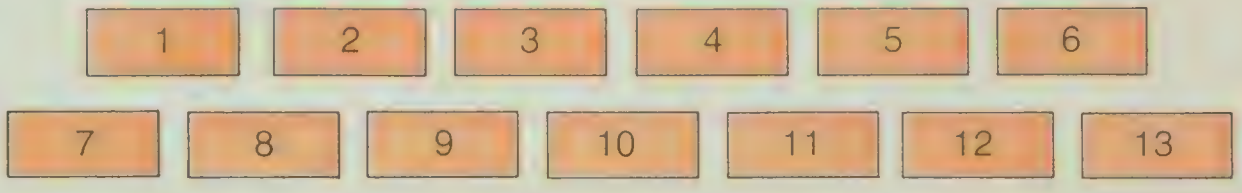
D The numbers 3, 4, and 5 are called **Pythagorean numbers** because

$$3^2 + 4^2 = 5^2.$$

Can you find some other triples which are Pythagorean numbers? Look up this topic in a reference book to see how you can find many other triples.

A STACKING PROBLEM

Print the numerals 1 through 13 on index cards.



The problem is to devise a way of stacking the cards in such a way that when you place the stack in your hand face down, you can turn up the cards 1 through 13 in order following the rules below.

RULES:

1. With the stack of cards face down in your hand, turn the first card face up and place it on your desk. It must be card number 1.

2. Slide the next card (that is on top of the stack) to the bottom of the stack without turning it over.

3. Turn up the next (third) card and place it face up on your desk. It must be card number 2.

4. Slide the next card that is on top to the bottom of the stack.

5. Repeat the steps, alternating by turning up a card and then moving the following top card to the bottom of the stack.

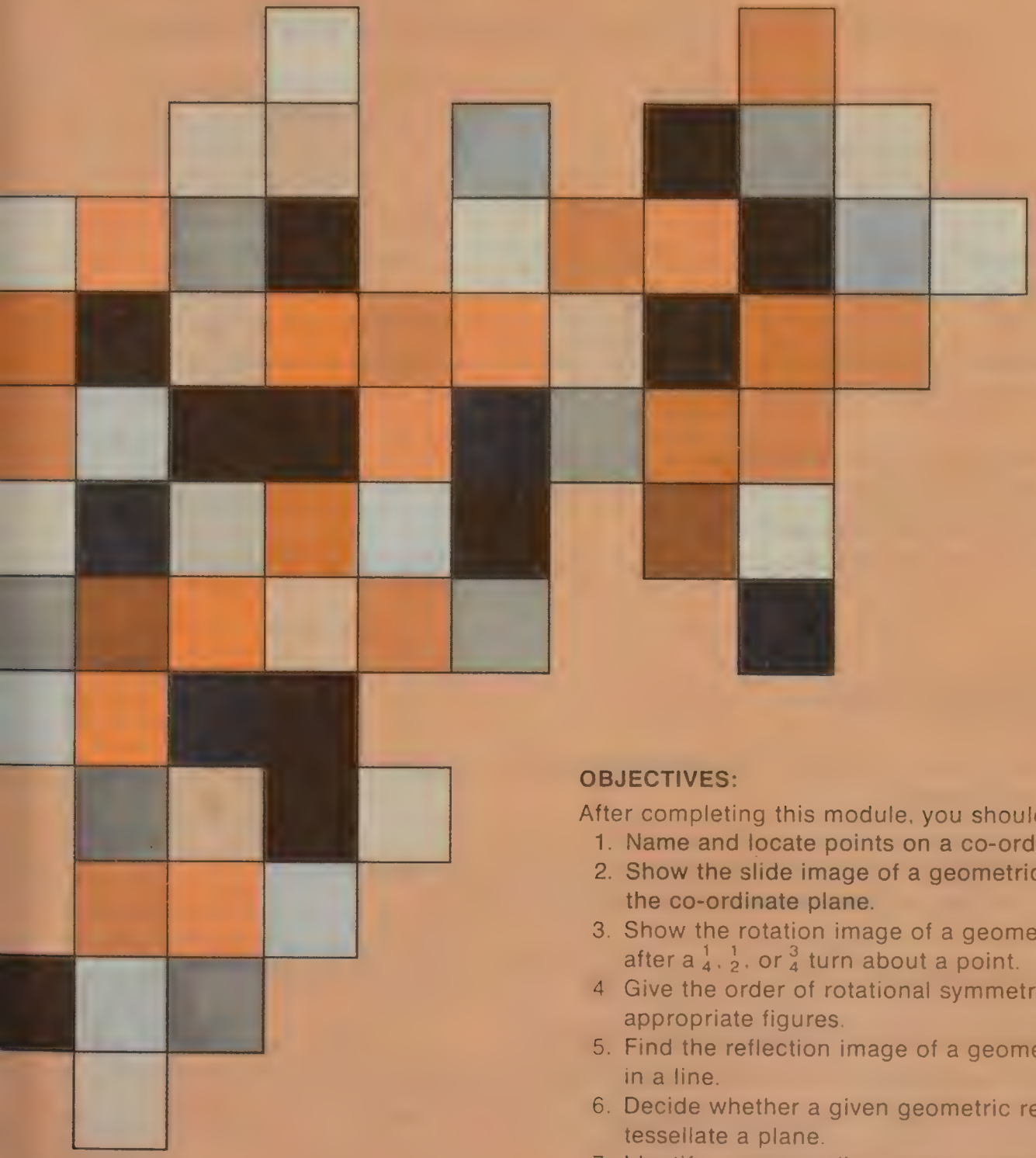
How should the cards be "stacked" so that they can turn up in 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 order?

MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

1. 34 mm **2.** 730 **3.** A $19\frac{1}{2}$ units; B $16\frac{7}{8}$ sq units **4.** 25 dm² **5.** A A: 9 sq units; B: 25 sq units; B C. 34 sq units **6.** A 481.95 m²; B 373.5 m² **7.** 50° **8.** A 119°8′; B 66°3′10″ **9.** A 70°47′; B 33°12′39″ **10.** 72°

MODULE 3: *Rigid Motions and Constructions*



OBJECTIVES:

After completing this module, you should be able to:

1. Name and locate points on a co-ordinate grid.
2. Show the slide image of a geometric figure in the co-ordinate plane.
3. Show the rotation image of a geometric figure after a $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ turn about a point.
4. Give the order of rotational symmetry of appropriate figures.
5. Find the reflection image of a geometric figure in a line.
6. Decide whether a given geometric region will tessellate a plane.
7. Identify corresponding angles and sides of congruent triangles.
8. Make the following constructions using reflections or ruler-and-compass method:
   - copy an angle
   - copy a triangle
   - bisect an angle
   - construct perpendicular lines
   - construct perpendicular bisector of a segment
   - construct a triangle given at least two segments

Investigating the Ideas

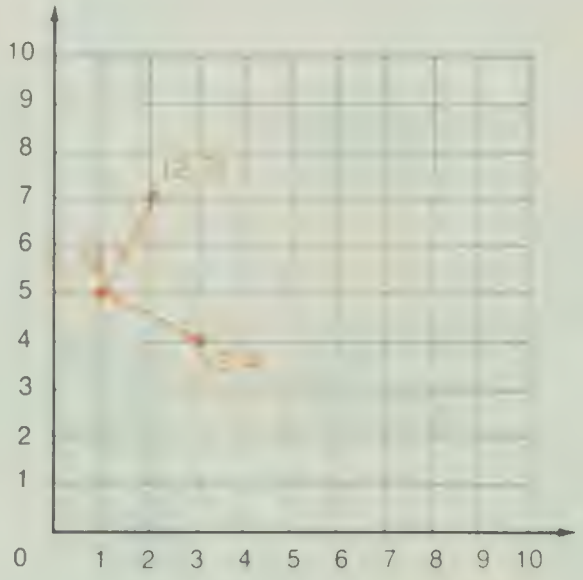
How many of the situations below can you show on a co-ordinate grid?

A A square with three of its vertices at (1,5), (2,7), and (3,4).

B The segment with an endpoint at (7,5) and its midpoint at (8,7).

C A circle with its centre at (5,5) and passing through (7,3).

D A rectangle with one vertex at (0,0), a height of one unit, and an area equal to the area of the square in part **A**.



Discussing the Ideas

1. What are the co-ordinates of the fourth vertex of the square for part A above?

2. Explain how you found the other endpoint of the segment for part B. What are the co-ordinates of this endpoint?

3. Can you give other co-ordinates of points on the circle for part C?

4. **A** What is the area of the square in part A?
   B What is the length of the rectangle for part D?
   C What are the co-ordinates of all the vertices of the rectangles that you drew for part D?

5. **A** Give the co-ordinates of a square whose sides are each twice as long as the square for part A and which also has one of its vertices at (1,5).
   B How does the area of this square compare with the area of the square for part A?

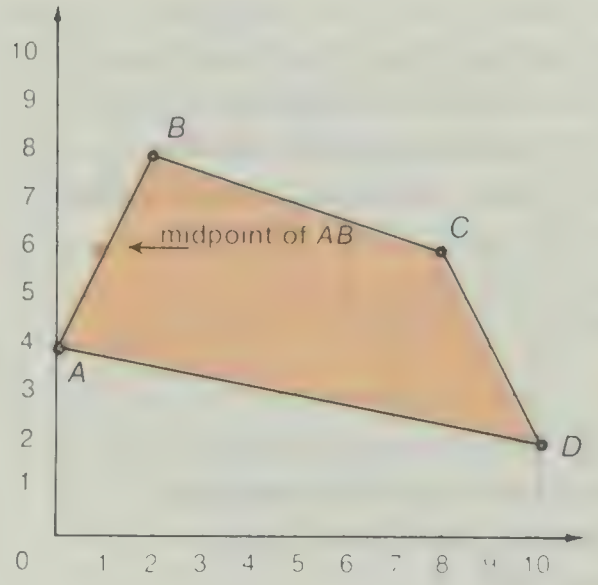
Refer to quadrilateral *ABCD* on the grid below for Exercises 1 through 4.

1. Give the co-ordinates of each vertex of quadrilateral *ABCD*.

2. Give the co-ordinates of the **midpoint** (middle point) of each side of the quadrilateral.

3. Find the co-ordinates of the midpoint of each diagonal.

4. Estimate the co-ordinates of the point of intersection of the diagonals.



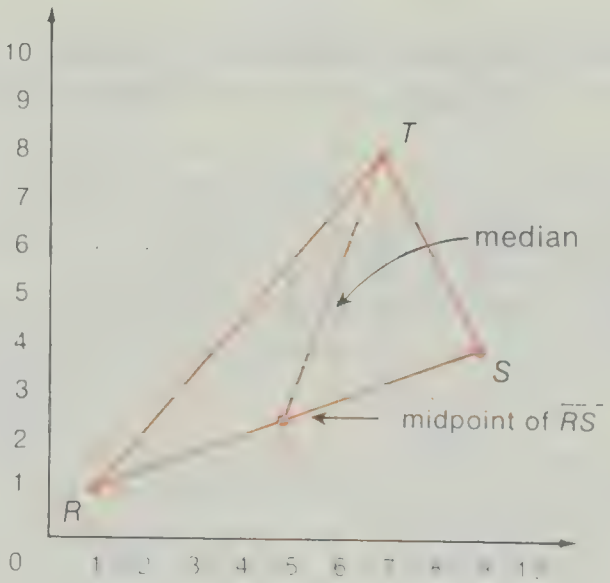
Refer to triangle *RST* on the grid below for Exercises 5 through 8.

5. Give the co-ordinates of the vertices *R*, *S*, and *T*.

6 Give the co-ordinates of the midpoint of each side of the triangle

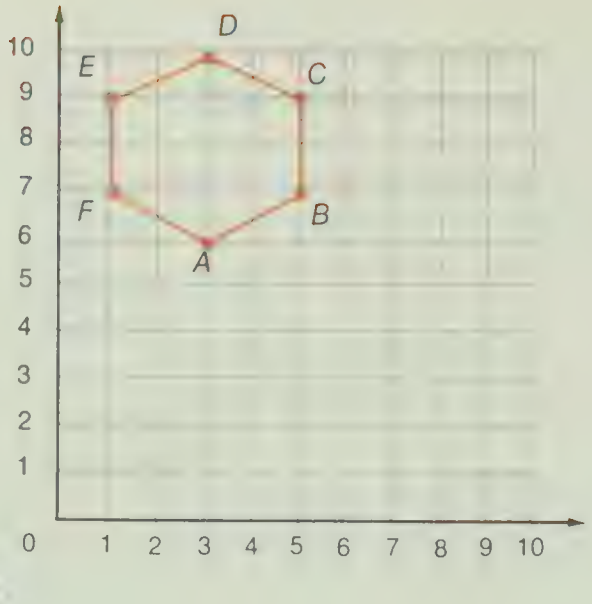
7. A **median** of a triangle is the segment from a vertex to the midpoint of the opposite side of the triangle.
Draw $\triangle RST$ on graph paper. Then draw the three medians of the triangle.

8. The three medians of a triangle intersect at one point called the **centroid** of the triangle.
Estimate the co-ordinates of the centroid of $\triangle RST$.



Investigating the Ideas

Draw a hexagon, like the one shown, on a 10 by 10 co-ordinate grid. List the co-ordinates for the vertices of the hexagon. Make another list of co-ordinates in which you add 4 to the first number of each pair and subtract 5 from the second number of each pair.



Can you draw the polygon formed by joining the new number pairs with line segments?

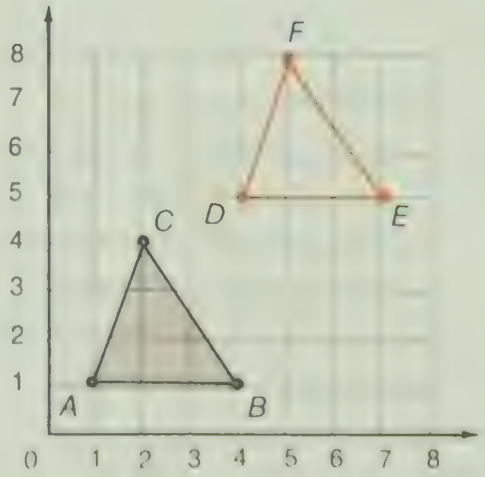
Discussing the Ideas

1. Describe the effect of "adding 4 and subtracting 5" to the coordinates of the original hexagon.

2. The motion produced by "adding 4 and subtracting 5" to the co-ordinates of every point in the plane is called a **slide** or **translation**. Describe some other translations.

3. A motion is a **rigid motion** if it does not change the size and shape of a figure. Do you think that a translation is a rigid motion?

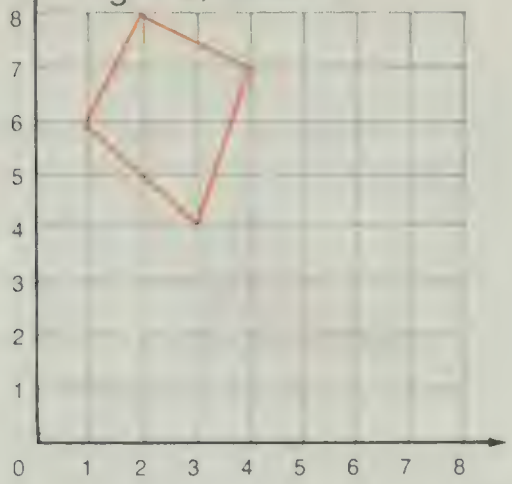
4. The figure that you drew in the Investigation is called a **slide image** of the original hexagon. It could be called a "right 4, down 5" translation. Draw some figures on your graph paper. Then show a "left 2, up 3" translation.



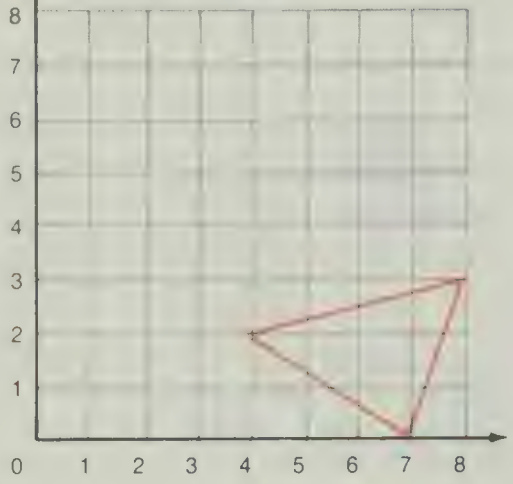
5. **A** Is $\triangle DEF$ a slide image of $\triangle ABC$? How can you tell?
   B The translation that slides $\triangle ABC$ to $\triangle DEF$ is "right ▮, up ▮."
   C How would you describe the translation that slides $\triangle DEF$ to $\triangle ABC$?

1. Copy each figure on a co-ordinate grid. Then show the slide image for the given translation.

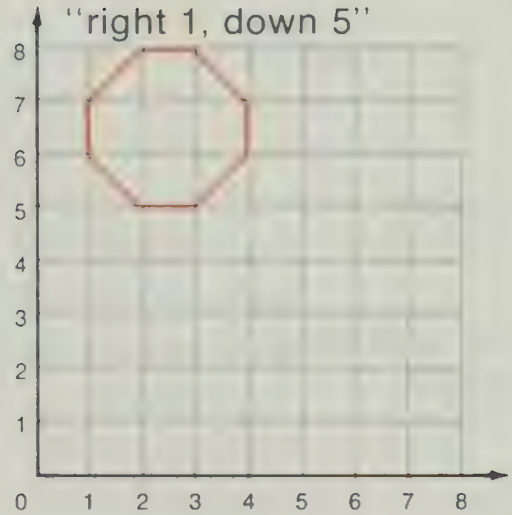
A "right 2, down 3"



B "left 3, up 3"



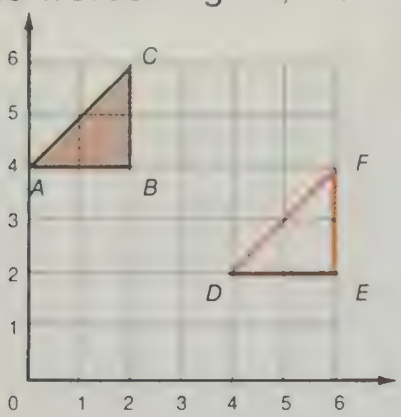
C "right 1, down 5"



D "right 4, down 2"

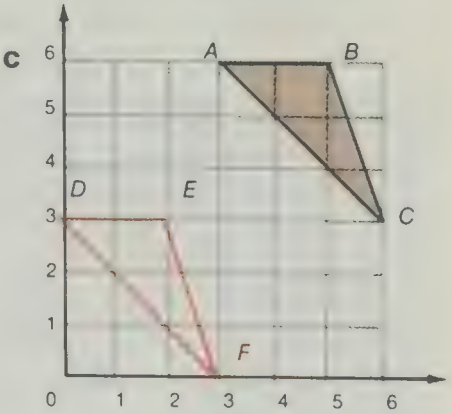
2. In each part, $\triangle DEF$ is the slide image of $\triangle ABC$. Describe the translation for each part using the words "right", "left", "up", or "down".

A



B

C

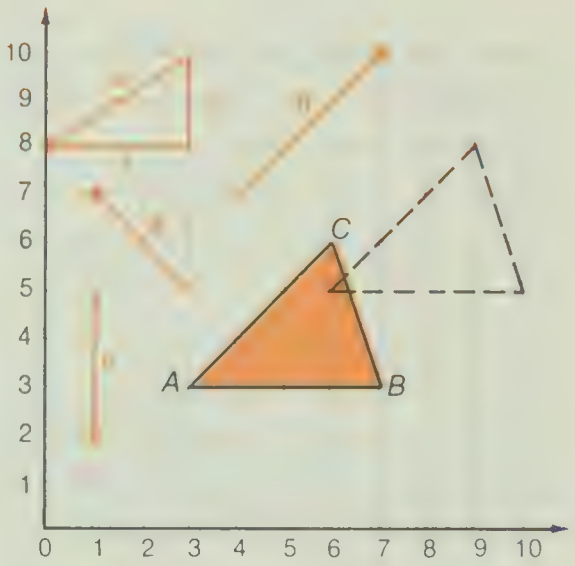


3. If a line is translated "right 2, up 1," what is the relation between the original line and its slide image?

Investigating the Ideas

The dotted triangle shows the slide image of $\triangle ABC$ when translated "right 3, up 2."

The arrow ***a*** is another way of indicating the translation "right 3, up 2."



Can you show how the triangle will be translated for arrows ***b***, ***c***, and ***d***?

Discussing the Ideas

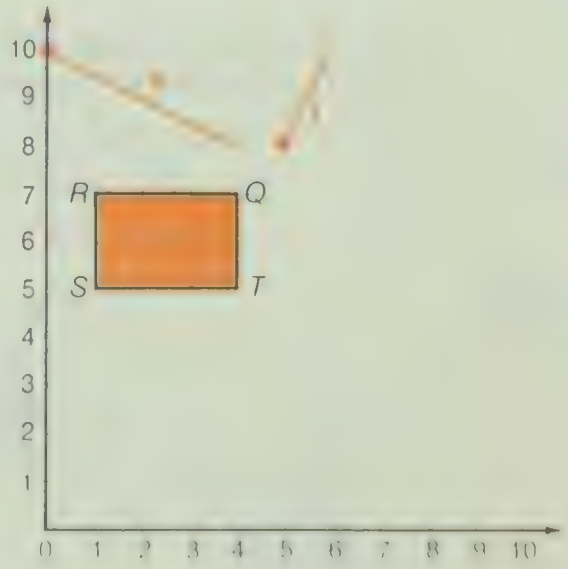
1. How would you describe the translation for arrow ***b***? arrow ***c***? arrow ***d***?

2. Suppose that rectangle *QRST* was translated first according to arrow ***e*** and the image then translated by arrow ***f***.

   A On your graph paper, show the location of the final image of rectangle *QRST*.

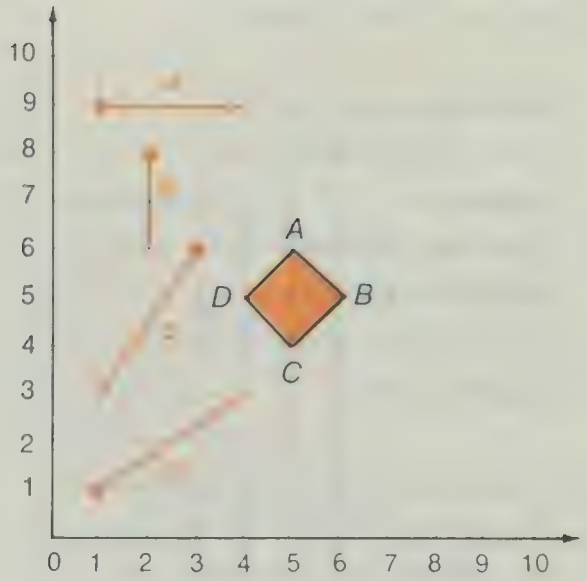
   B What single arrow would have translated *QRST* to its final position in only one move?

   C Suppose that *QRST* was first translated by arrow ***f*** and then by arrow ***e***. Is the final translation image in a different position than the one you drew for part **A**?

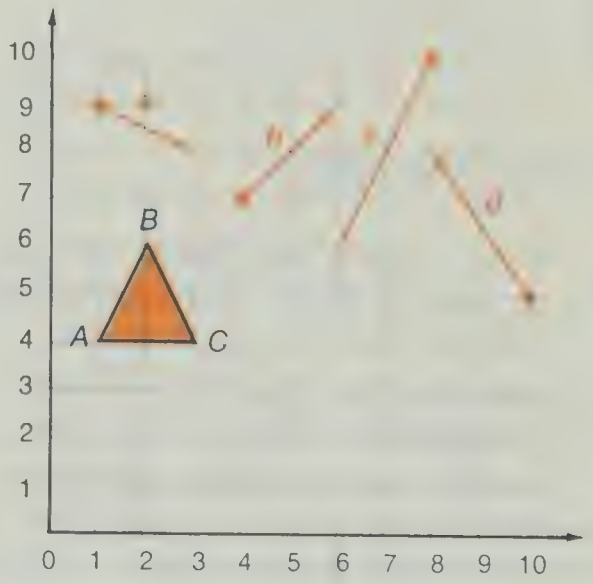


1. Show the slide image of square *ABCD* for each arrow.

2. Show the slide image of square *ABCD* for each combination of translations.
   - **A** ***a*** followed by ***b***
   - **B** ***a*** followed by ***c***
   - **C** ***b*** followed by ***d***
   - **D** ***c*** followed by ***d***
   - **E** ***b*** followed by ***a***
   - **F** ***c*** followed by ***a***



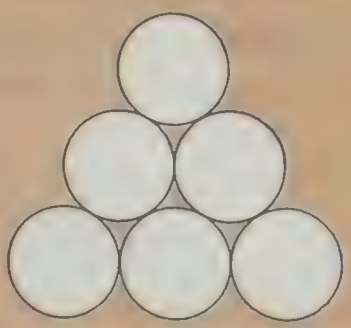
3. Show the slide image of triangle *ABC* for each combination of translations.
   - **A** ***a*** followed by ***d***
   - **B** ***b*** followed by ***c***
   - **C** ***b*** followed by ***d***
   - **D** ***a*** followed by ***b***
   - ★ **E** ***a*** followed by ***b***, followed by ***d***
   - ★ **F** ***a*** followed by ***b***, followed by ***c***, followed by ***d***



think

Fill the circles with the numerals 1, 2, 3, 4, 5 and 6 so that the sum along each three circles in a row is 9.

Can you arrange the numerals to get a sum of 10? A sum of 11? A sum of 12?

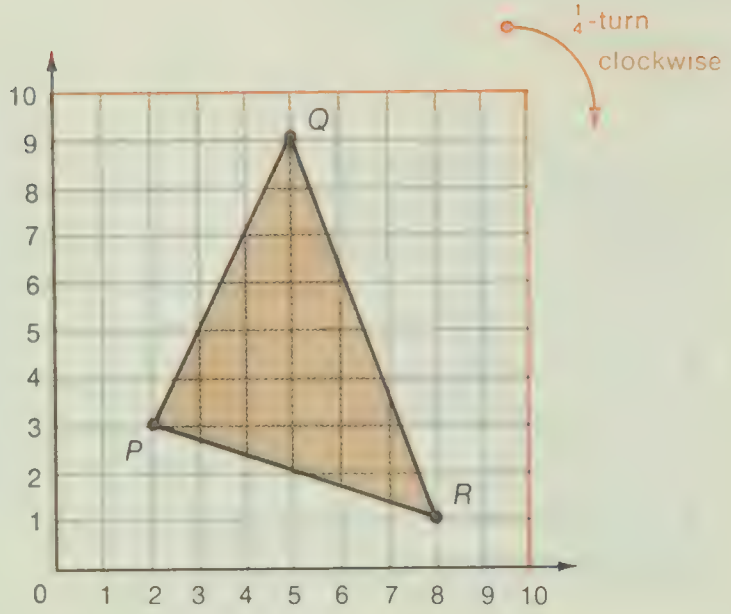


Investigating the Ideas

A Mark the sides of a 10 by 10 square grid in color.

B Mark the points P, Q, and R as shown.

C Cut out the 10 by 10 square grid.



Can you find the new co-ordinates of points P, Q, and R if the cut-out square is given a $\frac{1}{4}$ turn in a clockwise direction and placed back inside the colored square?

Discussing the Ideas

1. When the square was given a $\frac{1}{4}$-turn, all of the points on the grid, except one, moved to a new position. What are the co-ordinates of that point?

2. The turn that you gave the square in the Investigation is called a **rotation**. The point whose co-ordinates did not change (the centre of the square) is called the **centre of rotation**. The **rotation** image of the point P, whose co-ordinates are (2, 3), is the point whose co-ordinates are (3, 8).

   A Give the rotation images of some other points for a $\frac{1}{4}$-turn rotation.

   B Can you find the co-ordinates of some rotation images without actually rotating the square?

3. Do you think that a rotation is an example of a **rigid motion**?

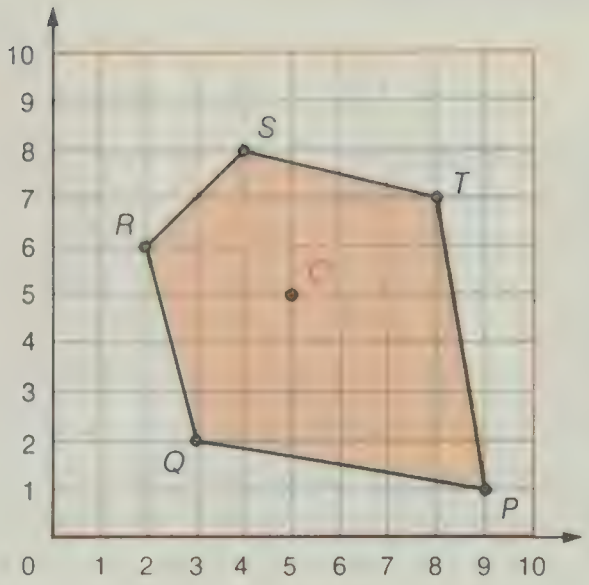
4. What are the new co-ordinates of points P, Q, and R for each of these rotations?

   A $\frac{1}{2}$-turn clockwise **B** $\frac{3}{4}$-turn clockwise **C** full turn

1. Suppose that the 10 by 10 square is given a $\frac{1}{4}$-turn clockwise with centre at point O. What are the co-ordinates of the rotation images of each of these points?

   A P **C** R **E** T

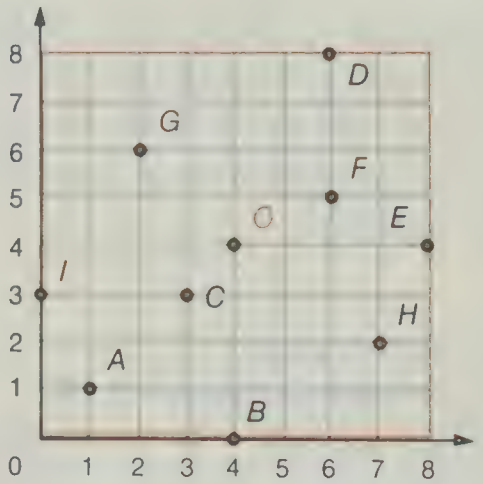
   B Q **D** S **F** O



2. Give the rotation images of each of the points P, Q, R, S, and T if the square is given a $\frac{1}{2}$-turn clockwise.

3. Repeat Exercise 2 for a $\frac{3}{4}$-turn clockwise of the square.

4. An 8 by 8 square with centre O at (4,4) is given a $\frac{1}{4}$-turn clockwise around O. Give the co-ordinates of the rotation images for the points with the co-ordinates shown.



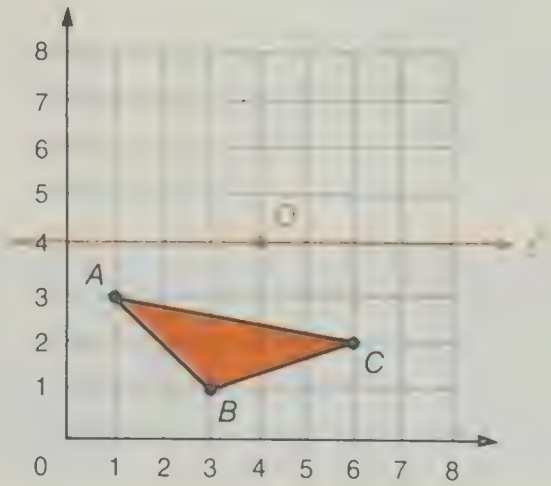
5. Repeat Exercise 4 for a $\frac{1}{2}$-turn clockwise.

6. Repeat Exercise 4 for a $\frac{3}{4}$-turn clockwise.

★ 7. **A** What are the co-ordinates of the vertices of the reflection image of $\triangle ABC$ in the line ℓ?

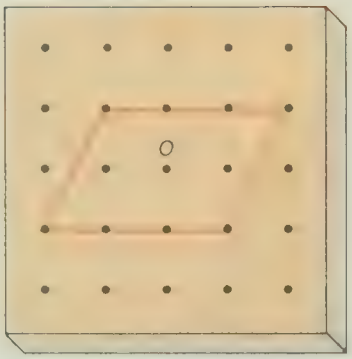
   B What are the co-ordinates of the vertices of $\triangle ABC$ after a $\frac{1}{2}$-turn clockwise about point O?

   C Is the half turn about O the same motion as a reflection in line ℓ?



Investigating the Ideas

Make a parallelogram like the one shown and in the same position on your geoboard or dot paper.

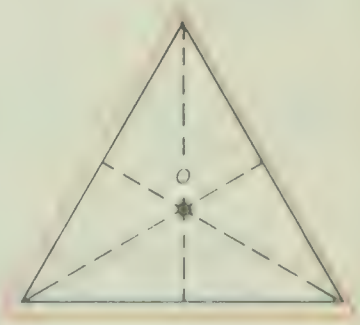


What is the smallest rotation with centre O that you can make so that the parallelogram will appear in the same position on the geoboard?

Discussing the Ideas

1. The parallelogram is said to have **rotational symmetry** with respect to a rotation about point O because all the points of the parallelogram fall on the parallelogram again after a $\frac{1}{2}$ turn. What are some other geometric figures that have rotational symmetry?

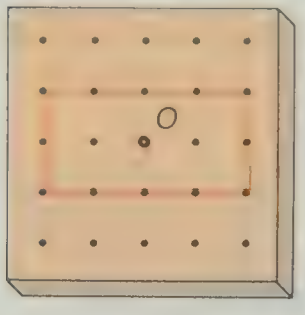
2. A square is said to have **rotational symmetry of order 4** because the square will fit into its outline 4 times in one complete turn about point O. What is the rotational symmetry order for the equilateral triangle about its centre O?

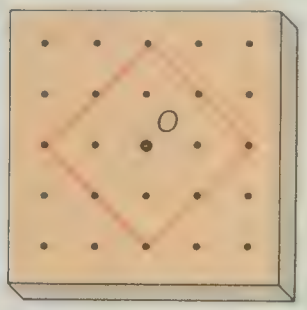


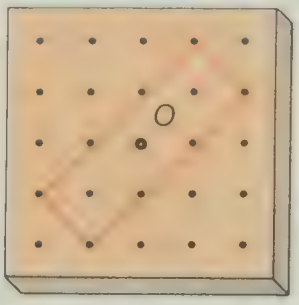
3. What is the order of rotational symmetry for the parallelogram shown in the Investigation?

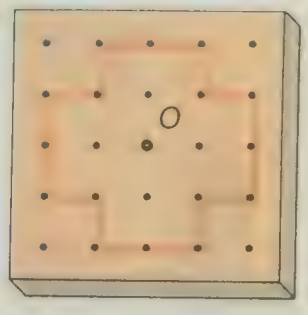
4. Form some polygons on your geoboard or on dot paper. See if they have rotational symmetry of any order.

1. Each figure on the geoboards has rotational symmetry about centre *O*. Give the **order** of the symmetry.

A 

B 

C 

D 

2. Give the order of rotational symmetry for each regular polygon with centre at point *O*.

A

B

C

D

3. Suppose square *ABCD* is rotated about centre *O*. Complete the table for each clockwise rotation.

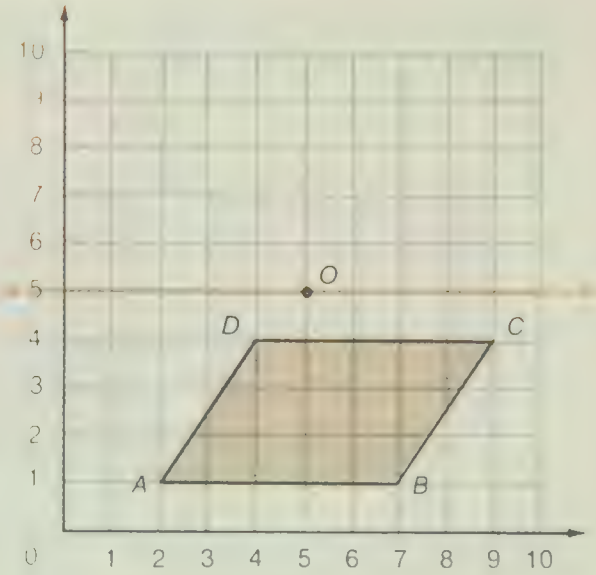
ROTATION	CO-ORDINATES OF POINTS			
	A	*B*	*C*	*D*
$\frac{1}{4}$	(4,7)			
$\frac{1}{2}$				
$\frac{3}{4}$				
1				

think

The quotient of **two consecutive even whole numbers** is $1\frac{1}{48}$.
Find this pair of whole numbers. $a \div b = 1\frac{1}{48}$

Investigating the Ideas

Suppose the colored line represents a mirror placed upright to the grid.



Can you find the reflection image of each point *A*, *B*, *C*, and *D* and then connect these points with line segments?

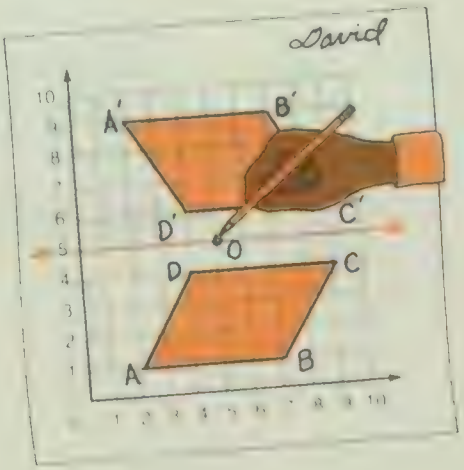
Discussing the Ideas

1. A How does the reflection image of figure *ABCD* compare with the original figure *ABCD*?
   B Do you think that reflections are examples of rigid motions?

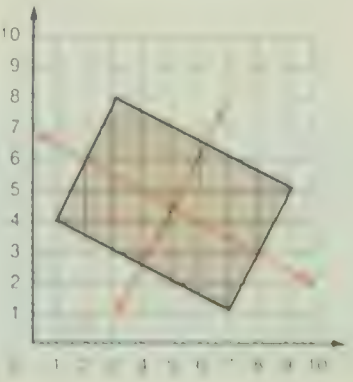
2. Suppose that the reflection image of *ABCD* is labeled *A′ B′ C′ D′*. Do you think there is a translation that would slide *ABCD* to *A′ B′ C′ D′*? Explain your answer.

3. David thought that a reflection in the colored line had the same effect as a $\frac{1}{2}$-turn about a point. In order to check this, he put a piece of tracing paper over his drawing, traced figure *ABCD*, then placed his pencil at point *O* and rotated his tracing a $\frac{1}{2}$ turn.

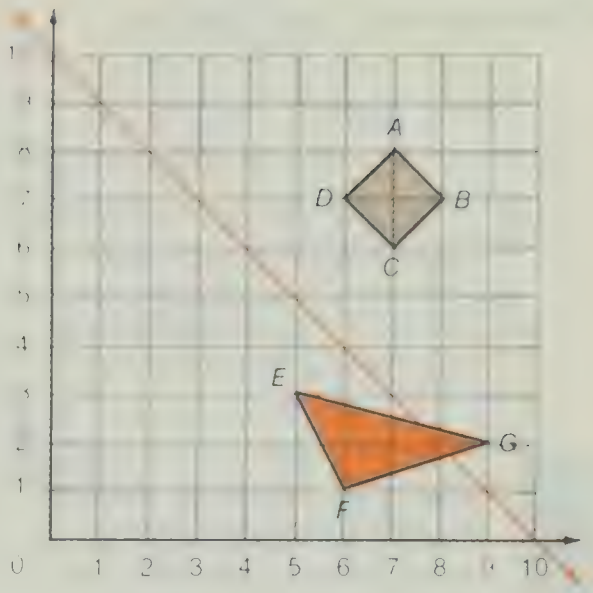
   What do you think that David found out about the rotation compared to the reflection?



4. Draw a figure on a co-ordinate grid paper that you think is a symmetric figure. Show one or more lines of symmetry for your figure.

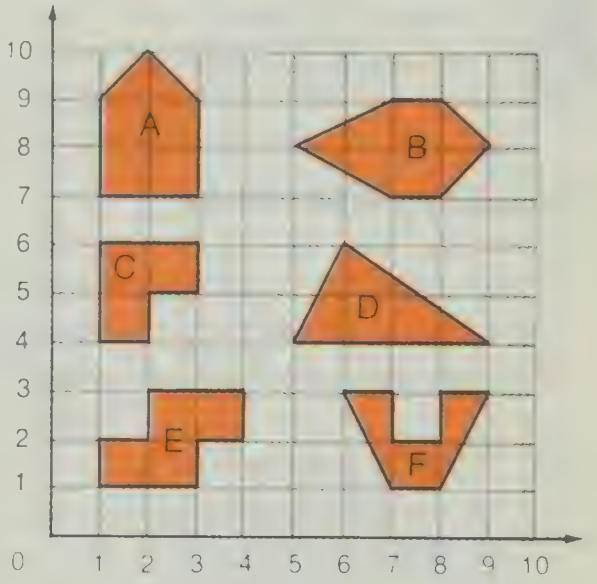


1. **A** Draw square *ABCD* as shown. Then find its reflection image in the diagonal line.
   B Give the co-ordinates of the reflection images for points *A*, *B*, *C*, and *D*.

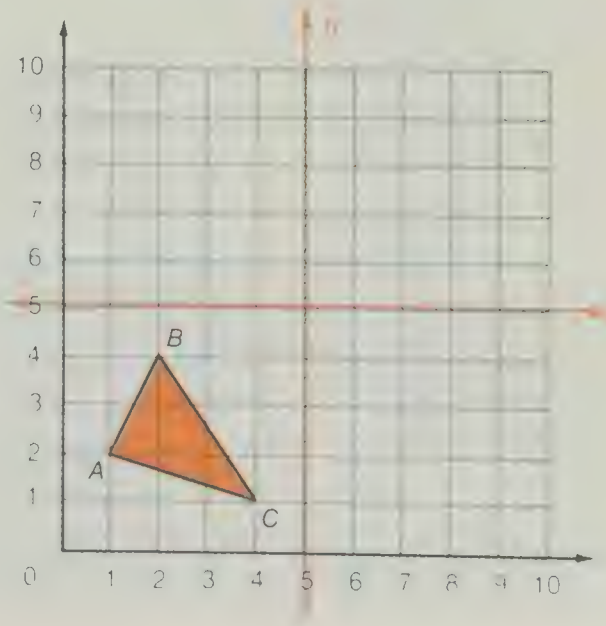


2. **A** Draw triangle *EFG* as shown. Find its reflection image in the diagonal line.
   B Give the co-ordinates of the reflection images for points *E*, *F*, and *G*.

3. **A** Draw each shape on a 10 by 10 co-ordinate grid.
   B Which shapes have a line of symmetry?
   C Draw the line(s) of symmetry for the symmetric figures.



4. **A** Draw $\triangle ABC$ in the position shown on a 10 by 10 co-ordinate grid at the right.
   B Find the reflection image of $\triangle ABC$ in line *a*. Call it $\triangle A'B'C'$.
   C Find the reflection image of $\triangle A'B'C'$ in line *b*. Call it $\triangle A''B''C''$.
   D Are $\triangle ABC$ and $\triangle A''B''C''$ reflection images of each other?
   E Can you find a rigid motion so that $\triangle ABC$ and $\triangle A''B''C''$ will be images of each other?

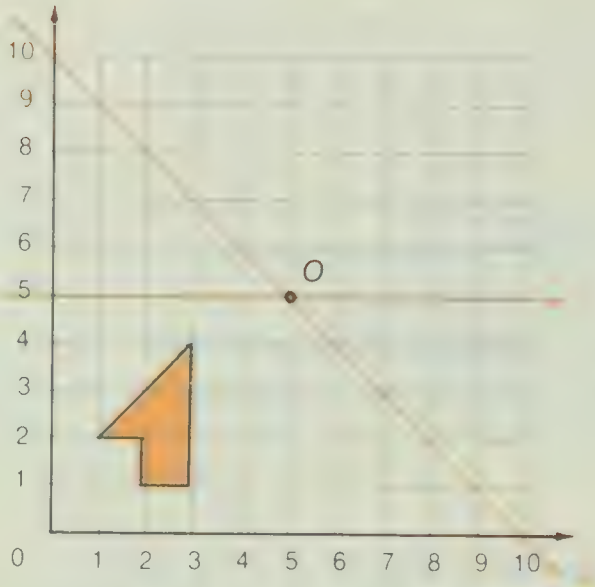


Investigating the Ideas

Draw a region like the one shown on the co-ordinate grid.

A Find the reflection image of the figure in line m.

B Reflect the image found for part **A** in line ℓ.



Can you describe a single translation, reflection, or rotation that will give the same final image as the combination of reflections **A** and **B**?

Discussing the Ideas

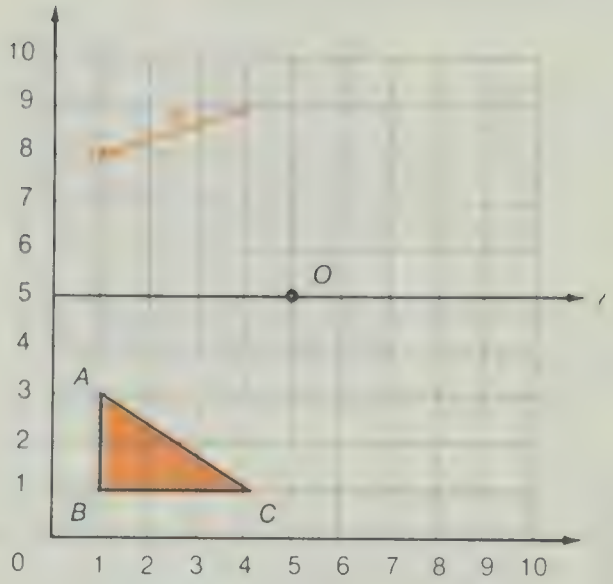
1. Would the final reflection image be in the same final position if the first reflection were in line ℓ and the second reflection in line m?

2. **A** What would be the co-ordinates of the image of point P after a clockwise rotation of 90° $\left(\frac{1}{4}\text{-turn}\right)$ about O and then a reflection in line s?

   B Find the co-ordinates of the image of points R and S after the motions given in part **A**.

3. Repeat the motions of Exercise 2, but perform them in reverse order. Do you get the same final image points?

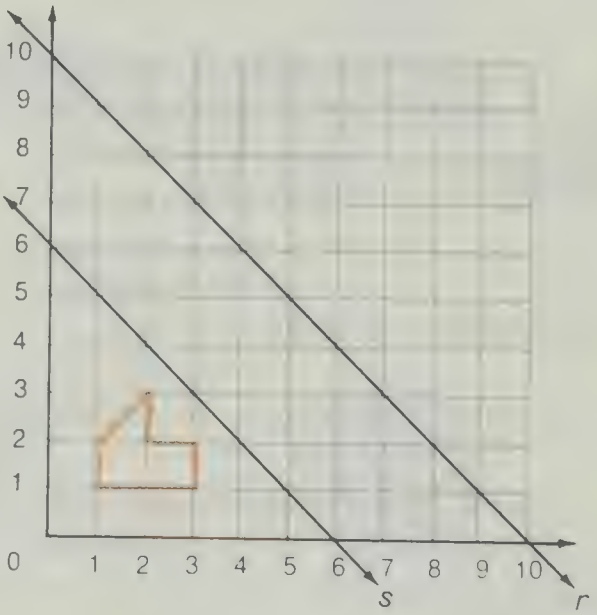
Refer to the co-ordinate grid containing $\triangle ABC$ for Exercises 1, 2, and 3.



1. Show the final image of $\triangle ABC$ after a translation by arrow ***a*** and a reflection in line ℓ.

2. Show the final image of $\triangle ABC$ after a clockwise rotation of $180°$ $\left(\frac{1}{2}\text{-turn}\right)$ about O and a reflection in line ℓ.

3. Show the final image of $\triangle ABC$ after a translation by arrow ***a*** and a clockwise rotation of $90°$.



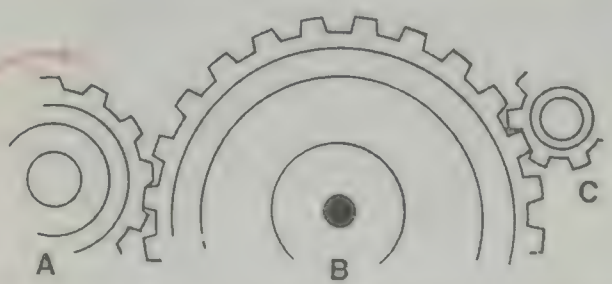
4. Draw parallel lines r and s and the region as shown.
   - **A** Find the reflection image of the region in line s followed by a reflection in line r.
   - **B** Can you find and describe one rigit motion that has the same effect as the two reflections for part **A**.

★ 5. Draw a geometric figure on a 10 by 10 co-ordinate grid. Choose any two motions (translation, rotation, reflection). Then, show the final image of your geometric figure after the two motions.

Think

Gear **A** has 24 cogs (teeth) and turns clockwise at 20 revolutions per second. Gear **B** has 48 cogs and gear **C** has 12 cogs.

How fast and in which direction do gears **B** and **C** turn?



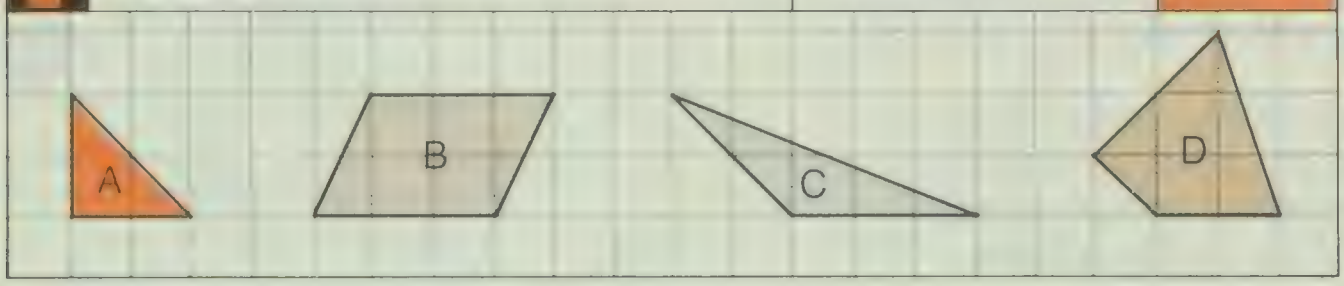
Investigating the Ideas

A **tessellation** is a repeated pattern of geometric figures which will completely "cover a plane" without any gaps or overlapping.

A checkerboard is an example of a tessellation. You can think of continuing the pattern endlessly.

Choose one of the regions below. Can you show if it could be used to make a tessellation?

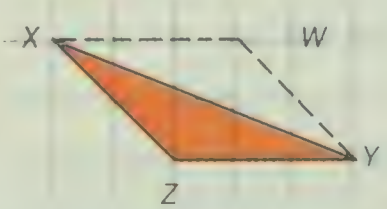
Use graph paper to show your tessellation.



Discussing the Ideas

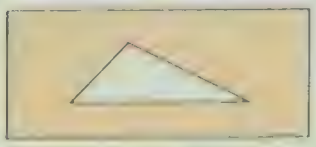
1. Do you think that every triangle can tessellate the plane?

2. In order to see that triangle C above will tessellate the plane, think about turning $\triangle XYZ$ until it is in the position of $\triangle XYW$.
How would you complete the tessellation?



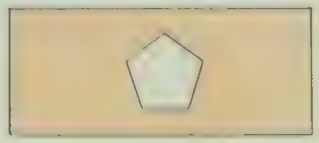
3. Some tessellations may be easier to construct if you would use a **template** or pattern to help you draw the shapes.
Cut out geometric shapes like those shown below from a small index card. Try each of these shapes to form a tessellation.

A



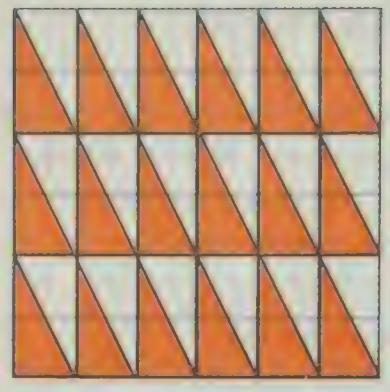
B

C



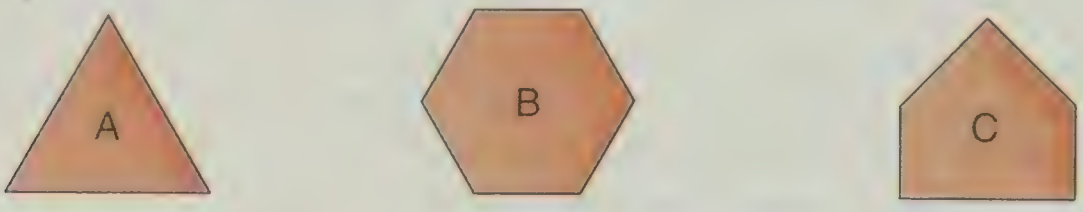
4. Which shape in Exercise 3 do you think will not tessellate the plane?

1. The tessellation shown at the right was made from a certain triangle. Graph paper was used to help make the tessellation.
Show a different tessellation using the same shape and size of triangle



2. Show that each of the shapes will tessellate the plane. Draw enough of the pattern to convince yourself that the pattern will form a tessellation.

3. Make a template for each figure. Use it to form a tessellation.



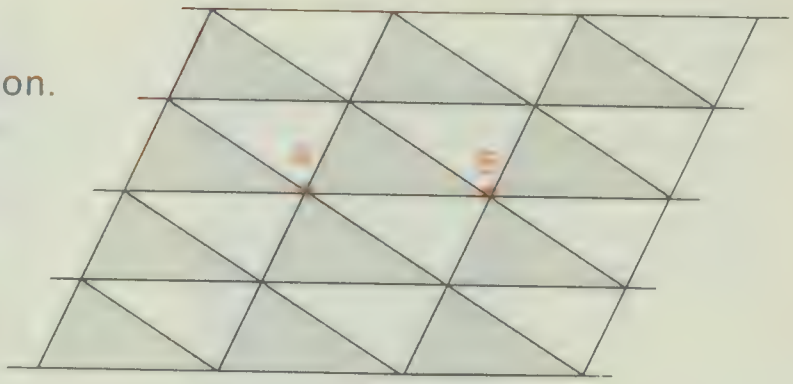
4. The figure below shows a tessellation made up of squares and octagons. Design a tessellation using squares and triangles

think

Can you make a tessellation with this shape?

Investigating the Ideas

Make a copy of this tessellation. on tracing paper. If you slide your copy from *A* to *B*, the tracing will again fit on the tessellation. (Assume that the tessellation covers the complete plane.)



?	How many slides in different directions along the lines can you find that will make the tracing fit the tessellation again?	Do not turn or flip the tracing!

Discussing the Ideas

1. The tessellation above has **translational symmetry** because certain slides (translations) of the points of the tessellation makes the tessellation pattern fit again on itself.
   Do you think that the tessellation has **reflectional symmetry**?

2. A Can you find a translation of the tessellation that is not along one of the lines so that the pattern falls again on itself? Describe the translation.
   B Can you find the shortest translation that is not along a line that makes the tessellation fall again on itself?

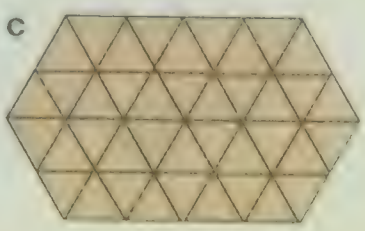
3. A Is point *A* a centre of rotation for a turn that will fit the tessellation back on itself? If it is, then the pattern has **rotational symmetry**.
   B What other centres of rotational symmetry can you find?

4. What kind of symmetry (**rotational**, **translational**, **reflectional**) can you find in these tessellations?

A

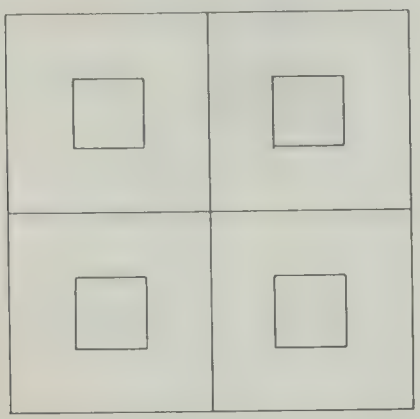
B

C

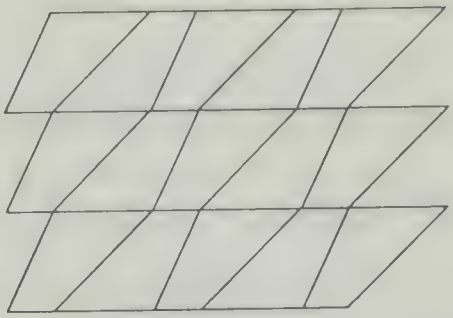


A small portion of a tessellation is shown for Exercises 1–6. Tell which kinds of symmetry (**rotational**, **translational**, or **reflectional**) each figure has.

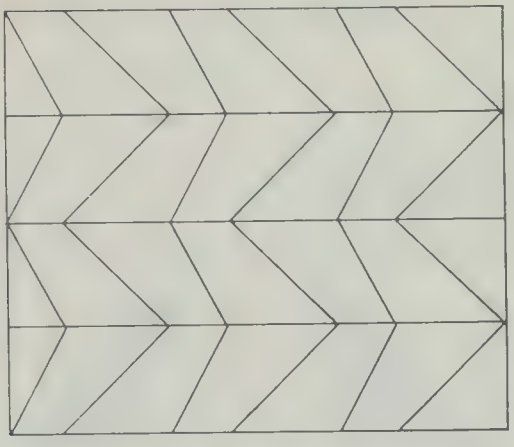
1.



2.

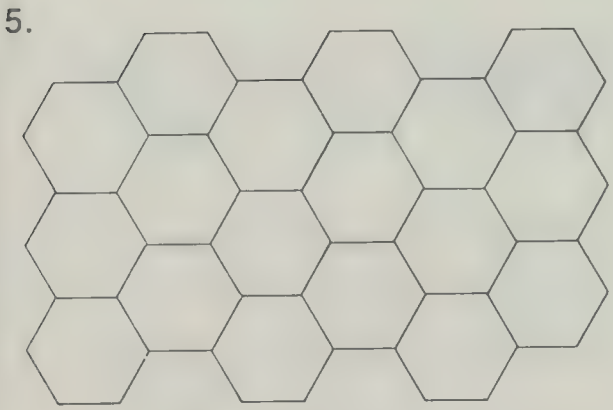


3.

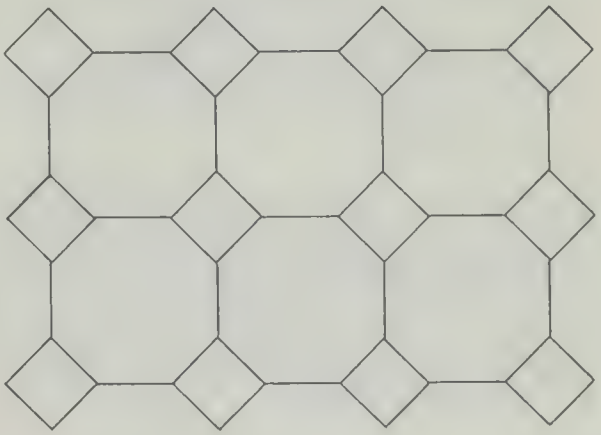


4.

5.



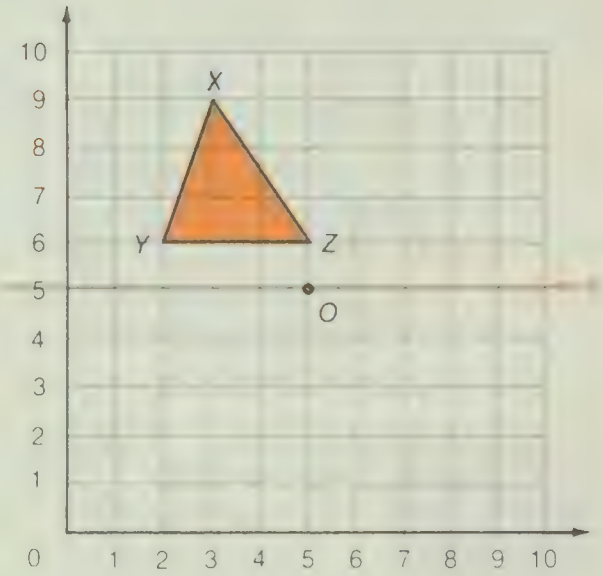
6.



★ 7. Design and color some tessellations of your own. Find what kind(s) of symmetry your tessellations have.

Investigating the Ideas

Draw $\triangle XYZ$ and line ***a*** through point O on a 10 by 10 grid.



Can you find the images of $\triangle XYZ$ for each of the rigid motions below?

1. The reflection image in line ***a***.
2. The slide image after a "right 3, down 6" translation.
3. The rotational image after a $\frac{1}{4}$-clockwise turn about centre O.

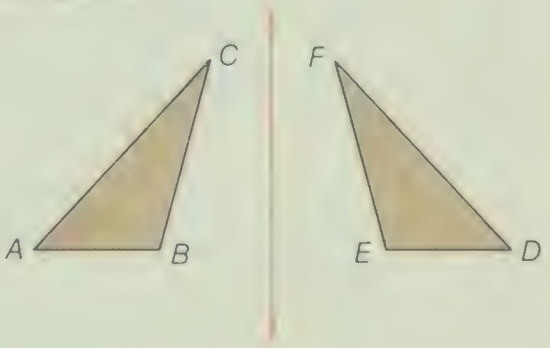
Discussing the Ideas

1. Are the three images that you found in the Investigation the same size and shape as $\triangle XYZ$?

2. Two geometric figures are **congruent** if one of the figures is the image of the other by one of the rigid motions, a reflection, a translation, or a rotation, or by any combination of rigid motions.

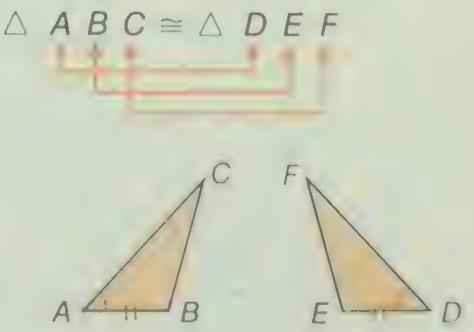
   A Which rigid motion makes $\triangle DEF$ an image of $\triangle ABC$?

   B What is the image of each vertex of $\triangle ABC$?



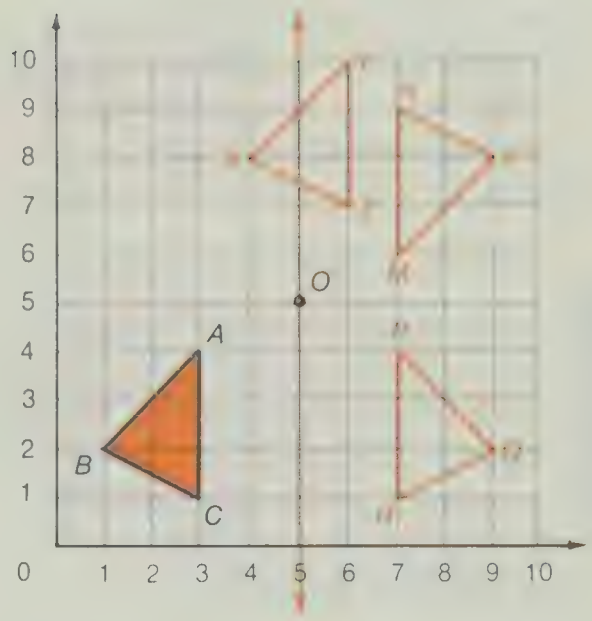
$\triangle ABC$ is congruent to $\triangle DEF$.
We write: $\triangle ABC \cong \triangle DEF$

3. When you write $\triangle ABC \cong \triangle DEF$, you pair the **corresponding points** that are images of each other. (See diagram at right.) Which points are images of each other if $\triangle RTS \cong \triangle QEK$?



4. If $\triangle ABC \cong \triangle DEF$, then **corresponding segments** and **corresponding angles** are congruent. Thus, $\overline{AB} \cong \overline{DE}$ and $\angle A \cong \angle D$. What other pairs of segments and angles are congruent?

Use the co-ordinate grid at the right for Exercises 1, 2, and 3.

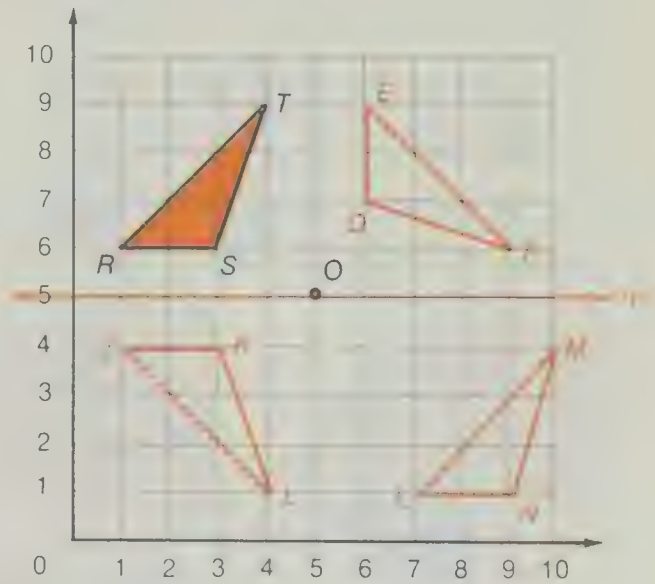


1. **A** $\triangle ABC \cong \triangle YXZ$ because $\triangle YXZ$ is a translation image of $\triangle ABC$ by a translation of "right ▒▒, up ▒▒."
   B $\angle A \cong \angle$▒▒ $\angle B \cong \angle$▒▒ $\angle C \cong \angle$▒▒
   C $\overline{AB} \cong$ ▒▒ $\overline{BC} \cong$ ▒▒ $\overline{AC} \cong$ ▒▒

2. $\triangle HKM$ is the rotation image of $\triangle ABC$ by a $\frac{1}{2}$-turn about point O.
   A Which pairs of segments are congruent?
   B Which pairs of angles are congruent?

3. $\triangle PQR$ is the reflection image in line ℓ of $\triangle ABC$.
   A Which pairs of segments are congruent?
   B Which pairs of angles are congruent?

4. Describe the rigid motion that makes
   A $\triangle JKL$ an image of $\triangle RST$.
   B $\triangle EDF$ an image of $\triangle RST$.
   C $\triangle LMN$ an image of $\triangle RST$.



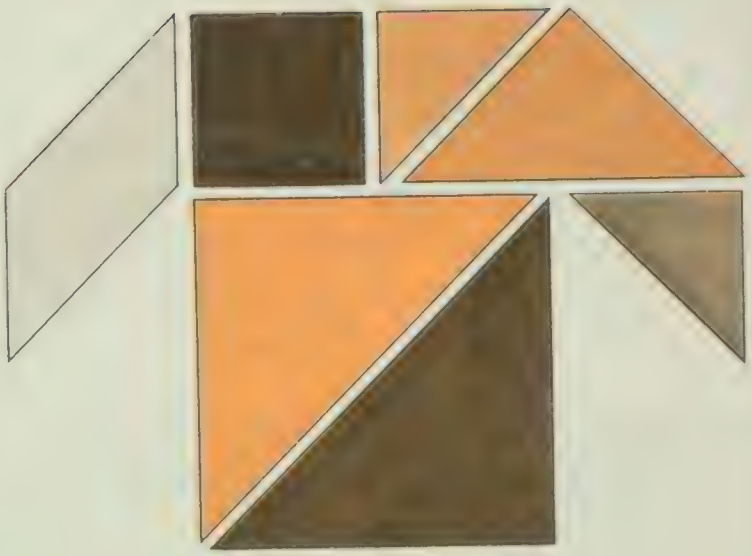
5. Write a statement of congruence for $\triangle RST$ and each of the other triangles.

★ 6. Draw a quadrilateral on a grid.
   A Show a translation image.
   B Show a rotation image about a point.
   C Show a reflection image in a line.
   D Write a statement of congruence between the quadrilateral and its images.

Investigating the Ideas

These seven regions are the pieces of an ancient puzzle called the Tangram Puzzle.

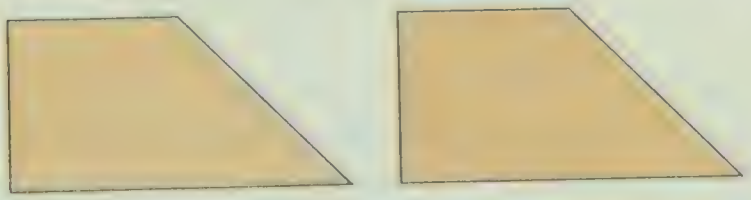
Trace the pieces and cut them out.



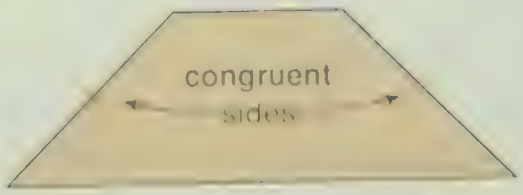
How many different ways can you find to make a triangle that is congruent to one of the large triangles?

Discussing the Ideas

1. Are the two large triangles congruent? How can you decide?
2. What other pair of pieces of the tangram puzzle are congruent?
3. Which pieces will form a square congruent to the small square piece?
4. Can you make a figure congruent to the parallelogram? Which pieces must you use?
5. Form a triangle congruent to the middle-sized triangle. Which pieces must you use?
6. A trapezoid has a pair of parallel sides. Use the tangram pieces to show two congruent trapezoids like the ones below.



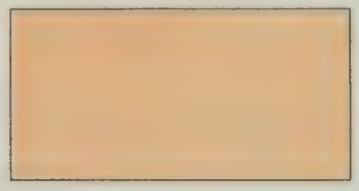
7. Can you form an **isoceles trapezoid** from some of the tangram pieces?



Isoceles trapezoid

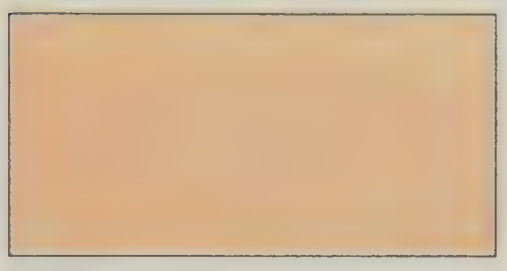
1. Make shapes congruent to each of these below using the tangram pieces that you cut out for the Investigation. Make a sketch to show your solution.

A



B

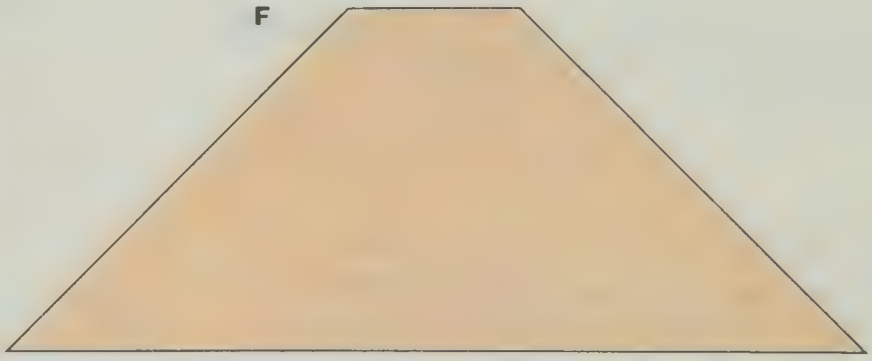
C



D

E

F

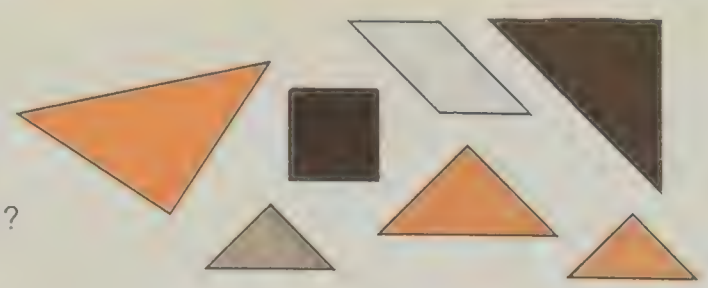


2. Use one of the small triangles and the square to form a shape. Use other pieces to form a shape congruent to the first shape.

3. Make a square using the two large triangles. Then make a square congruent to it by using the remaining pieces.

Think

How many different-sized squares can you form using two or more of the tangram pieces for each square?

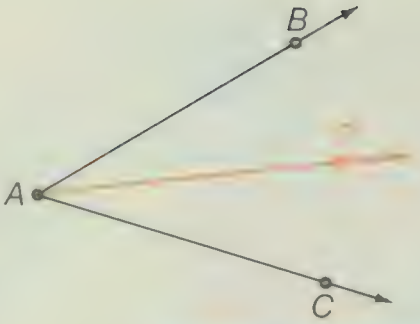


Investigating the Ideas

Ray AP is the **bisector of** $\angle BAC$ because it divides $\angle BAC$ into two congruent angles.

We say: $\overline{AP}$ bisects $\angle BAC$

Draw some angles on your paper.

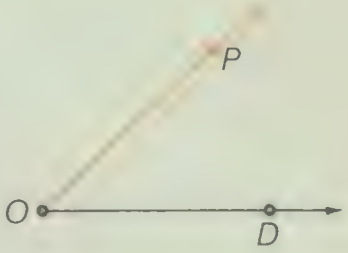


Can you find a way to use a plastic mirror and reflections to bisect the angles?

Discussing the Ideas

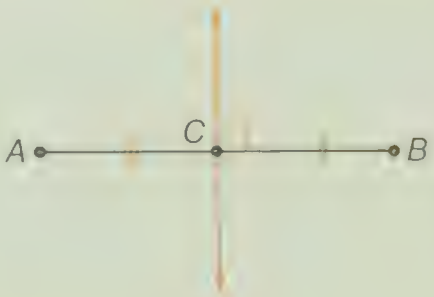
1. Explain how you found where to draw the bisector of your angle?

2. If $\overrightarrow{OP}$ is the bisector of $\angle DOH$, how can you use reflections to find the other ray, $\overrightarrow{OH}$, of $\angle DOH$?

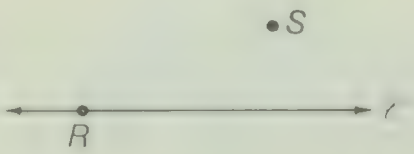


3. Line ℓ is the **perpendicular bisector** ($\perp$ bis) of AB at point C because $\ell \perp AB$ and $AC \cong BC$.

   Draw a segment. Then explain how to use reflections to construct the "$\perp$ bis" of the segment.



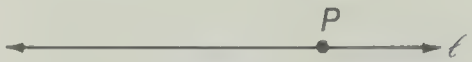
4. Explain how to use reflections to make these constructions:
   - **A** Construct a line through point R which is perpendicular to ℓ.
   - **B** Construct a line through point S which is perpendicular to ℓ.



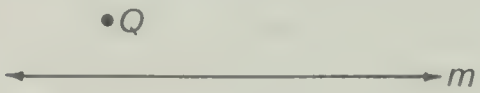
1. Draw a segment. Use reflections to construct the perpendicular bisector of the segment.

2. Draw an angle. Find the bisector of the angle by using reflections.

3. Construct a line through point P which is perpendicular to line ℓ.

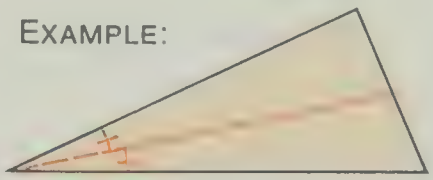


4. Construct a line through point Q which is perpendicular to line m.



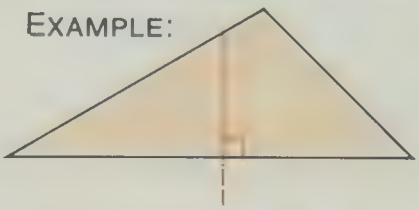
5. Draw a triangle. Find the three bisectors of the angles using reflections.

EXAMPLE:



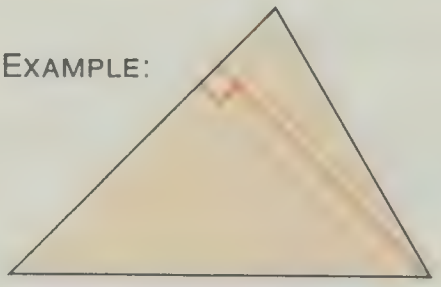
6. Draw a triangle. Construct the perpendicular bisectors of the sides using reflections.

EXAMPLE:

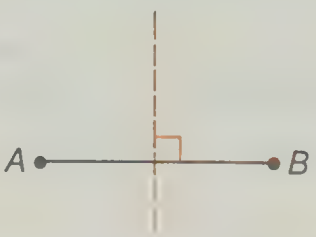


7. Draw a triangle. Construct the three altitudes of the triangle by using reflections. (An **altitude** of a triangle is a segment from one vertex perpendicular to the opposite side of the triangle.)

EXAMPLE:

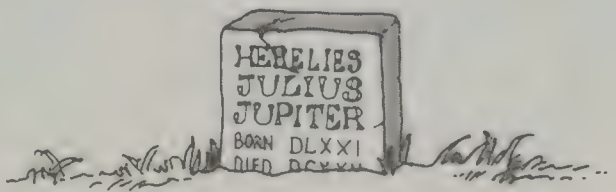


★ 8. Draw a segment and label it $\overline{AB}$. Then use reflections to construct an equilateral triangle with sides congruent to $\overline{AB}$. (HINT: First construct the $\perp$ bis of $\overline{AB}$.)



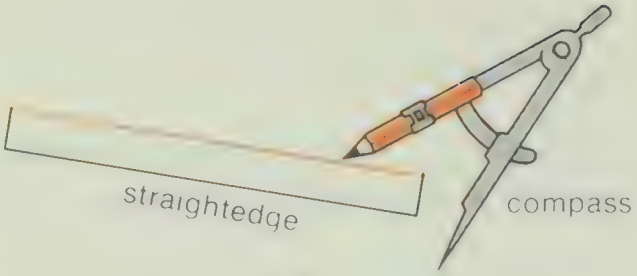
think

In what year did Julius Jupiter die? (Half of the date is missing.) How old was Julius when he died?

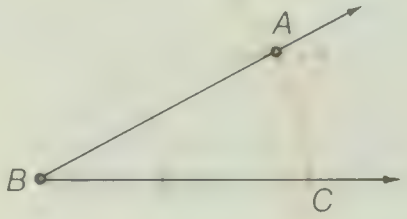


Investigating the Ideas

Drawings of geometric figures using only a straightedge and compass, and of course, a pencil are called **ruler-and-compass constructions**. A straightedge can be thought of as a ruler without units marked on it.



The numbered steps on the figure at the right shows how $\angle A'B'C'$ was constructed so that it is congruent to $\angle ABC$; that is, $\angle A'B'C' \cong \angle ABC$.

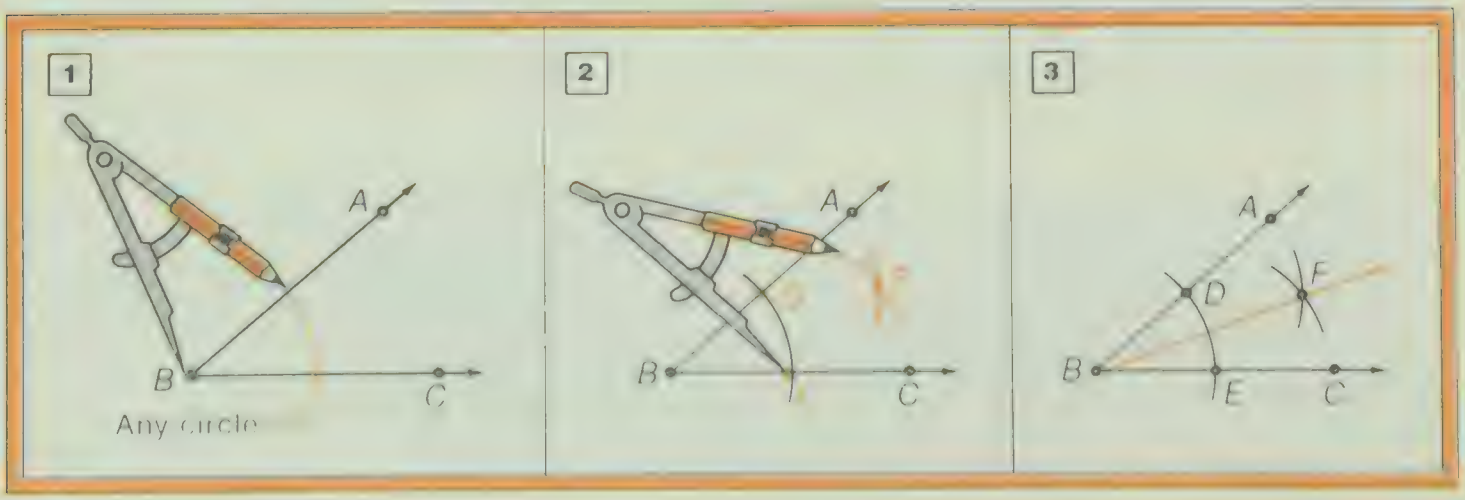


Can you draw an angle on your paper and then construct an angle congruent to it using a straightedge and compass?

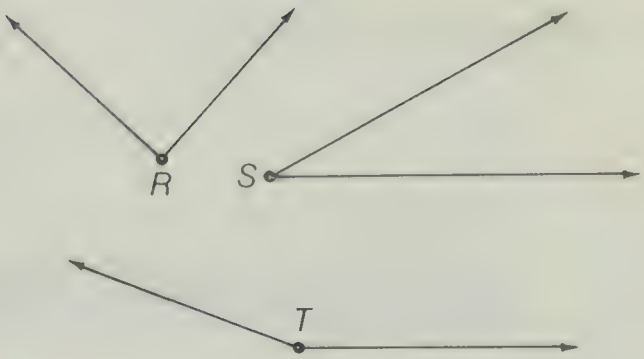
Discussing the Ideas

1. Explain the six steps for constructing an angle congruent to a given angle.

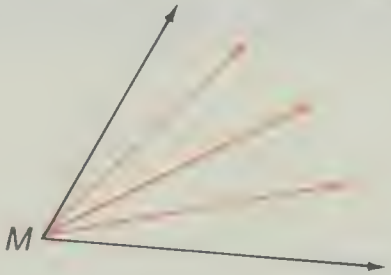
2. The illustration below shows three steps needed to bisect $\angle ABC$ using a straightedge and compass. Draw an angle on your paper and then bisect it with your straightedge and compass.



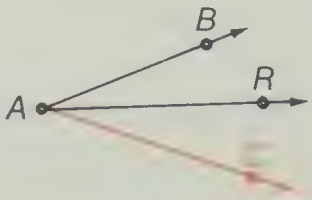
1. **A** Draw three angles about the same size as angles R, S, and T.
   B Construct angles R', S', and T' congruent to angles R, S, and T respectively.
   C Bisect each of the constructed angles R', S', and T'.



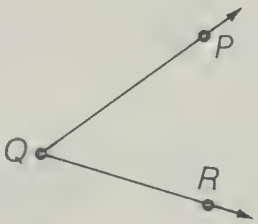
2. Draw an angle M.
   A Bisect it.
   B Bisect each of the two new angles formed.



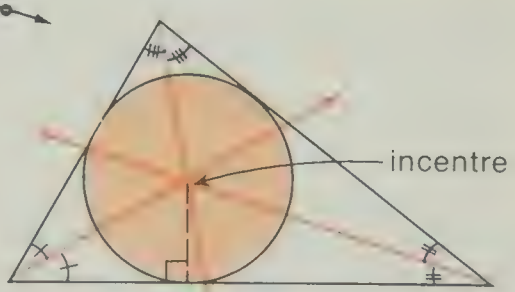
3. Ray AB is one side of $\angle BAC$.
   Ray AR is the bisector of $\angle BAC$.
   Show a construction that will give the other side ($\overrightarrow{AC}$) of $\angle BAC$.



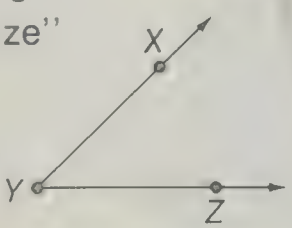
4. **A** Draw an angle about the size of $\angle PQR$, and label it $\angle STW$.
   B Construct an angle twice the "size" of $\angle STW$.



5. **A** Draw a large triangle on your paper.
   B Construct the bisector of each angle of the triangle.
   C If you make the constructions for part **B** carefully, you should find that the three bisectors intersect in one point called the **incentre** of the triangle. The incentre is the centre of a circle that just touches all three sides of the triangle. Draw that circle in your triangle.



6. **A** Draw $\angle XYZ$.
   B Construct an angle $1\frac{1}{2}$ times the "size" of $\angle XYZ$.

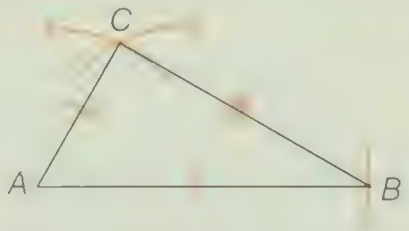


think

Arrange ten points in five rows with four points in each row.

Investigating the Ideas

In the construction shown below, $\triangle A'B'C'$ was drawn so that $\triangle A'B'C' \cong \triangle ABC$. Study and follow the steps to see how the construction of $\triangle A'B'C'$ was made.

Given triangle	Construction
	

Can you draw a triangle and then construct a triangle congruent to it using a straightedge and compass?

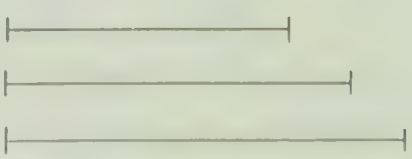
Discussing the Ideas

1. Draw a triangle like $\triangle XYZ$. Explain how to construct a triangle congruent to $\triangle XYZ$.

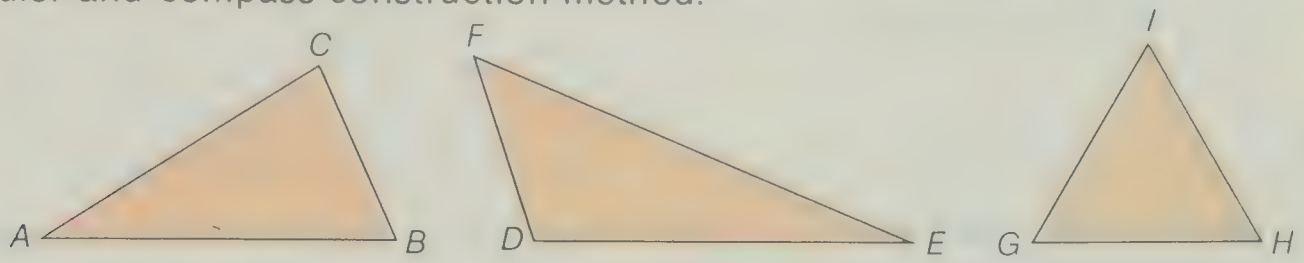
2. Study the construction shown. What kind of triangle is $\triangle PQR$?

3. Show how to draw an isoceles triangle.

4. Show and explain how to construct a triangle whose sides will be congruent to the three given segments.

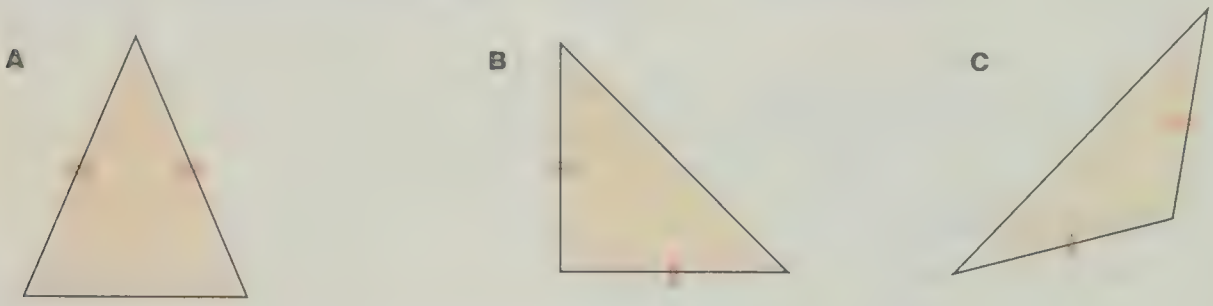


1. Draw three different-shaped triangles like those shown below. Then construct a triangle congruent to each of them using the ruler-and-compass construction method.



2. Construct an equilateral triangle. Make each side four centimetres long.

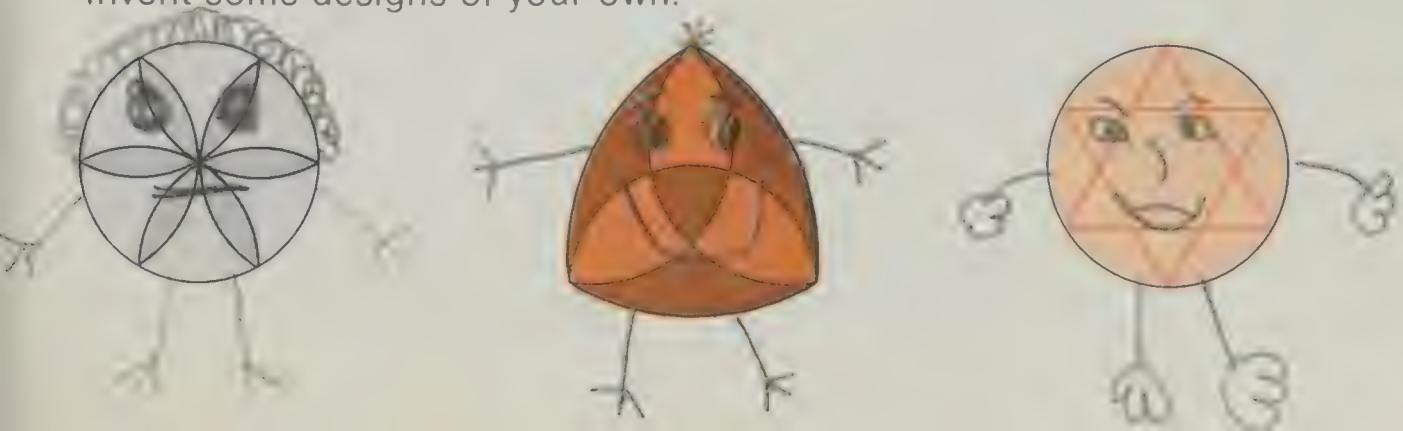
3. Construct isoceles triangles congruent to those below.



4. Construct triangle whose sides are congruent to the three given segments.

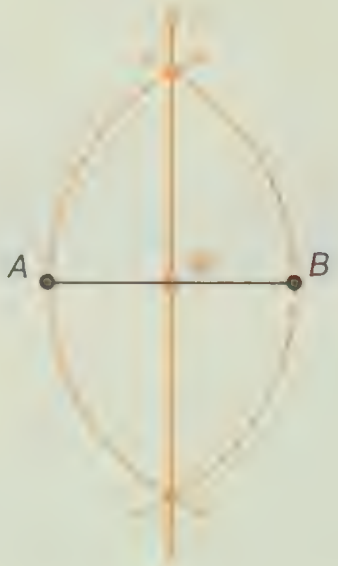
★ 5. Can you draw three segments that cannot be used as sides of a triangle?

★ 6. The designs below are based on equilateral triangles and circles. Make a copy of at least one of them. Invent some designs of your own.



Investigating the Ideas

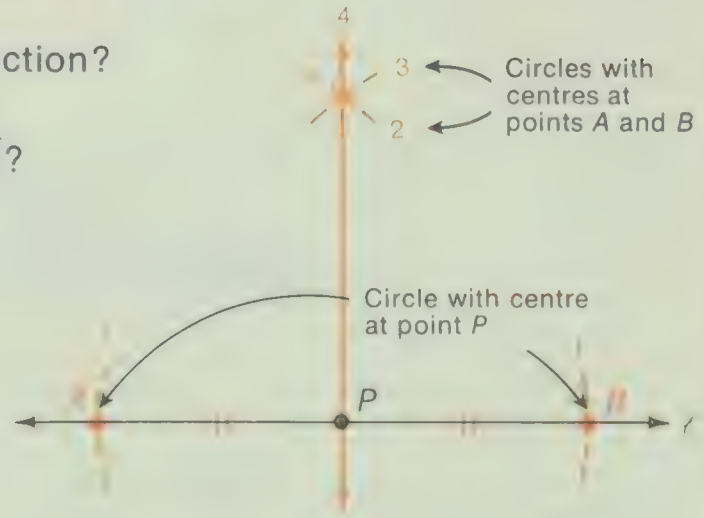
Study the construction at the right.



Can you start with a segment, $\overline{AB}$, and make a construction like this with your straightedge and compass?

Discussing the Ideas

1. The figure in the Investigation suggests a ruler-and-compass construction of a line that is **perpendicular** to a segment. The line also bisects the segment; thus, line ℓ is called the **perpendicular bisector** of $\overline{AB}$.
   - **A** Explain how you made your construction?
   - **B** What kind of angles were formed?
   - **C** What can be said about $\overline{AM}$ and $\overline{MB}$?

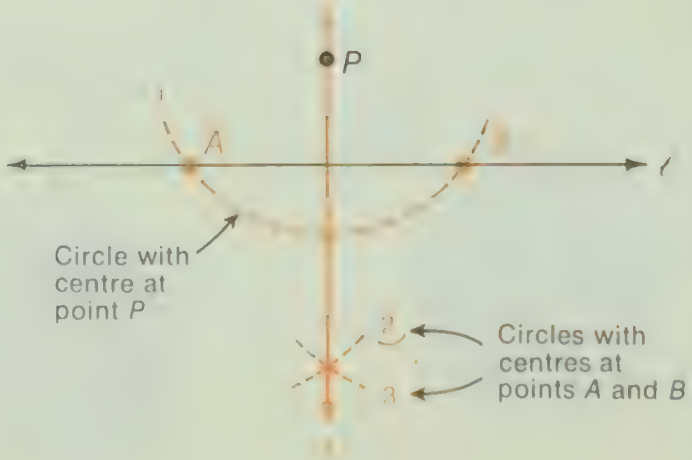


2. Study the construction for drawing "a perpendicular to a line ℓ through a point P *on the line.*"

   Make a construction like this one.

3. Study the construction for drawing "a perpendicular to a line ℓ through a point P which is **not** on the line."

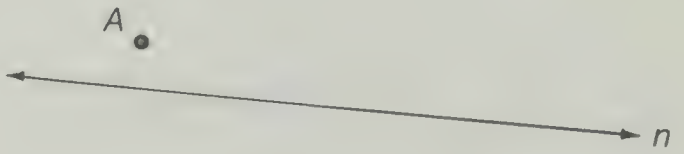
   Make a construction like this one.



1. Draw a segment. Then construct a perpendicular bisector of the segment.

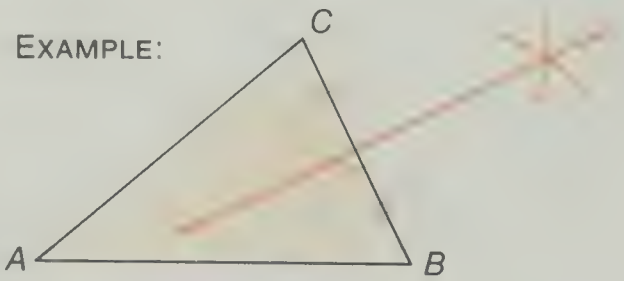
2. Draw a line and call it m. Mark a point Q on it and construct a line through Q that is perpendicular to line m.

3. Draw a line n and mark a point A not on n. Construct a line through A that is perpendicular to line n.



4. Draw a large triangle. Construct the perpendicular bisectors of each side of the triangle.

EXAMPLE:

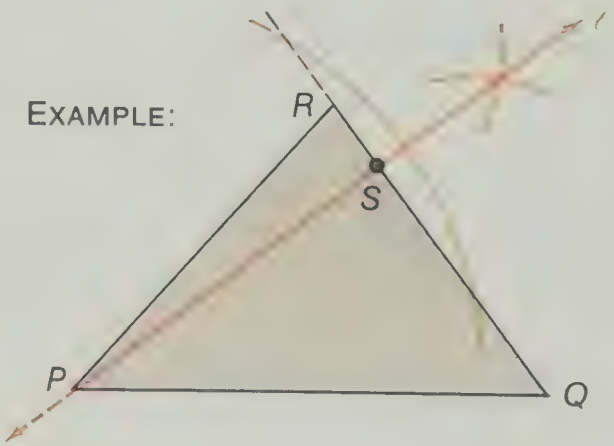


5. A Draw a triangle like $\triangle PQR$.

   B Construct a line ℓ through point P such that $\ell \perp \overleftrightarrow{QR}$. Let point S be the point where line ℓ intersects $\overline{QR}$. $\overline{PS}$ is one of the **altitudes of** $\triangle PQR$.

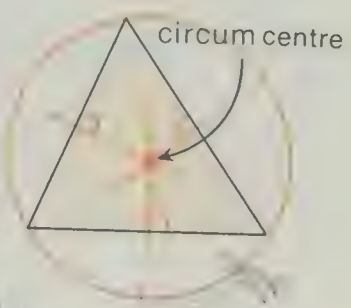
   C Construct the other two altitudes of $\triangle PQR$.

EXAMPLE:



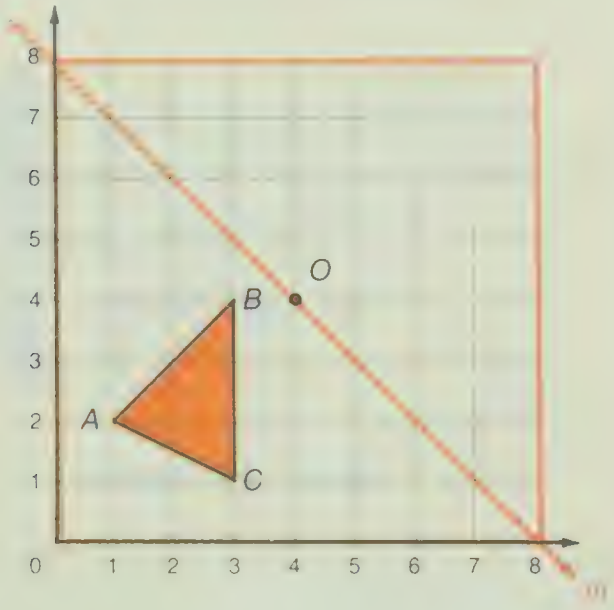
6. Construct a right triangle with legs congruent to these segments.

7. The intersection of the perpendicular bisectors of the sides of a triangle (see Exercise 4) is the centre of a circle which will contain the vertices of the triangle. The centre of the circle is the **circumcentre** and the circle is said to **circumscribe** the triangle. Draw a triangle and find the circumcentre. Then construct the circle that circumscribes the triangle.



REVIEWING THE IDEAS

For each Exercise 1, 2, 3, and 4, refer to the figure on the grid in its original position.



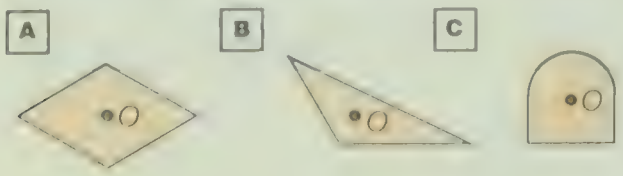
1. Give the co-ordinates of the vertices of $\triangle ABC$.

2. Give the co-ordinates of the images of A, B, and C after a translation of "right 2, up 1."

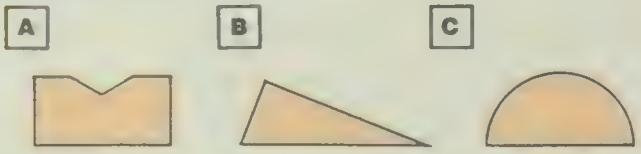
3. Give the co-ordinates of the images of A, B, and C after the square grid has been given a $\frac{1}{4}$ clockwise rotation with center at point O.

4. Give the co-ordinates of the reflection images of A, B, and C of $\triangle ABC$ in line m.

5. Which figure has rotational symmetry?



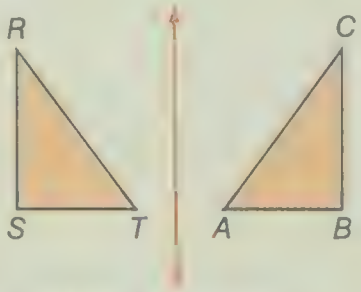
6. Which region could be used to tessellate a plane?



7. Complete the statement:

   If two geometric figures are images of each other by some kind of rigid motion, then the two figures are ?

8. $\triangle RST$ is the reflection image of $\triangle ABC$.

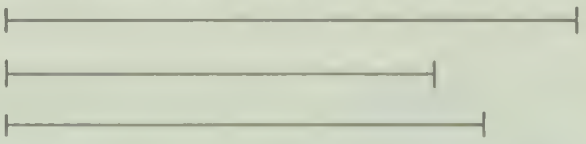


   Write a statement of congruence about the triangles.

9. Suppose $\triangle JKL \cong \triangle GEF$.
   - A What pairs of sides are congruent?
   - B What pairs of angles are congruent?

10. Draw an angle, then find its bisector using
    - A reflections.
    - B ruler-and-compass construction.

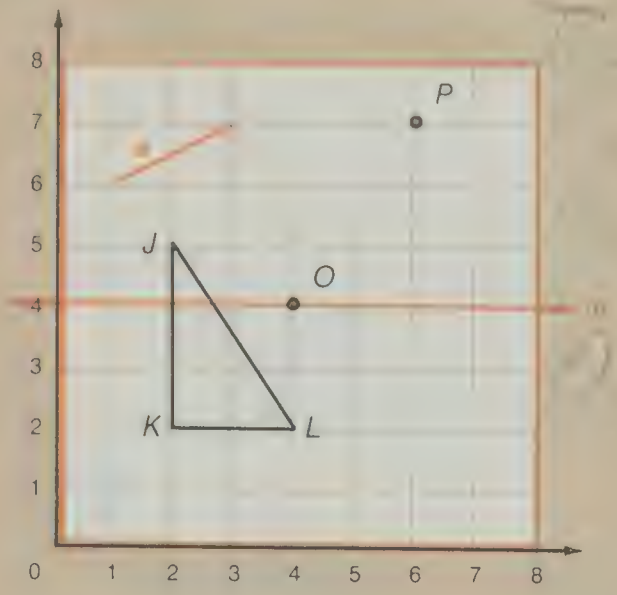
11. Construct a triangle whose sides are congruent to these three segments.



12. Draw a segment. Then construct the perpendicular bisector of the segment using
    - A reflections.
    - B ruler-and-compass construction.

TEST YOURSELF

Refer to the grid for Exercises 1–4.



1. Give the co-ordinates of the vertices of $\triangle JKL$.

2. Give the co-ordinates of the images of J, K, and L after a translation by arrow **a**.

3. Give the co-ordinates of the image of point P after a $\frac{1}{2}$ turn with point O as the centre of rotation.

4. Give the co-ordinates of the reflection image of point P in line m.

5. What is the order of rotational symmetry for the rectangle with centre at point O?

6. **True** or **false**:

   Any triangle can form a tessellation of the plane.

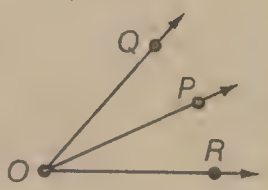
7. If $\triangle XYZ \cong \triangle MOP$, then

   A $\angle Z = \angle$ ▒▒ **B** $\overline{XY} \cong$ ▒▒

Complete the statements for Exercises 8, 9, and 10.

8. If A and B are reflection images of each other in line ℓ, then line ℓ is the ? ? of $\overline{AB}$.

9. If $\overrightarrow{OP}$ bisects $\angle QOR$, then $\overrightarrow{OR}$ is the ? image of $\overrightarrow{OQ}$ in $\overrightarrow{OP}$.



10. Geometric constructions can be made using a ? and straightedge.

RESEARCH PROJECTS

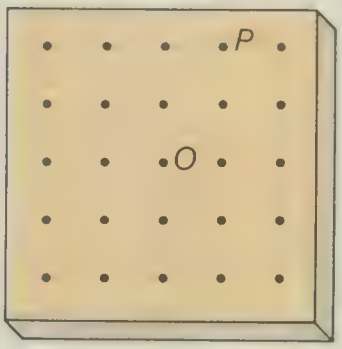
A Any angle can be bisected with a ruler and compass but **trisection** (dividing into three congruent parts) of a general angle using only a ruler and compass has been proved to be impossible. Other devices have been invented that enable us to trisect an angle. Make some models of the devices and report your findings to your class. (See *Mathematical Models* by H. Martyn Cundy and A. P. Rowlett; London: Oxford University Press, 1954.)

B Make ruler-and-compass constructions of as many different-sided regular polygons as you can. (See *Mathematical Recreations and Essays* by E. T. Ball and W. W. Rouse; New York: The Macmillan Company, 1962.)

C Make and color various designs (see C-95, Ex. 6) or tessellations based on geometric figures that have rotational, translational, or reflectional symmetry. Display them on the bulletin board.

A Geoboard Treasure Hunt

Try an imaginary treasure hunt on a 5 by 5 geoboard. Follow the steps below in order to locate the treasure.



1. Start at point P.
2. Rotate point P a $\frac{1}{4}$ turn clockwise with point O as the centre of rotation.
3. Translate the image of step 2 "down 1, left 1."
4. Translate the image of step 3 "up 3, left 1."
5. Rotate the image of step 4 using a $\frac{1}{2}$ turn clockwise with point O as the centre of rotation.
6. Translate the image of step 5 "right 2, up 3."
7. Reflect the image of step 6 in the horizontal line of symmetry of the geoboard.
8. The image of step 7 is the location of the hidden treasure. Where is it?

Make up your own treasure hunt on a geoboard. Give your puzzle to a classmate to solve.

MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

1. $J:(2,5)$, $K:(2,2)$, $L:(4,2)$ **2.** $J:(4,6)$, $K:(4,3)$, $L:(6,3)$
3. (2, 1) **4.** (6, 1) **5.** 4 **6.** true **7.** A $\angle P$; B $\overline{MO}$
8. perpendicular bisector **9.** reflection **10.** compass

MODULE 4: *Circles and Space Figures*

OBJECTIVES:

After completing this module, you should be able to:

1. Identify various parts of a circle such as radius, diameter, chord, and arc.
2. Identify central angles and inscribed angles in a circle, and to know that the measure of an inscribed angle is one-half the measure of a central angle subtending the same arc of a circle.
3. Recognize when a line is tangent to a circle.
4. Find the circumference of a circle.
5. Find the area of a circular region.
6. Recognize an ellipse.
7. Identify prisms, pyramids, cylinders, cones, and spheres.
8. Make nets and models of various space figures.
9. Find the volume of prisms, pyramids, cylinders, and cones.
10. Sketch a cross section of a space figure given a picture of a plane intersecting a space figure.

Investigating the Ideas

Can you make the construction following the directions below?

1. Use your compass to draw a **circle**.
2. Label the **centre** of the circle O.
3. Choose a point P on the circle. Draw and label segment $\overline{OP}$.
4. Draw a segment that contains point O and whose endpoints are on the circle. Label this segment $\overline{XY}$.
5. Choose two points on the circle. Connect these points with a segment. Label this segment $\overline{AB}$.

Discussing the Ideas

1. Give your own definition of a circle. Check your definition with the definition given in the glossary.

2. The segment OP that you drew in step 3 above was a **radius** of the circle. (The plural of radius is **radii**.)
   - A How many radii does a circle have?
   - B Are they all the same length?

3. Segments like XY (step 4) are called **diameters** of a circle. How does the length of a diameter compare to the length of a radius of a circle?

4. A segment like $\overline{AB}$ (step 5) is a **chord** of a circle. Is a diameter also a chord of a circle? Explain.

5. Suppose that a circle has a radius of 4 centimetres.
   - A If a point is in the **interior** of the circle, what can you say about its distance from the centre of the circle?
   - B If a point is 6.1 centimetres from the centre of the circle, is it in the interior or the **exterior** of the circle?

1. **A** Use a compass to draw a circle that has a radius of 3 centimetres
   B How long is the diameter of the circle?
   C Draw a chord of the circle that is 4.5 centimetres long.
   D Mark a point *X* that is 2.5 centimetres from the centre of the circle. Is *X* in the interior of the circle?

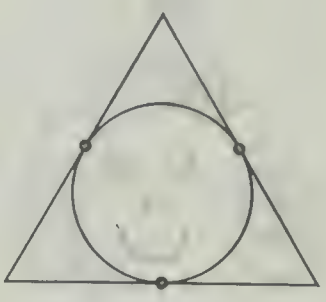
2. Draw a circle and two diameters that are not perpendicular. Label their endpoints *E*, *F*, *G*, and *H*. Connect the endpoints. What kind of figure is *EFGH*?

3. What kind of figure would you have had in Exercise 2 if the diameters had been perpendicular? Is it a regular polygon?

4. A polygon is **inscribed** in a circle if each of its vertices is on the circle. The figure suggests a way of inscribing a regular hexagon in a circle.

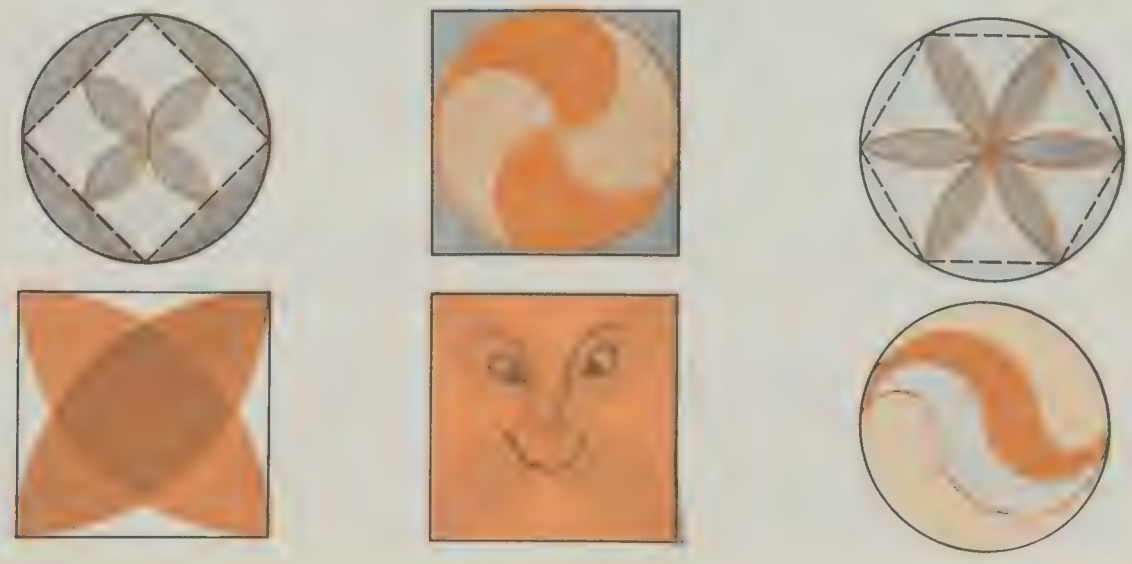
   Experiment with your ruler and compass to see if you can construct a regular inscribed hexagon.

5. A polygon is **circumscribed** about a circle if each of its sides touches the circle in just one point.



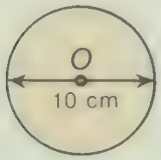
   Construct a square which circumscribes a circle.

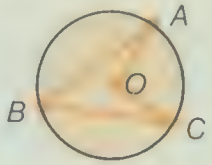
★6. By experimenting with your ruler and compass, you can invent many interesting designs. The figures below illustrate a few of them. Try to copy one of these and then invent some of your own.

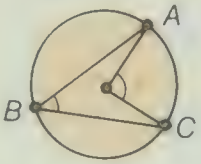


Investigating the Ideas

Draw a circle with a 10-cm diameter → Mark three points on the circle → Connect the points and the centre as shown → Label and measure angles 1 and 2 → m ∠ 1 = ▒▒ m ∠ 2 = ▒▒







Can you find a relationship between the measures of the two angles?

Try this experiment again using three other points on the circle.

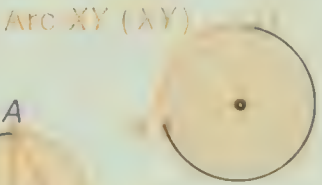
Discussing the Ideas

1. Study the definitions at the right.
   - **A** Which angles in the Investigation are central angles?
   - **B** Which angles in the Investigation are inscribed angles?

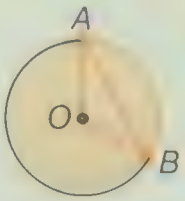
An angle with its vertex at the centre of the circle

An angle that contains three points of the circle and has its vertex on the circle

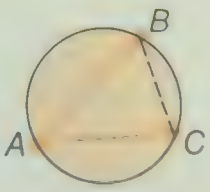
2. The portion of the circle shown in color is an **arc**. How would you define an arc of a circle?



3. Central angle $\angle AOB$, chord AB, and arc AB are shown in the figure at the right. Each of these figures is said to **subtend** the other. For example: $\overset{\frown}{AB}$ subtends $\angle AOB$ or $\angle AOB$ is subtended by $\overset{\frown}{AB}$.



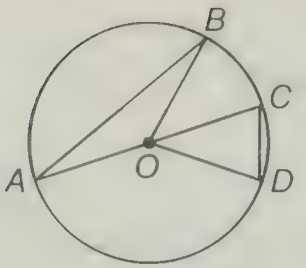
   - **A** What kind of angle is $\angle BAC$ with respect to the circle?
   - **B** Which arc subtends $\angle BAC$?
   - **C** Which chord subtends $\angle BAC$?



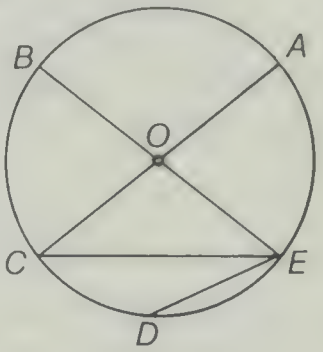
4. An arc that is exactly half of a circle is called a **semicircle**. What special chord of a circle does a semicircle subtend?

5. The Investigation suggests the following statement:
   If a central angle and an inscribed angle in a circle subtend the same arc, then the measure of the central angle is ? the measure of the inscribed angle.

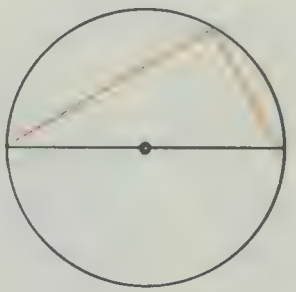
1. **A** Point O is the centre of the circle. Name four central angles of circle.
   B Name two inscribed angles.
   C Name two angles that subtend $\overset{\frown}{BC}$.
   D What angle subtends $\overline{CD}$?



2. **A** Name the central angle subtended by $\overset{\frown}{AE}$.
   B Name the central angle subtended by $\overset{\frown}{BC}$.
   C Name the inscribed angle subtended by $\overset{\frown}{BC}$.
   D Name the central angle subtended by $\overset{\frown}{CDE}$.
   E Name the central angle subtended by $\overset{\frown}{CE}$.
   F Name the arc subtended by $\angle BOA$.
   G Name the arc subtended by $\angle DEB$.
   H Name an inscribed angle that does not have one side as a diameter.

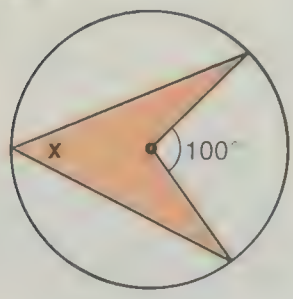


3. Draw a circle and its diameter.
   A Inscribe an angle that subtends the diameter. What is the measure of the angle?
   B Inscribe another angle subtending the same diameter. What is the measure of this angle?
   C If an inscribed angle subtends a diameter, what kind of an angle is it?

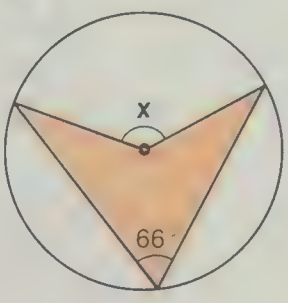


4. For each circle, give the degree measure for **x**.

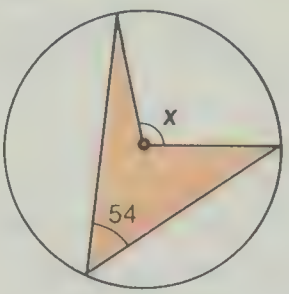
A



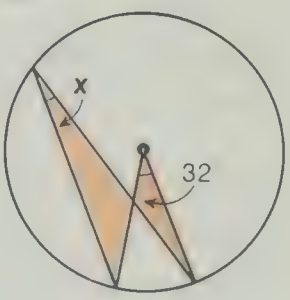
B



C



D



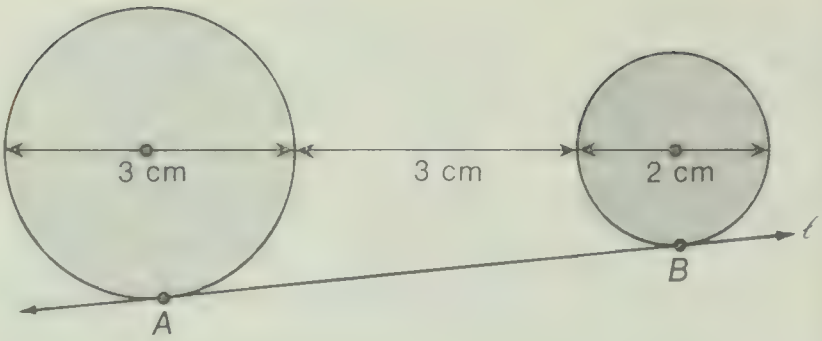
think

Can you unscramble each of these "words" to form a mathematical term that is in this lesson?

car glean recent dreamite dorch cleric undebts biscinder

Investigating the Ideas

Draw two circles like these about 3 centimetres apart and a line ℓ that touches both circles at one point on each circle



How many other lines like line ℓ can you draw so that each line touches both circles at one point on each circle?

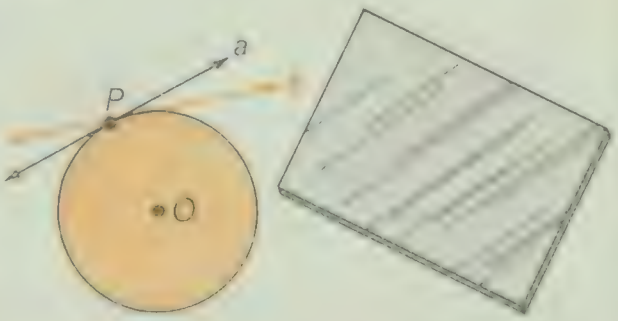
Discussing the Ideas

1. A line is **tangent** to a circle if it intersects the circle in just one point. In the figure, the line is tangent to the circle at point *A*. Point *A* is called the **point of tangency**.

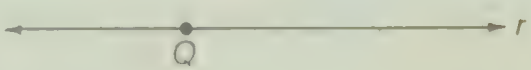
   - **A** How many lines are tangent to a circle at a point on the circle?
   - **B** What relationship do you think there is between a tangent to a circle and the radius of the circle at the point of tangency?

2. How many tangents to both circles did you find in the Investigation?

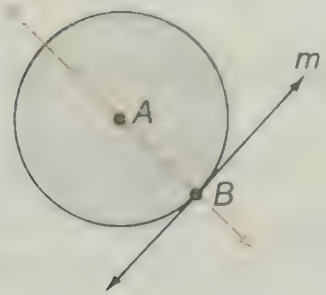
3. Lines *a* and *b* both intersect at point *P* on the circle. How can you use reflections and a plastic mirror to decide which line, if either, is a tangent to the circle?



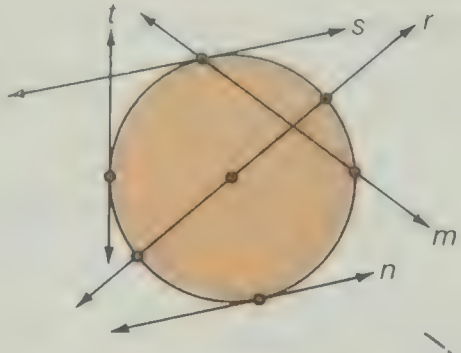
4. **A** How would you construct a circle tangent to a given line *r* at a given point *Q* on the line?
   - **B** Is there more than one circle tangent to the line at point *Q*?



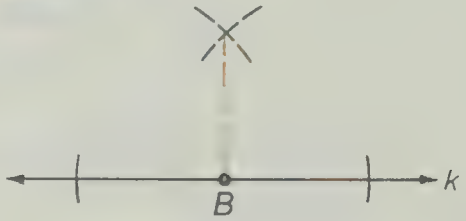
1. Line m is tangent to the circle with centre A.
   - **A** How are the two lines m and $\overleftrightarrow{AB}$ related?
   - **B** A line passing through the point of tangency and the centre of the circle is ? to the tangent line.



2. Which of the lines are tangent to the circle? All points of intersection are shown.

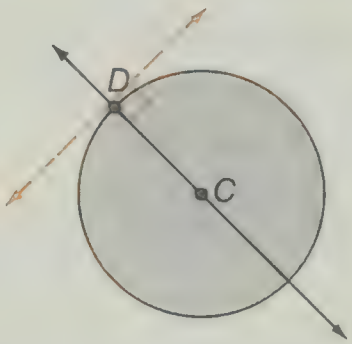


3. Draw a line k.
   Mark a point B on k.
   Construct the perpendicular to k at point B.
   Now draw three circles tangent to k at point B.

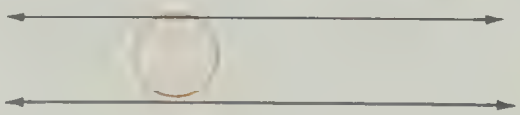


4. Draw a circle.
   Mark a point D on the circle.
   Label the centre C.
   Draw a line through C and D.
   Construct the perpendicular to $\overleftrightarrow{CD}$ at point D.

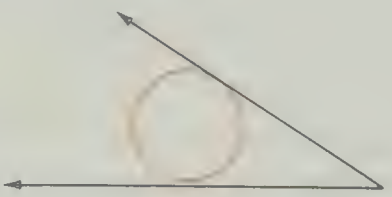
   Is the line perpendicular to $\overleftrightarrow{CD}$ tangent to the circle at point D?



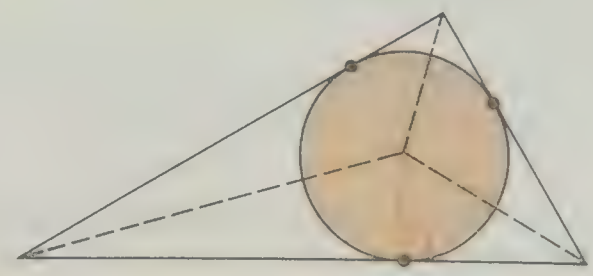
5. Draw two parallel lines.
   Construct a circle that is tangent to both lines.



6. Draw an acute angle.
   Construct a circle that is tangent to both sides of the angle.



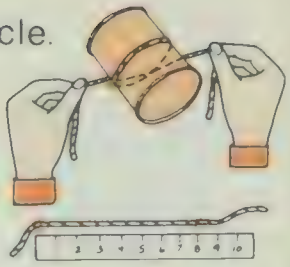
7. Draw a triangle.
   Bisect each angle.
   Then complete the construction for the circle inscribed in this triangle.



Investigating the Ideas

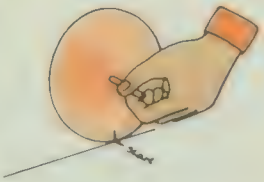
The distance around a circle is called the **circumference** of the circle.

Choose several different-sized circular objects. Then find a way to measure the circumference of each of them. The figures at the right suggest some ways to accomplish this.



A Find the length of a piece of string needed to go around this circular object.

B Roll a circular disk along a line for one full turn. Measure the distance along the line.



Can you find the ratio of the circumference to the diameter of each circle to the nearest hundredth?

Discussing the Ideas

1. Diane made this table to show her results for the Investigation.

   A Are her results similar to yours?

   B If you find the average of the ratios $C \div d$, what is this number to the nearest hundredth?

circle	*circumference (c)*	*diameter (d)*	$c \div d$
A	20.5 cm	6.5 cm	3.15
B	33.6 cm	10.7 cm	3.14
C	9 cm	2.8 cm	3.10

2. Since the actual measurement of physical objects is only an approximation, the ratio $\frac{c}{d}$ that you found was slightly different for each object measured. However, mathematicians have proved that the ratio $\frac{c}{d}$ is the same for all circles. This number, **the ratio of the circumference to the diameter**, is named by the Greek letter π (pronounced "pie"). The decimal for π is endless and nonrepeating. The first fifteen places are shown here: $\pi = 3.141592653589793\ldots$

   $\frac{C}{d} = \pi$

   $C = \pi \cdot d$

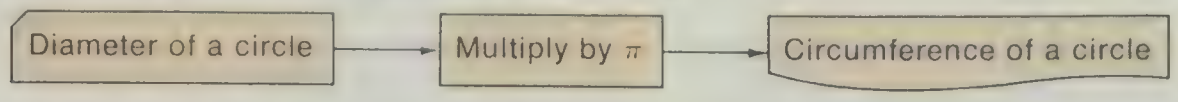
   A Find a decimal for $\frac{22}{7} = 3\frac{1}{7}$ to the nearest hundredth.

   B Is $3\frac{1}{7}$ a good approximation of π?

3. If you know the diameter of a circle, what would you do to compute the circumference of the circle?

4. If you knew the circumference of a circle, how would you find the diameter of the circle?

1. The flow chart can help you better understand the formula $C = \pi \cdot d$ for finding the circumference of a circle.



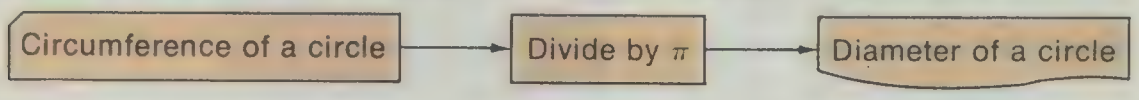
Use the fact $\pi \approx 3.14$ to find the approximate circumference of circles with the following diameters:

A 30 cm **B** 5 cm **C** 1.9 m **D** 16 mm **E** 10.8 cm

2. Use the fact $\pi \approx 3\frac{1}{7}$ to find the approximate circumference of each circle whose diameter is given below.

A 14 **B** $3\frac{1}{2}$ **C** $10\frac{1}{2}$ **D** $1\frac{3}{4}$ **E** $2\frac{1}{10}$ **F** $4\frac{1}{5}$

3. This flow chart is the inverse of the one shown in Exercise 1.



Use the inverse flow chart and the fact $\pi \approx 3.14$ to find the diameter (to the nearest tenth) of circles with the following circumferences:

A 15.7 cm **B** 38 mm **C** 8.4 m **D** 1.27 m **E** 0.19 m

4. Bill measured the distance around his head and found that it was 56 centimetres. If his head were perfectly round, what would be the diameter of his head?

5. A bicycle wheel has a diameter of about 66 centimetres.
   - **A** About how many metres will it travel in one revolution?
   - **B** About how many metres will it travel in ten revolutions?
   - **C** About how many metres will it travel in 120 revolutions?
   - **D** About how many revolutions will it make travelling one kilometre?

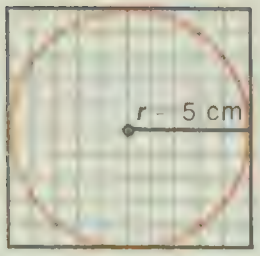
6. The radius of the circle is 10 millimetres.

   - **A** Use your centimetre ruler to find the perimeter of the small square.
   - **B** What is the perimeter of the large square?
   - **C** What is the average of the perimeters of the two squares?
   - **D** What is the circumference of the circle?
   - **E** What is the difference between your answers for parts **C** and **D**?

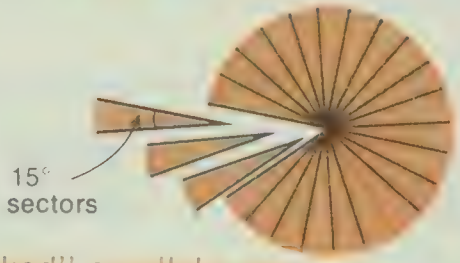
More practice, page S-27, Set 46

Investigating the Ideas

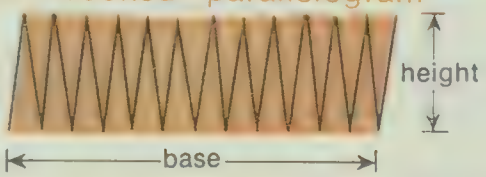
Draw a circle with a radius of five centimetres on graph paper. Then try each of these two methods for finding the area of the circular region.



1. Estimate the number of squares that the circular region covers.
2. Cut the circular region into sectors with angles of 15°. Tape or paste the sectors down as shown to form a "crooked" parallelogram. Measure the base and height, and compute the area of the parallelogram.



A "crooked" parallelogram



Which method do you think gives the best approximation of the area of a circular region?

Discussing the Ideas

1. A Is the height of the "crooked" parallelogram the same as the radius of the circle?

   B Explain why the base of the "crooked" parallelogram is $\frac{1}{2} \cdot C$, where C is the circumference of the circle.

2. Think about cutting a circular region into a very large number of sectors, then explain this statement:

   The area A of a circle with radius r and circumference C is given by the formula $A = \frac{1}{2} \cdot C \cdot r$.

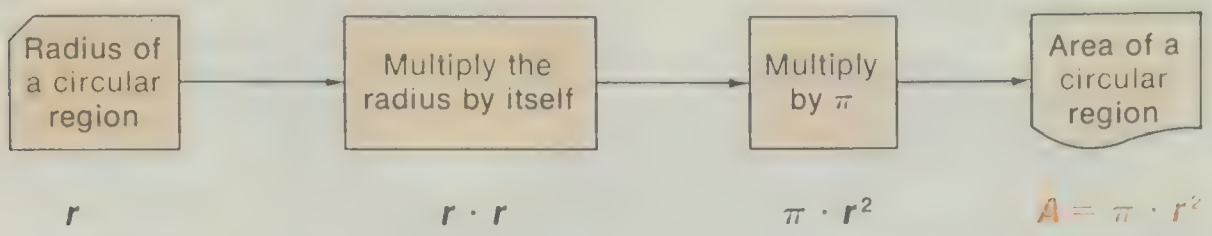
3. Since $C = \pi \cdot d = 2 \cdot \pi \cdot r$, then $\frac{1}{2} \cdot C = \pi \cdot r$.

   Therefore, $A = \frac{1}{2} \cdot (\pi \cdot r) \cdot r$ or

   Explain how to use this formula to find the area of a circular region whose radius is 2 centimetres.

4. Check your estimates of the area of the circular region in the Investigation using the area formula $A = \pi \cdot r^2$.

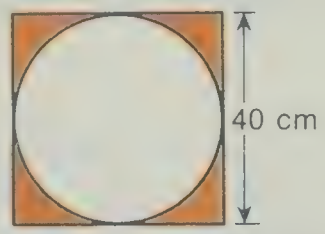
1. Use the flow chart to help you find the area of circular regions with the following radii (r) or diameters (d). Use $\pi \approx 3.14$.



A $r = 3$ cm **C** $d = 2$ m **E** $d = 10$ cm **G** $r = 24.1$ mm

B $r = 9$ m **D** $r = 60$ mm **F** $r = 2.3$ m **H** $d = 39$ cm

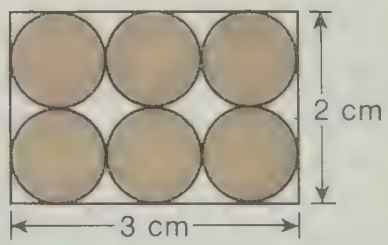
2. A circular region whose diameter is 40 centimetres is cut out of a square piece of tin 40 centimetres on a side. What is the area of the piece of tin which is left over? $\left(\text{Use } \pi \approx 3\frac{1}{7}.\right)$



3. A circular flower bed is surrounded by a walk two metres wide. The diameter of the flower bed is 20 metres. What is the surface area of the walk?

4. A cheese pizza with a diameter of 24 centimetres costs $1.55.
   A About how much per square centimetre does the pizza cost?
   B At this rate, about how much would a 30-centimetre round pizza cost?

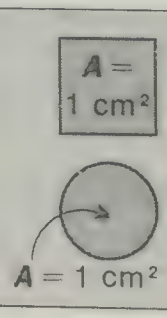
★ 5. The figure shows six circular disks, each of which has a diameter of one centimetre, placed on a rectangular region that is 3 by 2 centimetres. What percent of the rectangular region is covered by the six circular disks?



★ 6. If the rectangular region of Exercise 5 is covered by circular disks one millimetre in diameter and placed in a similar pattern, what percent is covered?

think

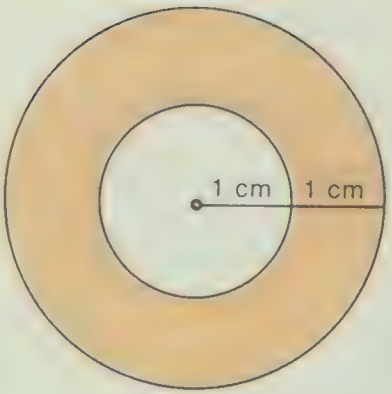
Which is larger, the perimeter of the square or the circumference of the circle?



More practice, page S-27, Set 47

Circle Problems

Refer to the figure at the right for Exercises 1 through 4.



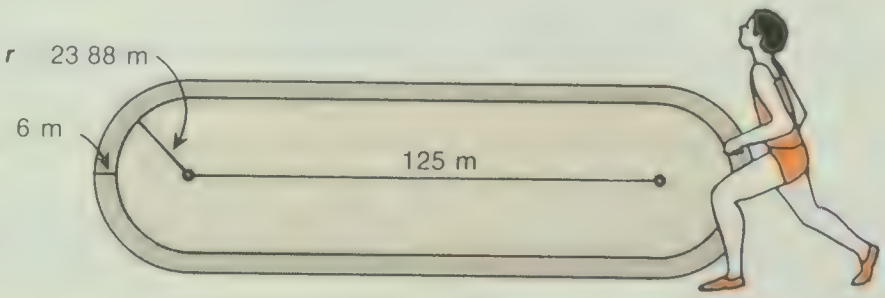
1. What is the circumference of the small circle?

2. What is the circumference of the large circle?

3. What is the area of the inner circular region?

4. What is the area of the shaded part?

The figure at the right represents an oval race track. The running track is 6 metres wide.



5. How far is it around the inside of the track? (Round your answer to the nearest metre.)

6. How much farther is it around the outside edge of the track than the inside?

7. During the 1968 Olympics in Mexico City, Lee Evans set a world record by running the distance of the track in 43.8 seconds. About how many metres per second (to the nearest tenth) did he run?

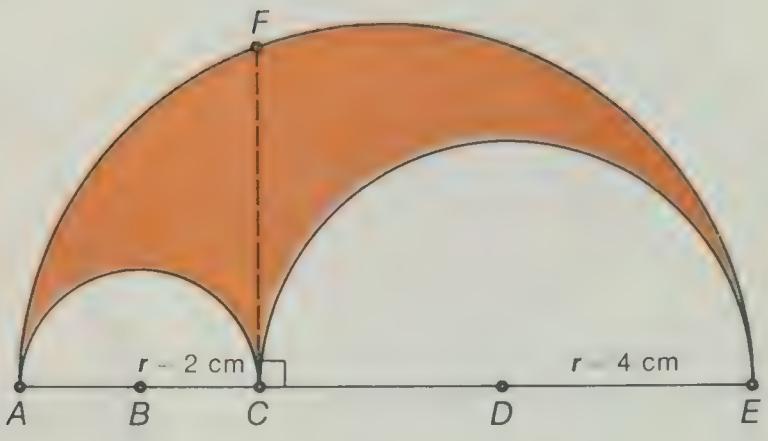
8. On July 30, 1970, Marilyn Neufville of Canada ran the distance of the track in 51.0 seconds to set a world record for women. What was her speed in metres per second (to the nearest tenth)?

9. What is the total area covered by the track and the infield?

10. How much greater is the area of the infield than the area of the running track?

THE SHOEMAKER'S KNIFE

The shaded region of the figure is called a "shoemaker's knife."



1. Make a full size drawing of the shoemaker's knife on your paper.

2. What is the distance around the large semicircle from A to E?

3. What is the distance from A to E going around the two smaller semicircles?

4. What is the area of the large semicircular region?

5. Find the area of the small semicircular region.

6. Find the area of the middle-sized semicircular region.

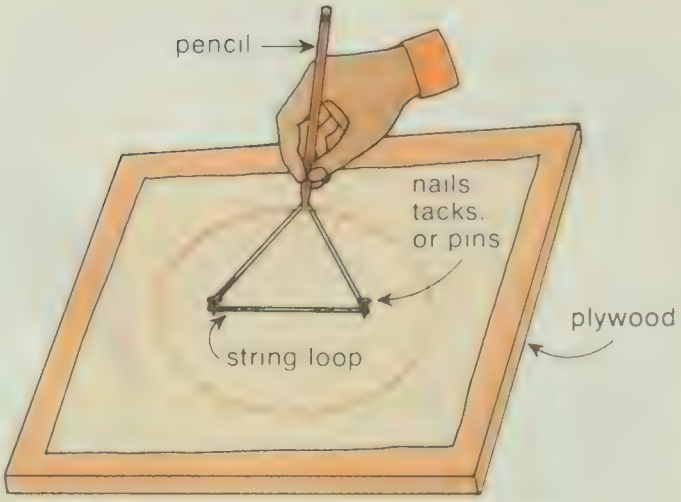
7. What is the area of the shoemaker's knife?

8. Draw $FC \perp AE$. Draw $\overline{AF}$ and $\overline{FE}$. Label the point where $\overline{AF}$ intersects the small semicircle as G and the point where $\overline{FE}$ intersects the middle-sized circle as H. Draw the quadrilateral $GFHC$. What kind of quadrilateral does it seem to be?

9. Using different-sized semicircles but keeping the length AE the same as before, draw other shoemaker's knives.
   - **A** Repeat Exercise 8. Do you get the same answer?
   - **B** If the two smaller semicircles are the same size, what is your answer?

Investigating the Ideas

A method is shown at the right for drawing an interesting geometric figure. It is drawn by keeping the loop stretched tightly against the two nails and drawing completely around the nails.



Can you make the drawing by studying and following the the illustration at the right?

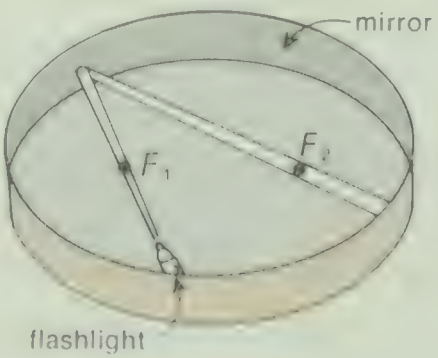
Discussing the Ideas

1. The figure that you drew for the Investigation is called an **ellipse**. What are some objects that are elliptical in shape?

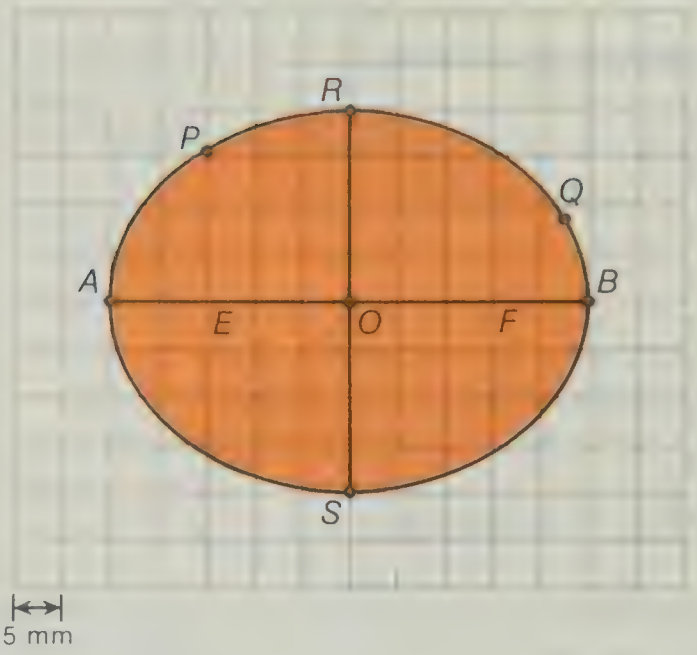
2. The two points marked by the nails (pins, tacks) are called **focal points** of the ellipse.
   - A What happens when you draw another ellipse using the same loop of string in the Investigation but move the two focal points closer together?
   - B What happens if you move the focal points farther apart?

3. Does an ellipse have any lines of symmetry? If so, describe them.

4. An ellipse has an interesting reflecting property. Suppose an elliptical region were bounded by a curved mirror. Then any ray of light from one focal point would be reflected through the other focal point. Show several of these rays on the ellipse that you drew for the Investigation.



Refer to the ellipse shown for Exercise 1 through 8 E and F are the focal points of the ellipse.



1. Find the length of the polygonal path from E to P to F.

2. Find the length of the polygonal path from E to Q to F.

3. Find the length of the polygonal path from E to S to F.

4. The **major axis** of the ellipse is $\overline{AB}$. What is the length of $\overline{AB}$?

5. How does the length of the major axis $\overline{AB}$ compare with the length of a polygonal path from one focal point to a point on the ellipse to the other focal point?

6. The **minor axis** of the ellipse is $\overline{RS}$. What is the length of $\overline{RS}$?

7. A Estimate the area of the ellipse. (Each small square is 25 mm^2.)

   B Compute the area using the following formula:

   $$\text{Area} = (\text{length } \overline{OB}) \cdot (\text{length } \overline{OS}) \cdot \pi$$

   Compare this answer with your estimate.

8. The **eccentricity** of an ellipse is defined to be the ratio:

   $$\frac{\text{length of } \overline{OF}}{\text{length of } \overline{OB}}$$

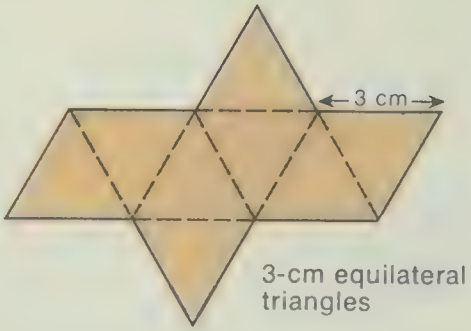
   What is the eccentricity of the ellipse above?

think

Arrange six sticks all of the same length (or toothpicks) in such a way as to form exactly four triangles having the same size and shape.

Investigating the Ideas

Here is the **net** or **pattern** for a space figure called an **octahedron**.



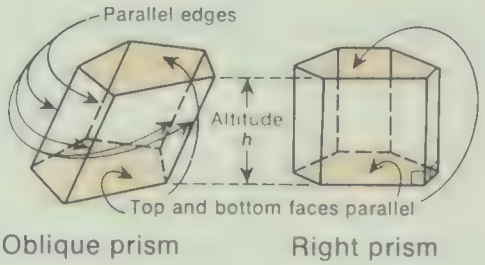
Can you make a copy of the net, cut it out, and fold it to form a model of an octahedron?

Discussing the Ideas

1. A **polyhedron** is the surface of a solid whose faces are polygonal regions. The word polyhedron means "many faces." Why do you think that the polyhedron in the Investigation is called an octahedron?

2. How many faces, edges, and vertices does an octahedron have?

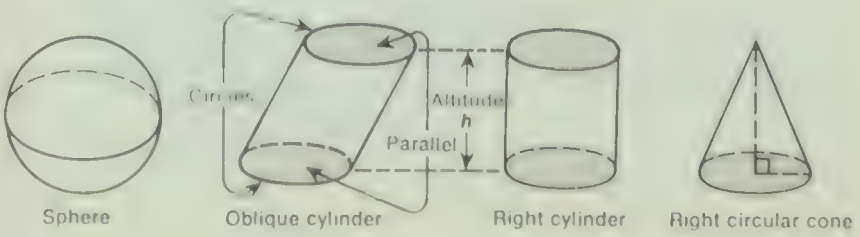
3. The illustration at the right shows an **oblique prism** and a **right prism**.
   - **A** In what way does an oblique prism differ from a right prism?
   - **B** What are some ways in which a prism differs from an octahedron?
   - **C** Explain why the oblique prism could be called a **pentagonal prism**.



4. The pyramid shown is a **pentagonal pyramid** because the base is a pentagon. Describe or draw the following:
   - **A** a square pyramid
   - **B** a triangular pyramid
   - **C** a hexagonal pyramid
   - **D** an octagonal pyramid

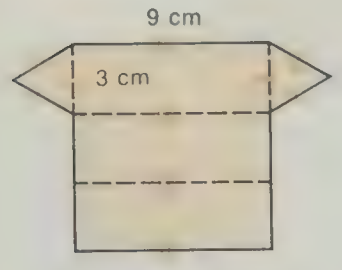
Pyramid

5. Some other space figures have some surfaces which are not flat like polyhedrons. Can you give some examples of physical objects like the four figures at the right?



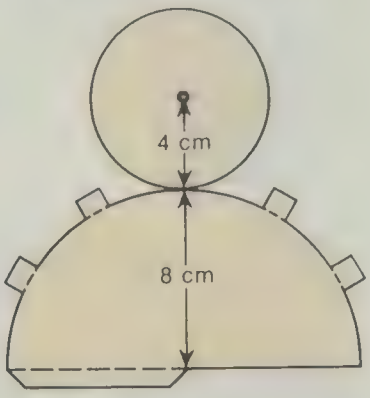
Make a net of each space figure below on light posterboard or tagboard. Fold along the dotted lines and tape the edges together. Congruent segments are marked by the same number of slashes.

1.



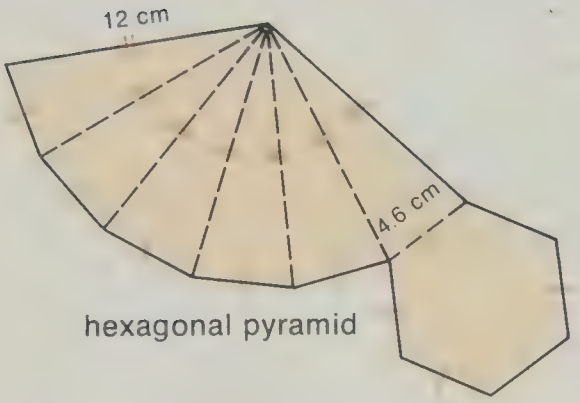
triangular prism

2.



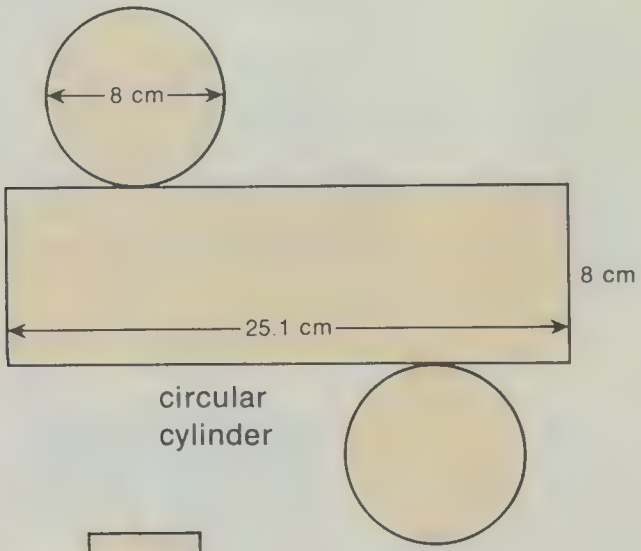
cone

3.



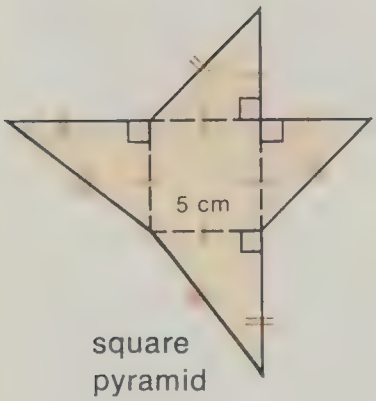
hexagonal pyramid

4.



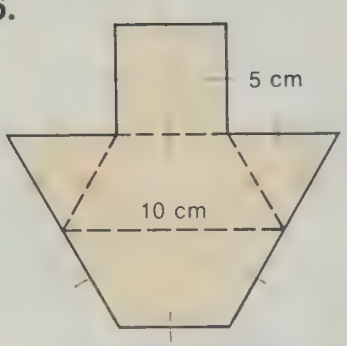
circular cylinder

5.



square pyramid

6.

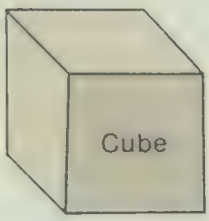


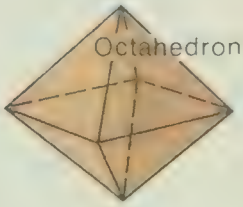
pentahedron

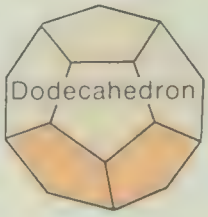
7. Show how three of the square pyramids (Exercise 5) can be placed together to form a cube.

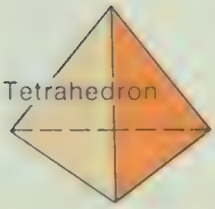
8. Place two of the pentahedrons (Exercise 6) together to form a **regular tetrahedron** (triangular pyramid).

Patterns for Regular Polyhedrons



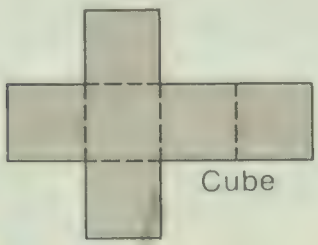


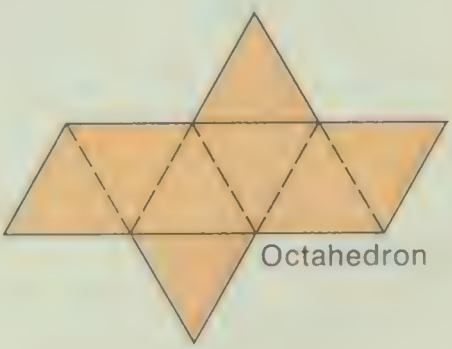


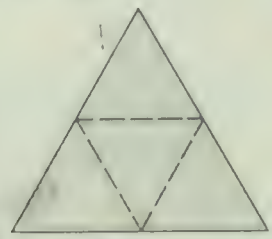


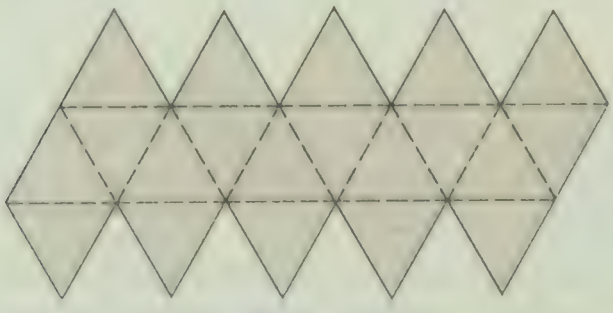
The five **regular polyhedrons** are shown above. The faces of each polyhedron are congruent regular polygonal regions. These polyhedrons are sometimes called the **Platonic solids** named after the Greek philosopher Plato. The ancient Egyptians were probably familiar with all of the regular polyhedrons except the dodecahedron.

A net for each of the regular polyhedrons is given on this page. Make a larger net for each polyhedron using light weight posterboard. Cut out the net, fold along the dotted lines, and tape the edges together to make a model of each of them.

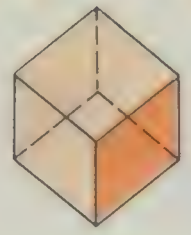


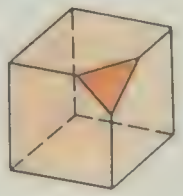


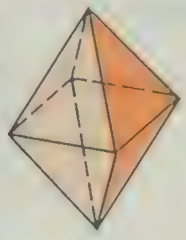


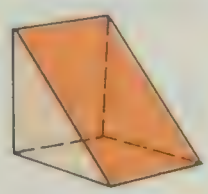


FACES, EDGES, AND VERTICES

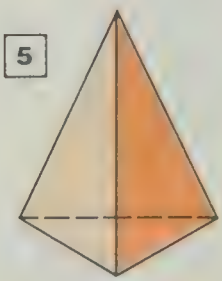
1 

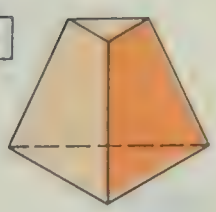
2 

3 

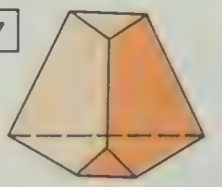
4 

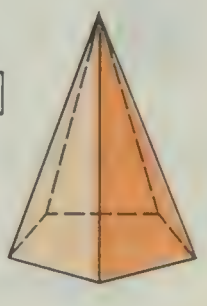
There is an interesting relationship between the number of faces, edges, and vertices of polyhedrons such as the ones pictured. You can discover this relationship called **Euler's Theorem** (named after Leonhard Euler, 1707–1783) by copying and completing the table.

5 

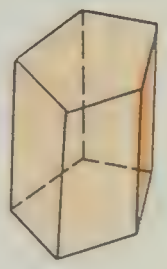
6 

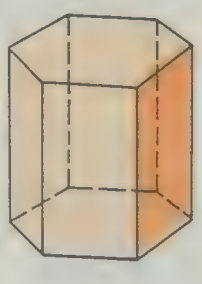
Polyhedron	V	F	E	$V + F$	$(V + F) - E$
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

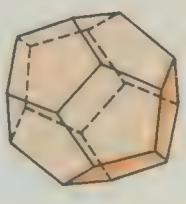
7 

8 

9

10 

11 

12 

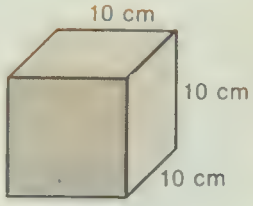
13 In general, for polyhedrons such as these, $(V + F) - E =$?.

Comparing Volumes of Space Figures

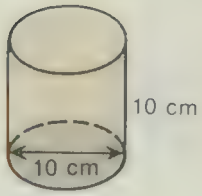
Investigating the Ideas

1. Which has the greater volume, the cube or the cylinder?

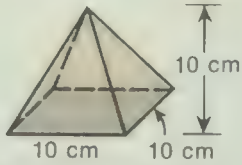
2. Is the volume of the pyramid more or less than the volume of a cone



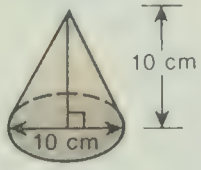
Cube



Cylinder



Square pyramid



Cone

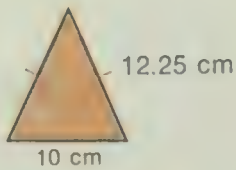
Can you guess how the volumes of the cube and the pyramid compare? Can you guess how the volumes of the cylinder and the cone compare?

Find a way to check your guesses.

Discussing the Ideas

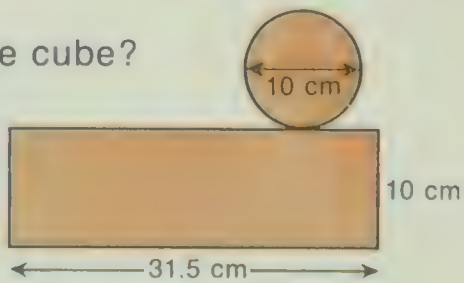
1. Make a cardboard model of the cube pictured above. Leave one face (the top) open.

2. Make a model of the pyramid without its base. It has the same height as the cube. You will need four isoceles triangles with the dimensions shown at the right.



3. Fill the pyramid with rice, beans, or sand. How many pyramidfuls does it take to fill the cube?

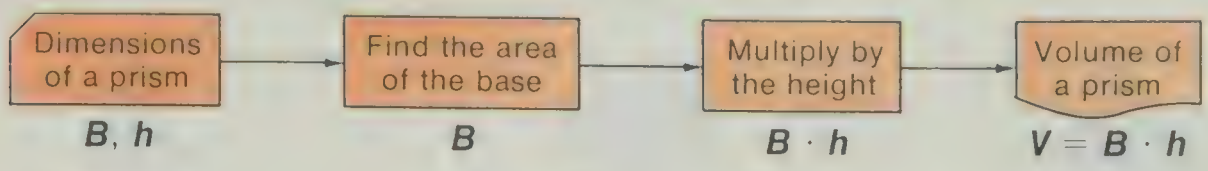
4. Make a model of the cylinder omitting one of its bases.

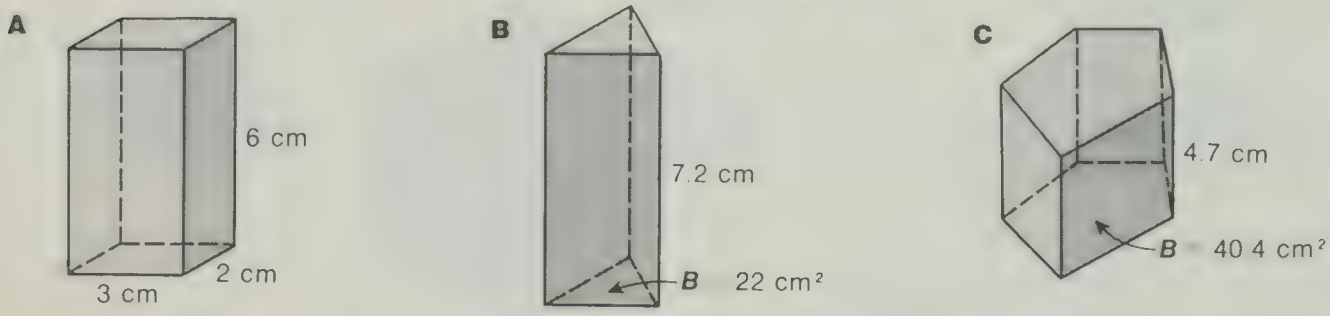


5. Make a model of the cone that has the same height as the cylinder. Omit the base. Use a pattern like this one.

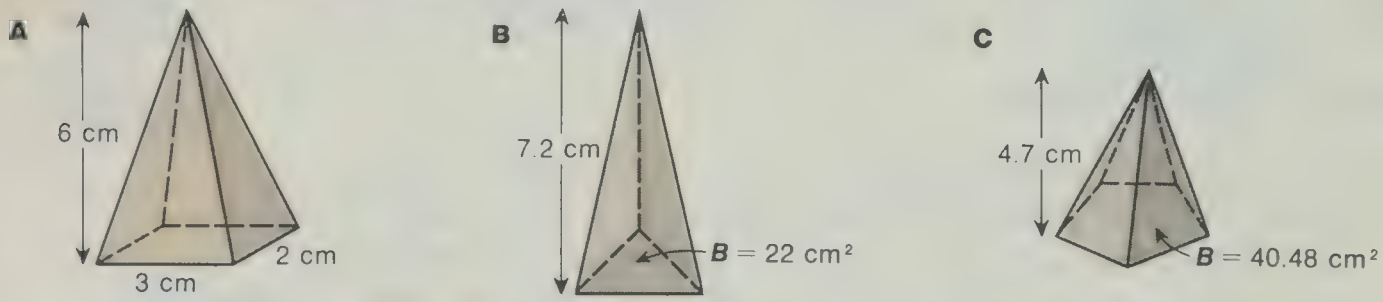
6. How many conefuls of rice, beans, or sand does it take to fill the cylinder?

1. Find the volume of each prism. The flow chart will help you.

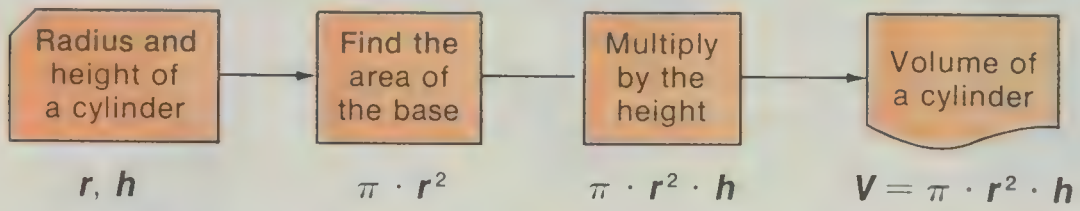


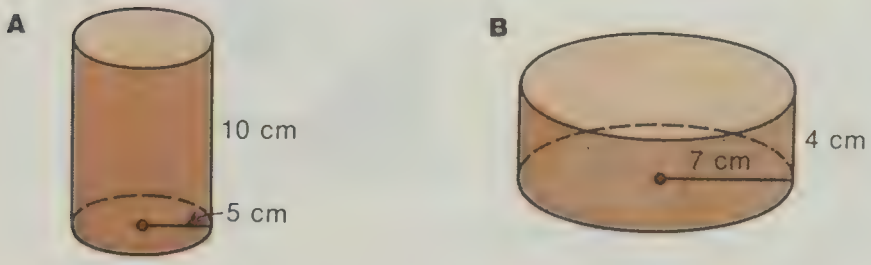


2. The volume of a pyramid is one-third the volume of a prism having the same base and height. Use the formula $V = \frac{1}{3} \cdot B \cdot h$ to find the volume of each pyramid.



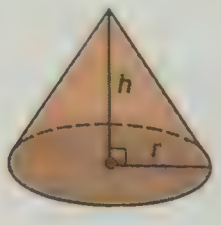
3. Find the volume of each cylinder with radius r and height h. The flow chart will help you. Use $\pi \approx 3.14$.





C $r = 0.8$ cm, $h = 3.2$ cm
D $r = 1.3$ m, $h = 3.8$ m
E $r = 4.2$ cm, $h = 4.2$ cm
F $r = 305$ mm, $h = 460$ mm

4. The volume of a cone is one-third the volume of a cylinder having the same base and height. Use the formula $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$ to find the volume.



A $r = 5$ cm, $h = 10$ cm
B $r = 1$ cm, $h = 1$ cm
C $r = 2.5$ cm, $h = 4$ cm
D $r = 0.4$ cm, $h = 2.1$ cm
E $r = 2$ m, $h = 3.9$ m
F $r = 18$ mm, $h = 42$ mm

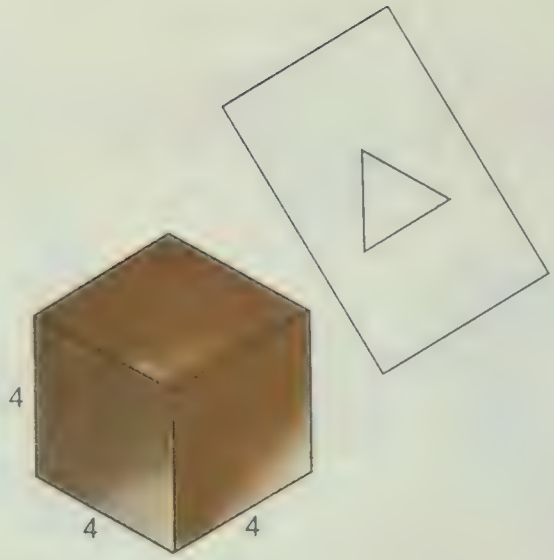
More practice, page S-28, Sets 48 and 49

Investigating the Ideas

You will need a small cube with 4-centimetre edges and some index cards to carry out this investigation.

Draw an equilateral triangle on the index card and cut out this region to form a triangular hole.

Find a way to place the card on the cube so that the triangular hole "fits" on the cube, without bending the card.

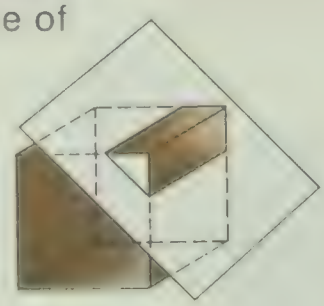


Can you find other differently-shaped holes that "fit" on the cube?

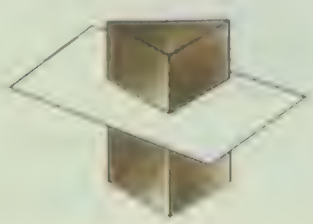
Discussing the Ideas

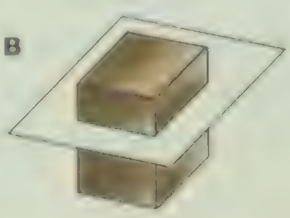
1. When the index card with the triangular hole is placed on the cube so that it just "fits," you have made a model of a plane intersecting a cube. This particular intersection of the plane and the cube is an equilateral triangle. The intersection of a cube and a plane is called a **cross section** of the cube.
   - A Show a cross section of the cube using a triangular hole which is not equilateral.
   - B Do you think that there are some triangles which cannot be cross sections of the cube? Show this with one of your index cards.

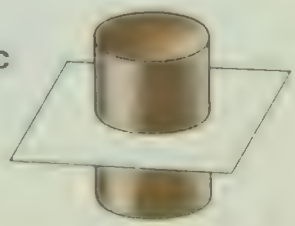
2. A What is the cross section of the cube illustrated in the drawing?
   - B What other shaped cross sections of a cube can you find? Show them.



3. Other space figures also have cross sections. What is the shape of the cross section in each figure below?

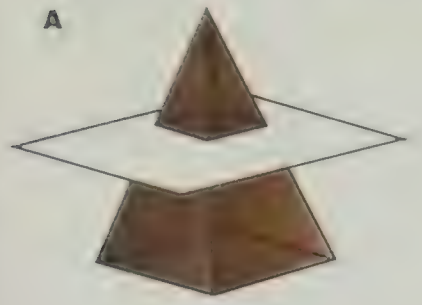
A 

B 

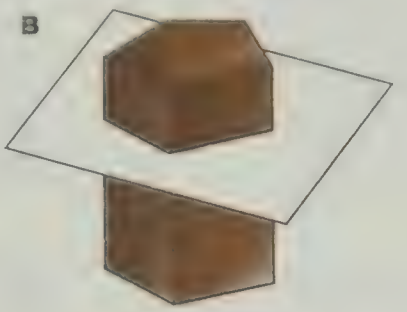
C 

1. Make a sketch of the cross section formed by the intersection of the plane and each space figure.

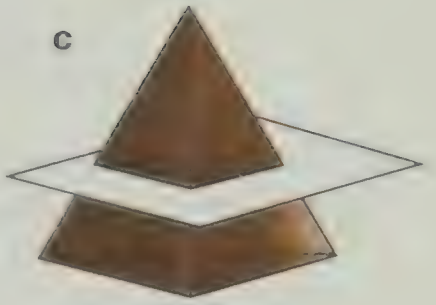
A



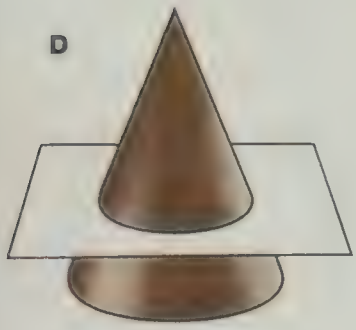
B



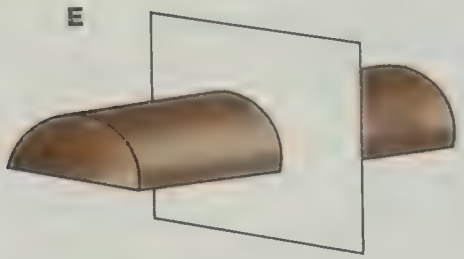
C



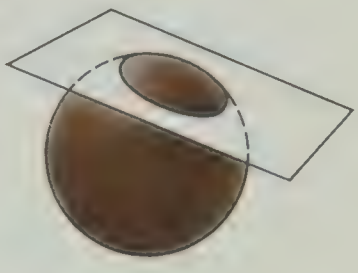
D



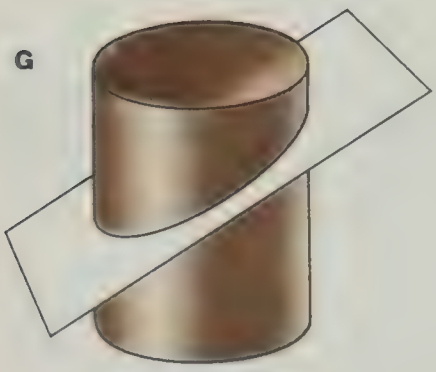
E



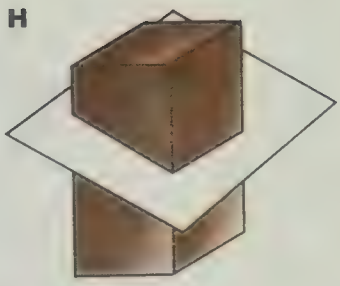
F



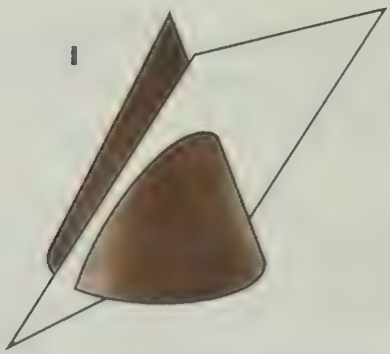
G



H

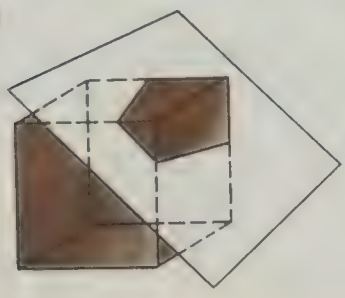


I

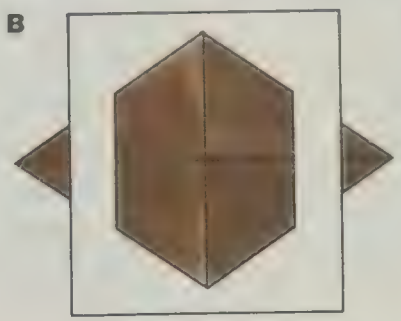


2. Draw the approximate shape of each cross section for each figure.

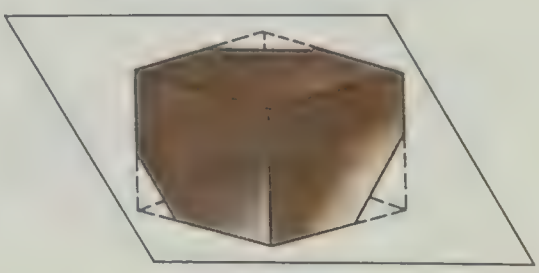
A



B



C



REVIEWING THE IDEAS

1. **A** Name the diameter.

B $\overline{AO}$ is a ? of the circle.

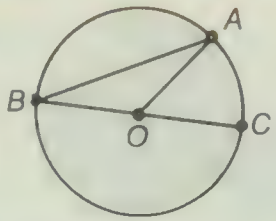
C Name a chord that is not a diameter.

D Name the arc subtended by $\angle AOC$.

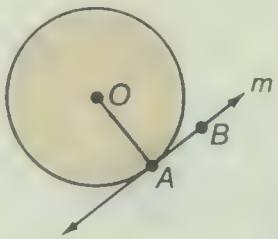
E Name the central angle subtended by $\overset{\frown}{AC}$.

F Name the inscribed angle subtended by $\overset{\frown}{AC}$.

G If $m\angle CBA = 25°$, then $m\angle COB =$? .



2. Line m is tangent to circle O at point A. What can you say about $\angle OAB$?



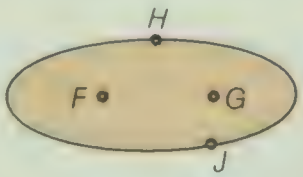
3. If a diameter of a circle is 10.6 cm, how long is a radius?

4. What is the decimal approximation for π to the nearest hundredth?

5. What is the circumference of a circle having a diameter of 21 cm? (Use $\pi \approx \frac{22}{7}$.)

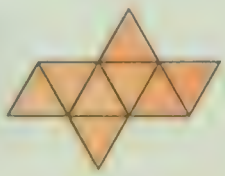
6. What is the area of a circular region having a radius of 3.5 cm? (Use $\pi \approx 3.14$.)

7. If F and G are the focal points of an ellipse, how do the lengths of the polygonal paths FHG and FJG compare?

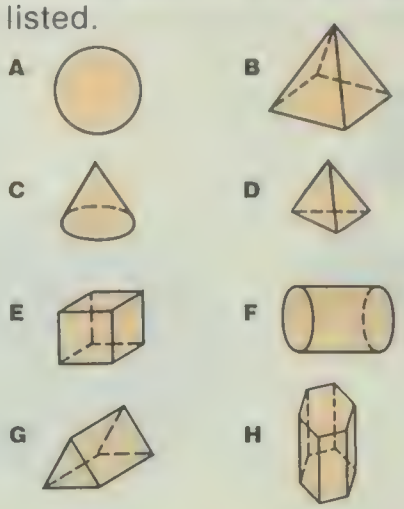


8. This is a net for a

- **A** tetrahedron
- **B** octahedron
- **C** icosahedron
- **D** dodecahedron



9. Identify each figure using the words listed.



circular cylinder
cone
cube
hexagonal prism
sphere
square pyramid
tetrahedron
triangular prism

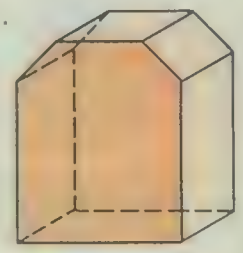
10. Look at the polyhedron.

A How many vertices?

B How many faces?

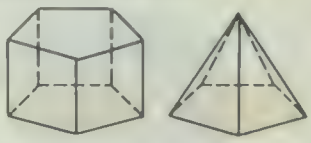
C How many edges?

D What is the $(V + F) - E$?



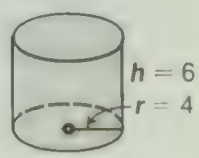
11. The base of a prism has an area of 42 cm^2 and a height of 9 cm.

A What is the volume of the prism?

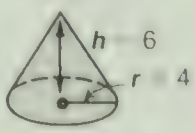


B What is the volume of a pyramid whose base area and height are the same as the prism?

12. **A** What is the volume of the cylinder?

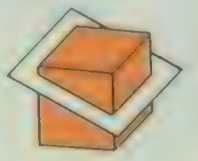


B What is the volume of the cone?



13. The cross section formed by the intersection of the plane and the cube is a

- **A** square
- **B** rectangle
- **C** parallelogram



TEST YOURSELF

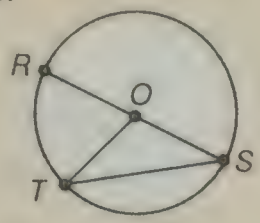
1. Give the name for each part of the circle listed.

   A $\overline{RS}$

   B $\overline{ST}$

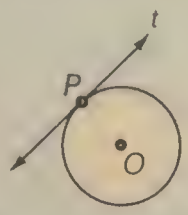
   C $\overline{OS}$

   D $\overset{\frown}{RT}$



2. Using the figure in Exercise 1, if $m\angle ROT = 80°$, what is $m\angle RST$?

3. Line t intersects the circle only at point P. Line t is a ? of the circle.



4. Find the circumference of a circle whose diameter is 6.2 cm. (Use $\pi \approx 3.14$.)

5. Find the area of a circle whose radius is 1.6 m. (Use $\pi \approx 3.14$.)

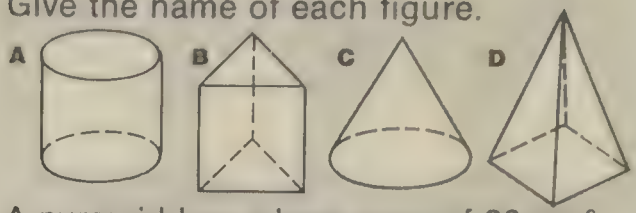
6. This is a net for a

   A tetrahedron

   B hexahedron

   C octahedron

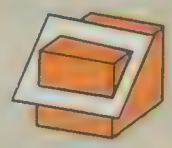
7. Give the name of each figure.



8. A pyramid has a base area of 36 cm^2 and a height of 10 cm. What is the volume of the pyramid?

9. A cone has a volume of 120 cm^3. If a cylinder has the same base and height as the cone, what is the volume of the cylinder?

10. What is the shape of the cross section shown by this figure?



RESEARCH PROJECTS

A Find the sizes and prices of pizzas. Compare the area of the pizzas and decide which size is the best buy for the money.

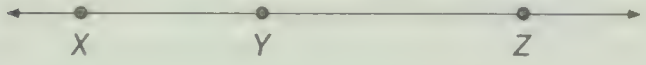
B A cube can be used to tessellate space. That is, a stack of cubes can be arranged so that no gaps occur in the stack. What are some other polyhedra that could be used to tessellate space? (See *Geometry in the Classroom: New Concepts and Methods* by H. A. Elliot, James R. McLean, and Janet M. Jorden; Holt, Rinehart and Winston of Canada Limited, 1968.)

C Find as many different sided polygons that can be cross sections of a cube. Make models to show these cross sections. You should be able to find triangles, squares, rectangles, trapezoids, a pentagon, and a hexagon.

D There are 13 semiregular polyhedra called the **Archimedian solids**. Find some patterns for these solids and make models of some of them. (See *Polyhedra Models for the Classroom* by Magnus J. Winniger; Washington, D.C.: National Council of Teachers of Mathematics, 1966.)

CUMULATIVE REVIEW

1. Name three segments using the points given.



2. Using the drawing in Exercise 1, Which ray does not contain point Z?

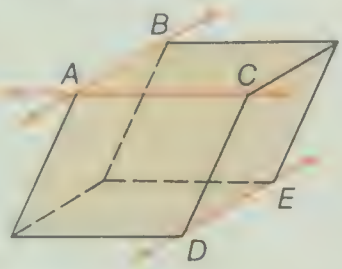
[A] $\overrightarrow{XY}$ [B] $\overrightarrow{YX}$ [C] $\overrightarrow{ZX}$

3. Complete each sentence with the words **intersecting**, **parallel**, or **skew**.

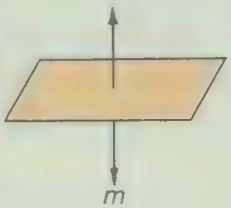
A AB and DE are ? lines.

B $\overline{AB}$ and $\overline{AC}$ are ? lines.

C $\overline{AC}$ and $\overline{DE}$ are ? lines.



4. Which phrase best describes the picture?



[A] a plane parallel to line m

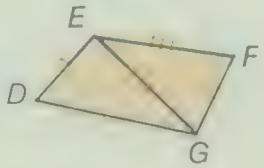
[B] a plane containing line m

[C] a line intersecting a plane

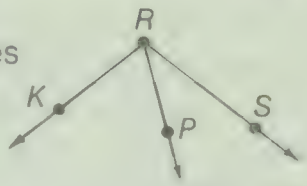
5. Complete the sentence:

All the points in a plane that are on one side of a line in the plane is called a ? .

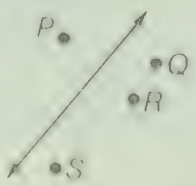
6. Name two pairs of congruent segments in the figure.



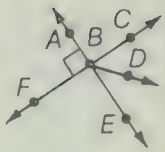
7. Name three angles shown by the drawing.



8. Which point Q, R, or S is the reflection image of point P in the line?



9. Use the words listed below to complete each sentence about the figure.



A $\angle DBE$ is a(n) ? angle.

B $\angle ABC$ and $\angle FBE$ are ? angles.

C $\angle ABE$ is a(n) ? angle.

D $\angle FBD$ is a(n) ? angle.

E $\angle CBD$ and $\angle DBF$ are ? angles.

F $\overleftrightarrow{AE}$ is ? to $\overleftrightarrow{FC}$.

G $\angle ABC$ is a(n) ? angle.

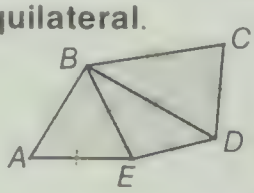
acute right perpendicular obtuse straight vertical supplementary

10. Tell whether each triangle is **scalene**, **isoceles**, or **equilateral**.

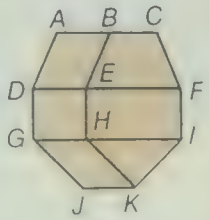
A $\triangle ABE$

B $\triangle BCD$

C $\triangle BDE$



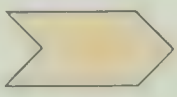
11. Name the types of polygons using the letters shown in the figure.



A $ABED$ C $BCFE$ E $BCFIHE$

B $DEHG$ D $HEFI$ F HIK

12. Is this a convex polygon?

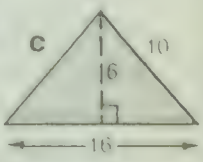


13. If a segment is 38 cm long, then it is ? mm long or ? m long.

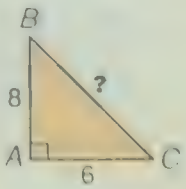
14. Find the perimeter and area of each figure.

A

C

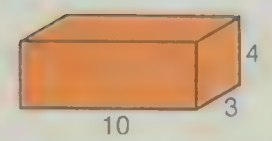


15. Using the Pythagorean Theorem, find the length of BC.



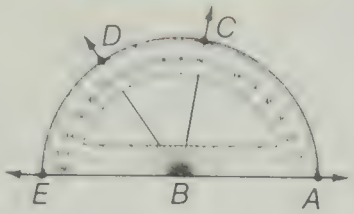
16. A Find the volume of the figure.

B Find the surface area.



17. A $m\angle ABD$ is ? .

B $m\angle$? is 45°.



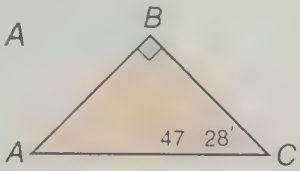
18. A Find the sum.

$$\begin{array}{r} 124°58'19'' \\ +\ 55°23'26'' \\ \hline \end{array}$$

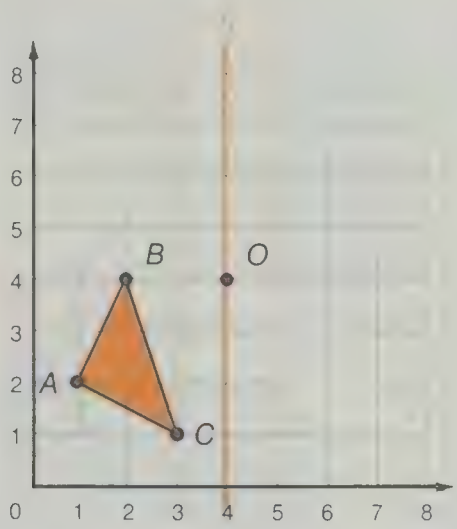
B Find the difference

$$\begin{array}{r} 83°46'37'' \\ -\ 45°18'49'' \\ \hline \end{array}$$

19. What is $m\angle A$ of $\triangle ABC$?



Refer to the grid for Exercises 20 through 23.



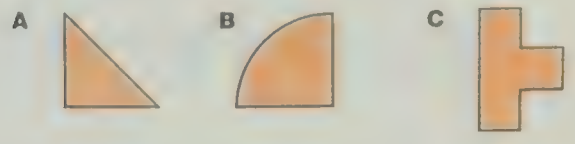
20. Give the co-ordinates of the vertices of $\triangle ABC$.

21. Give the co-ordinates of the images of A, B, and C after a translation of "right 4, up 3."

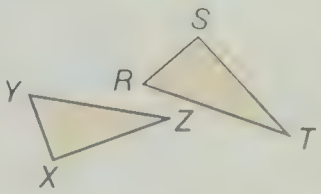
22. Give the co-ordinates of the images of A, B, and C after a $\frac{1}{2}$ turn with point O as the centre of rotation.

23. Give the reflection image of $\triangle ABC$ in line n.

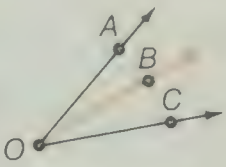
24. Which region(s) could tessellate a plane?



25. If $\triangle RST \cong \triangle XYZ$, name the pairs of angles and sides that are congruent to each other.



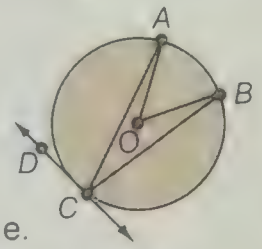
26. If $\overline{OB}$ is the bisector of $\angle AOC$, how do $\angle AOB$ and $\angle BOC$ compare?



27. A Name an inscribed angle in the figure.

B Name a central angle.

C If $m\angle AOB = 40°$, then $m\angle ACB =$? .



28. In the figure in Exercise 27, $\overline{CD}$ is a ? to the circle.

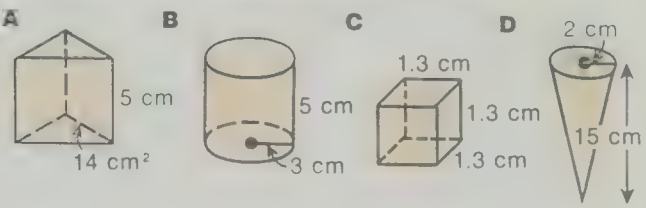
29. How many faces, edges, and vertices does this figure have?

30. A small table has a circular top with a diameter of 60 cm.

A What is the circumference of the table top?

B What is the area of the table top?

31. Find the volume of each figure.



32. Which space figure cannot have a circular cross section?

A sphere B pyramid C cone

NICKEL PROBLEMS

1. Which name best describes the shape of a nickel?
   - **A** circle
   - **B** sphere
   - **C** circular region
   - **D** solid cylinder

2. What is the diameter of a nickel to the nearest millimetre?

3. What is the thickness of a nickel to the nearest millimetre?

4. About how many nickels would be in a stack in order to be as tall as one standing on edge?

5. You can find the volume of a nickel by finding the product of the area of its base and its height. That is,

   Volume = area of base · height

   $V = \pi \cdot r^2 \cdot h$

   A What is the area of the base?
   B What is the height?
   C What is the volume?

6. What is the volume, to the nearest cubic centimetre, of a dollar's worth of nickels?

7. A nickel is composed of 75% copper and 25% nickel. About how many cubic centimetres of copper are in a dollar's worth of nickels (see Exercise 6)?

8. **A** If the smallest possible square were drawn around four adjacent (touching) nickels, what would be the area of the square in square centimetres?
   B About what part of the square would be covered by the nickels?

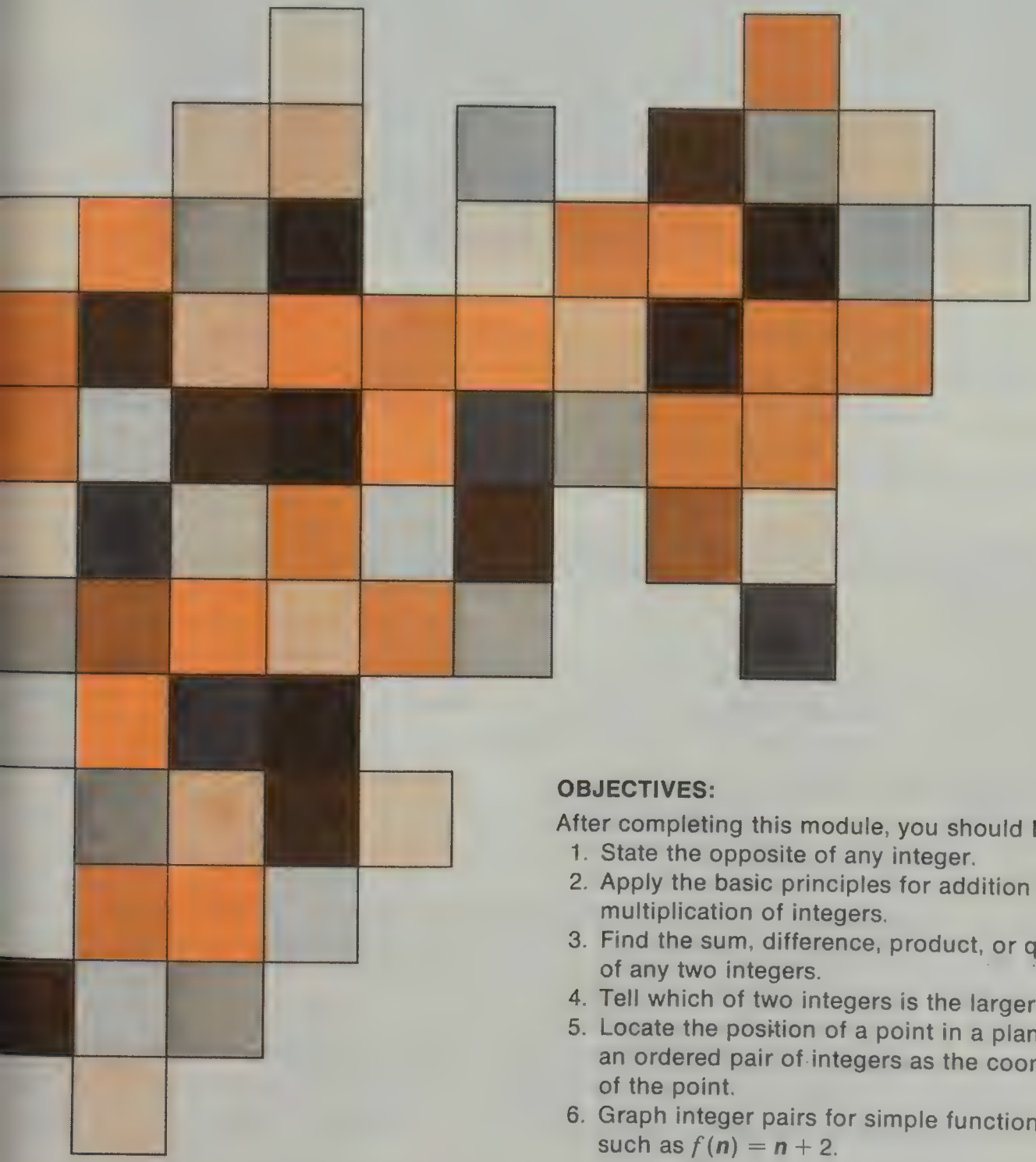
MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

1. **A** diameter; **B** chord; **C** radius; **D** arc 2. 40° 3. tangent 4. 19.468 cm 5. 8.0384 m² 6. A 7. **A** circular cylinder; **B** triangular prism; **C** cone; **D** square pyramid 8. 120 cm³ 9. 360 cm³ 10. rectangle

UNIT D: *Integers, Probability, and Statistics*

MODULE 1: *Integers*



OBJECTIVES:

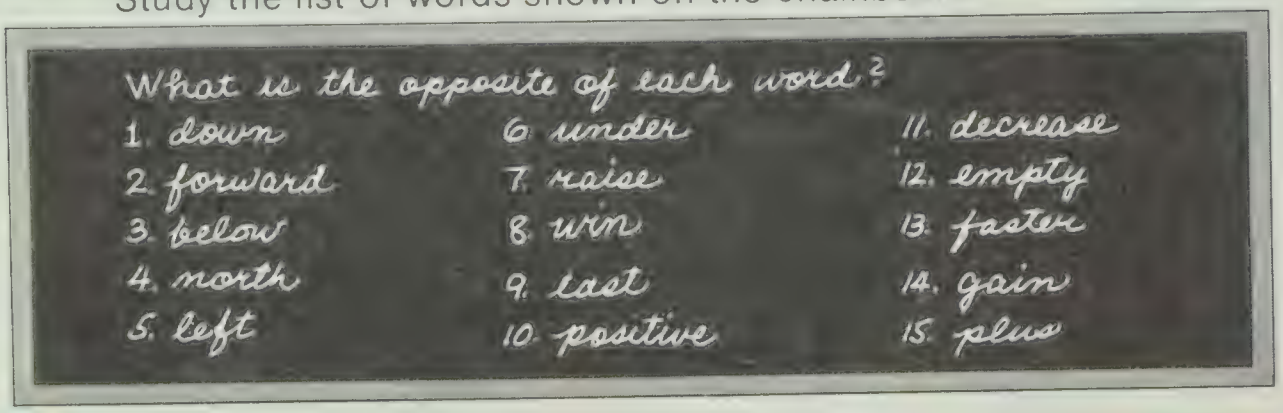
After completing this module, you should be able to:

1. State the opposite of any integer.
2. Apply the basic principles for addition and multiplication of integers.
3. Find the sum, difference, product, or quotient of any two integers.
4. Tell which of two integers is the larger.
5. Locate the position of a point in a plane given an ordered pair of integers as the coordinates of the point.
6. Graph integer pairs for simple function rules such as $f(n) = n + 2$.

What are Opposites?

Investigating the Ideas

Study the list of words shown on the chalkboard.

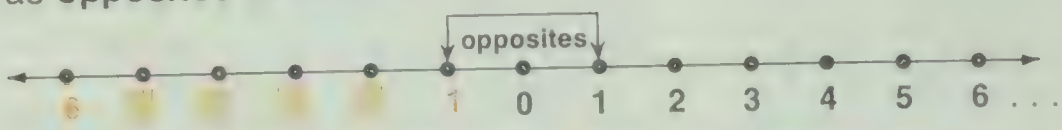


Can you write a word that has an "opposite" meaning for each of the words above?

Discussing the Ideas

1. Think of two words that are opposite in meaning. Tell a classmate one of the words and then see if he can name its opposite.

2. Just as some words are opposites, so are some numbers opposites of each other. We can think of the numbers for the points on opposite sides of zero on the number line as **opposites** of each other.



For the point **opposite 1**, we write [redacted] and call it [redacted]
We say that 1 is the opposite of [redacted] or [redacted] is the opposite of 1.
Give the other missing numbers on the number line.

3. Each nonzero whole number (positive integer) has an opposite. The opposite of a positive integer is a negative integer. Positive and negative integers, together with zero, form a set of numbers called the **integers**.

The set of integers = {0, 1, [redacted], 2, [redacted], 3, [redacted], 4, [redacted], 5, . . .}

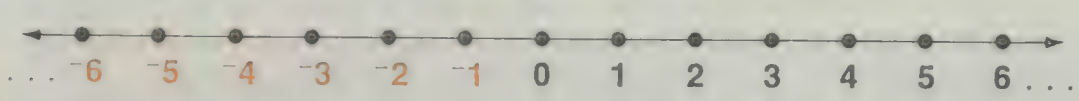
Give the opposite of each integer.

A 7 **B** 21 **C** $^{-}2$ **D** $^{-}19$ **E** 47 **F** 0

1. Give the opposite of each integer.

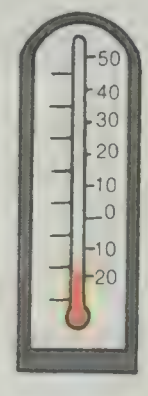
A 9	D $^{-}4$	G 25	J 586	M $^{-}5936$
B 7	E $^{-}9$	H 59	K $^{-}1079$	N 100
C 5	F $^{-}11$	I $^{-}86$	L 1 687 943	O $^{-}863\,650$

2. Think about the integers on the number line in order to give the integers described in each part.

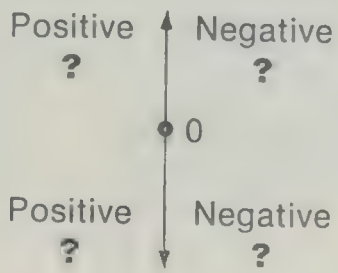


A The opposite of 5
B The opposite of $^{-}3$.
C The integer 2 units left of zero
D The integer 4 units right of zero
E The integer 3 units left of $^{-}1$
F The integer 3 units right of $^{-}5$
G The integer 3 units right of $^{-}3$
H The integer 3 units left of 3
I The integer 10 units right of $^{-}5$
J The integer 6 units left of $^{-}2$
K The opposite of the opposite of 4
L The opposite of zero

3. A If 17° represents "17 degrees **above** zero," what integer represents "17 degrees **below** zero?"
B If $^{-}8$ represents a loss of 8 points in a game, what integer represents a win of 8 points?
C If $^{-}15$ represents a discharge of 15 amperes of electrical current, what integer represents a charge of 15 amperes?
D If 6 denotes 6 seconds after blastoff for a space craft, what integer denotes 6 seconds before blastoff?



4. You may wonder why the positive numbers "go to the right" and the negative numbers "go to the left." It is simply a matter of choice.
A Draw a horizontal picture of the number line. Choose a zero point. Choose the positive number direction to the left. Mark points 1 cm apart. Label the number line.
B Draw a vertical picture of the number line. Choose a zero point. Choose a positive number direction either up or down and mark points 1 cm apart. Label the number line.

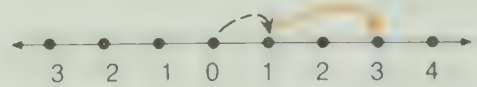


Discussing the Ideas

The table shows the basic principles that we accept for addition of integers.

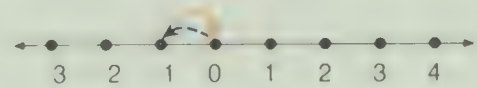
Opposites principle	Zero principle
The sum of each integer and its opposite is zero.	The sum of any integer and zero is that integer.
Commutative principle	**Associative principle**
When finding the sum of any two integers, you can change the order of the addends and the sum is the same For integers a and b, $a + b = b + a$	When finding the sum of any three integers, you can change the grouping of the addends and the sum is the same. For integers a, b, and c, $(a + b) + c = a + (b + c)$
Since you can change both the **order** and the **grouping** of the addends, you can rearrange addends in any way that is convenient.	

1. Figure A reviews whole number addition on the number line. Explain each jump and give the sum.

A 

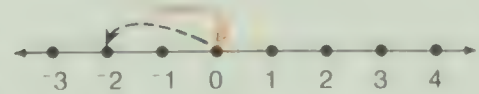
$1 + 2 = n$

2. Figure B suggests a way to think about the sum $^-1 + 1$. Explain each jump and give the sum.

B 

$^-1 + 1 = n$

3. Explain each jump in Figure C and give the sum.

C 

$^-2 + 2 = n$

4. Which basic principles do Figures B and C illustrate?

5. What is the sum in each part? Which basic principle must you use?
   A $2 + 0$ **B** $^-5 + 0$ **C** $0 + ^-7$ **D** $0 + 8$

6. Which basic principles do Examples A, B, and C illustrate?

EXAMPLES: **A** $^-4 + ^-3 = ^-3 + ^-4$
B $5 + ^-5 = 0$
C $(^-2 + 5) + ^-5 = ^-2 + (5 + ^-5)$

1. Find the sums.

   A $^{-}5 + 5$ C $^{-}23 + 23$ E $^{-}3 + 0$ G $^{-}18 + 0$

   B $^{-}8 + 8$ D $^{-}967 + 967$ F $^{-}9 + 0$ H $^{-}76 + 0$

2. Find the integer for ***a***. Then find the integer for ***b***. The opposites and zero principles will help you complete these exercises.

   A $^{-}1 + 1 = a \rightarrow 1 + {}^{-}1 = b$

   B $^{-}7 + 7 = a \rightarrow 7 + {}^{-}7 = b$

   C $^{-}56 + 56 = a \rightarrow 56 + {}^{-}56 = b$

   D $^{-}8 + 0 = a \rightarrow 0 + {}^{-}8 = b$

   E $^{-}72 + 0 = a \rightarrow 0 + {}^{-}72 = b$

   F $^{-}963 + 0 = a \rightarrow 0 + {}^{-}963 = b$

3. Find the missing integer.

   A ▒▒ is the integer that adds to 8 to give the sum 0.

   B ▒▒ is the integer that adds to $^{-}17$ to give the sum 0.

   C ▒▒ is the integer that adds to 58 to give the sum 0.

   D ▒▒ is the integer that adds to $^{-}867$ to give the sum 0.

   E ▒▒ is the integer that adds to 1083 to give the sum 0.

4. Find the integer for ***a***. Then find the integer for ***b***.

   A $2 + (3 + {}^{-}3) = a \rightarrow (2 + 3) + {}^{-}3 = b$

   B $6 + (4 + {}^{-}4) = a \rightarrow (6 + 4) + {}^{-}4 = b$

   C $^{-}3 + (^{-}5 + 5) = a \rightarrow (^{-}3 + {}^{-}5) + 5 = b$

   D $^{-}26 + (^{-}8 + 8) = a \rightarrow (^{-}26 + {}^{-}8) + 8 = b$

5. Solve the equations. The variables represent integers.

   A $^{-}4 + x = {}^{-}4$ D $t + {}^{-}12 = 0$ G $n + (^{-}5 + {}^{-}8) = (6 + {}^{-}5) + {}^{-}8$

   B $7 + n = 0$ E $9 + {}^{-}9 = n + 9$ H $^{-}13 + (t + 4) = (^{-}13 + {}^{-}8) + 4$

   C $r + 0 = {}^{-}66$ F $^{-}56 + k = 0 + {}^{-}56$ I $(^{-}72 + 4) + {}^{-}45 = {}^{-}72 + (4 + s)$

6. Find the integer for ***a***. Then find the integer for ***b***. Remember that the commutative and associative principles together suggest that you can rearrange addends in any way you choose.

   A Since $(4 + {}^{-}4) + (5 + {}^{-}5) = a$, then $(4 + 5) + (^{-}4 + {}^{-}5) = b$.

   B Since $(6 + {}^{-}6) + {}^{-}8 = a$, then $(6 + {}^{-}8) + {}^{-}6 = b$.

   C Since $17 + (9 + {}^{-}9) = a$, then $(^{-}9 + 17) + 9 = b$.

7. Solve the equations.

   A $(6 + {}^{-}7) + {}^{-}6 = n$ C $(n + {}^{-}8) + {}^{-}4 = {}^{-}8$ E $(n + {}^{-}2) + {}^{-}4 = 0$

   B $3 + (^{-}7 + 4) = n$ D $(^{-}7 + n) + 7 = 15$ F $5 + (^{-}3 + {}^{-}2) = n$

Investigating the Ideas

You can find the sum of two integers by using physical examples and logical reasoning.

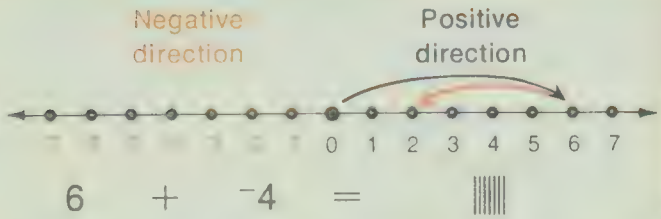
A Pay a bill for $3.00. Receive a cheque for $8.00. Richer or poorer? How much richer or poorer?

$^{-}3 + 8 =$ ▮

B From 0°, the temperature fell 4°. Then it rose 7°. Above or below 0°? How far?

$^{-}4 + 7 =$ ▮

C



$6 + {}^{-}4 =$ ▮

D $9 + {}^{-}5 = (4 + 5) + {}^{-}5 =$ ▮

? How many of the sums in A, B, C, and D can you find?

Discussing the Ideas

1. Explain how you found the sum for A.
2. What is the sum for B? Explain your answer.
3. Explain how to use the number line in order to find sum for C.
4. Can you show and explain some other integer sums using the number line?
5. How can the basic principles be used to find the sum for D?
6. The number line suggests a way of finding the sum of two negative integers. What is the sum?

Negative direction
Positive direction
0
Jump 3 units left
Jump 4 more units left
End up to left or right of starting point? How far?
3 + 4 = ▮

1. Think about the cheques and bills in order to find the sums.

Pay a bill for $4.	Receive cheque for $10.	How much richer or poorer?		Pay a bill for $3	Pay a bill for $6.	How much richer or poorer?		Pay a bill for $7.	Receive cheque for $10	How much richer or poorer?
↓	↓	↓		↓	↓	↓		↓	↓	↓
A $^{-}4 + 10 = n$			**B**	$^{-}3 + {}^{-}6 = n$			**C**	$^{-}7 + 10 = n$		

2. Think about a thermometer in order to find the sums.

20° above 0°.	Temperature falls 5°.	New temperature?		12° below 0°.	Temperature falls 7°.	New temperature?		10° above 0°.	Temperature falls 6°.	New temperature?
↓	↓	↓		↓	↓	↓		↓	↓	↓
A $20 + {}^{-}5 = n$			**B**	$^{-}12 + {}^{-}7 = n$			**C**	$10 + {}^{-}6 = n$		

3. Think about jumps on the number line starting at zero.

Jump 4 units right.	Jump 2 units left.	How far left or right of zero?		Jump 5 units left.	Jump 3 more units left.	How far left or right of zero?		Jump 8 units right.	Jump 5 units left.	How far left or right of zero?
↓	↓	↓		↓	↓	↓		↓	↓	↓
A $4 + {}^{-}2 = n$			**B**	$^{-}5 + {}^{-}3 = n$			**C**	$8 + {}^{-}5 = n$		

4. Solve the equation to find the sum of the addends in the colored screen. Because of the basic principles, you can add any two addends first.

A $4 + {}^{-}3$
$1 + 3 + {}^{-}3 = n$

B $8 + {}^{-}6$
$2 + 6 + {}^{-}6 = n$

C $5 + {}^{-}2$
$3 + 2 + {}^{-}2 = n$

D $^{-}3 + 7$
$^{-}3 + 3 + 4 = n$

E $^{-}9 + 15$
$^{-}9 + 9 + 6 = n$

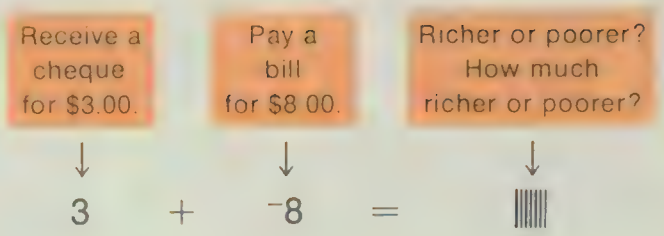
F $28 + {}^{-}12$
$16 + 12 + {}^{-}12 = n$

5. Find the sums. Use any method you choose.

A $3 + {}^{-}3$
B $10 + {}^{-}7$
C $12 + {}^{-}6$
D $12 + {}^{-}4$
E $5 + {}^{-}3$
F $^{-}7 + {}^{-}9$
G $21 + {}^{-}11$
H $^{-}2 + {}^{-}3$
I $^{-}10 + {}^{-}5$
J $^{-}7 + {}^{-}3$
K $8 + {}^{-}2$
L $^{-}2 + 1$
M $^{-}7 + 5$
N $4 + 2$
O $7 + {}^{-}8$
P $10 + {}^{-}17$
Q $({}^{-}8 + {}^{-}5) + {}^{-}7$
R $({}^{-}4 + {}^{-}6) + 7$
S $5 + ({}^{-}8 + {}^{-}2)$
T $^{-}6 + (2 + 4)$
U $^{-}8 + 2 + {}^{-}3$
V $({}^{-}6 + {}^{-}3) + {}^{-}10$
W $85 + {}^{-}37$
X $82 + {}^{-}75$
Y $71 + {}^{-}49$
Z $^{-}1000 + {}^{-}712$

Discussing the Ideas

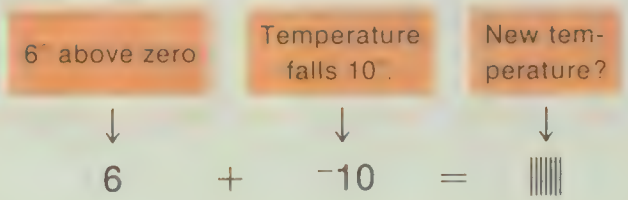
1. Think about cheques and bills. What is this sum?



2. Explain how the number line can help you find this sum.
$^{-}6 + 3 =$ ▥

3. Think about the number line to find these sums.
A $5 + {}^{-}9$ **B** $^{-}7 + 2$ **C** $2 + {}^{-}6$ **D** $^{-}8 + 7$

4. Think about a thermometer to find this sum.



5. Solve the equations in order to find the sum of the addends in the colored screen. Because of the basic principles, you can add any two addends first.

A $^{-}9 + 2$ **B** $5 + {}^{-}6$ **C** $^{-}10 + 4$

$^{-}7 + {}^{-}2 + 2 = n$ $5 + {}^{-}5 + {}^{-}1 = n$ $^{-}6 + {}^{-}4 + 4 = n$

6. If two integers are both positive, is their sum positive or negative?

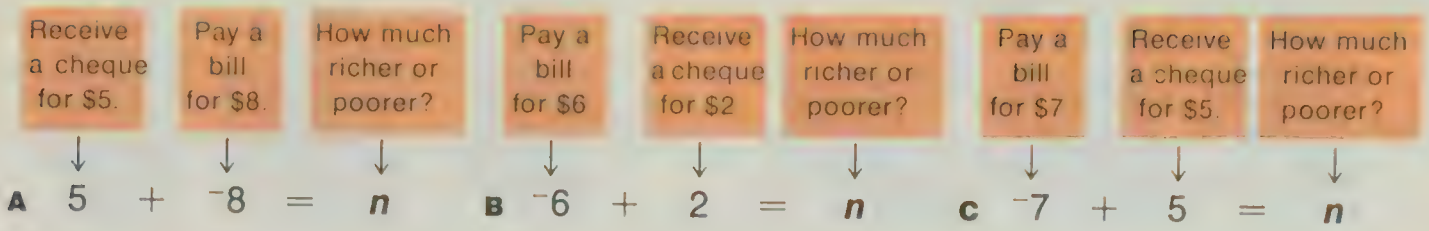
7. If two integers are both negative, is their sum positive or negative?

8. If one integer is positive and one is negative, how can you decide whether the sum is a positive integer, a negative integer, or zero?

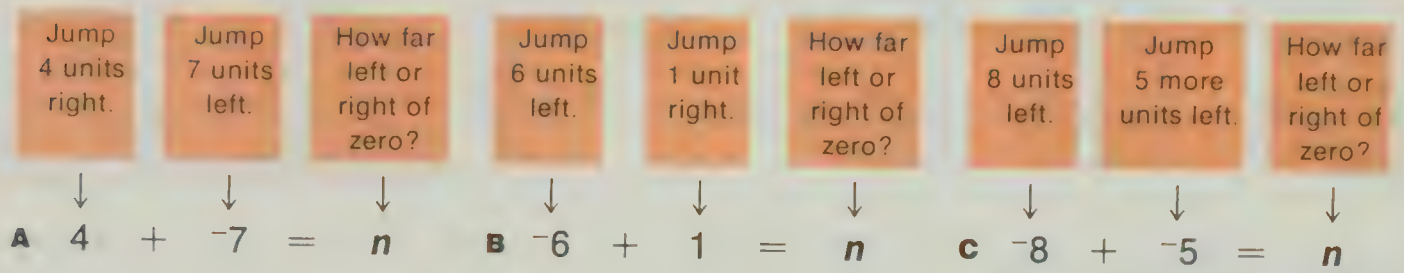
9. If the sum of two integers is zero, what can you say about the integers?

10. If one of the two integers is zero, what is the sum?

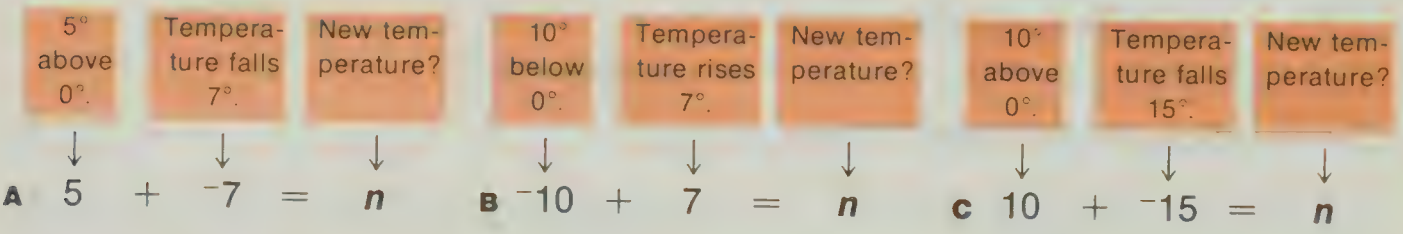
1. Solve the equations. Think of cheques and bills.


Receive a cheque for $5.
Pay a bill for $8.
How much richer or poorer?
Pay a bill for $6
Receive a cheque for $2
How much richer or poorer?
Pay a bill for $7
Receive a cheque for $5.
How much richer or poorer?
A 5 + ⁻8 = n
B ⁻6 + 2 = n
C ⁻7 + 5 = n

2. Solve the equations. Think about jumps on the number line starting at zero.


Jump 4 units right.
Jump 7 units left.
How far left or right of zero?
Jump 6 units left.
Jump 1 unit right.
How far left or right of zero?
Jump 8 units left.
Jump 5 more units left.
How far left or right of zero?
A 4 + ⁻7 = n
B ⁻6 + 1 = n
C ⁻8 + ⁻5 = n

3. Solve the equations. Think about a thermometer.



4. When you find the number for *n* in the cloud, you have found the sum of the two integers in the gray screen. Copy the equation, giving the correct sum.

A $-6 + 2 = n$ (n + 2)

C $-12 + 8 = n$ (n + 8)

E $7 + -10 = n$ (7 + n)

G $2 + -7 = n$ (⁻2 + n)

B $-9 + 8 = n$ (n + 8)

D $-20 + 6 = n$ (n + 6)

F $6 + -8 = n$ (6 + n)

H $-42 + 38 = n$ (n + 38)

5. Find the sums. Use any method you choose.

A $-4 + 3$	F $2 + -8$	K $-2 + -3$	P $-1 + 6$	U $15 + -9$
B $-7 + 6$	G $7 + -10$	L $7 + -7$	Q $-5 + 5$	V $4 + -13$
C $-9 + 2$	H $-8 + 7$	M $8 + -1$	R $6 + -4$	W $-8 + -7$
D $-8 + 4$	I $-12 + 9$	N $4 + -9$	S $-7 + 0$	X $17 + -9$
E $6 + -10$	J $4 + -11$	O $-4 + 9$	T $-1 + -9$	Y $-14 + 7$

Finding Missing Addends

Investigating the Ideas

What is the input number for each flow chart?

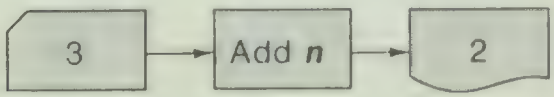
Discussing the Ideas

1. Each input number in the flow chart above is a **missing addend** in an addition equation.

   An equation for flow chart A is → $n + {}^{-}3 = 5$ (Addends: n, $^{-}3$; Sum: 5)

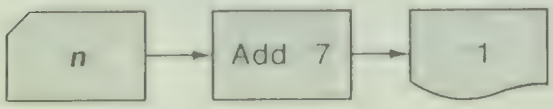
   Write an equation for each of the other flow charts.

2. A What is the number for n in the flow chart?



   B Write an equation for the flow chart.

3. A Write an equation for this flow chart.



   B Solve the equation for n.

4. A What equation is suggested by this word problem?
   When $^{-}2$ is added to a certain number, the sum is 8. What is the number?

   B Solve the equation for part A.

1. Find the number for n in each flow chart.

A n → Add $^{-}6$ → $^{-}2$

B n → Add 7 → $^{-}4$

C $^{-}10$ → Add n → $^{-}1$

D $^{-}7$ → Add n → $^{-}13$

2. Write and solve an equation for each flow chart.

A n → Add $^{-}5$ → 4

B n → Add $^{-}9$ → $^{-}4$

C 8 → Add n → $^{-}8$

D $^{-}6$ → Add n → 1

3. Solve the equations by finding the missing addends.

A $n + {}^{-}1 = {}^{-}5$	F $n + 6 = 3$	K $n + {}^{-}3 = {}^{-}5$	P $12 = n + {}^{-}3$
B $n + {}^{-}3 = {}^{-}4$	G $^{-}2 + n = 5$	L $8 + n = {}^{-}2$	Q $8 = {}^{-}6 + n$
C $n + 7 = 0$	H $n + {}^{-}4 = 2$	M $^{-}3 + n = {}^{-}7$	R $^{-}3 = n + 2$
D $5 + n = 3$	I $n + {}^{-}3 = 5$	N $n + {}^{-}6 = {}^{-}10$	S $^{-}7 = {}^{-}4 + n$
E $n + 2 = 1$	J $n + 4 = {}^{-}1$	O $9 + n = 2$	T $^{-}9 = 9 + n$

4. Write and solve an equation for each part.

A When $^{-}9$ is added to a certain number, the sum is 3. What is the number?

B During a certain winter day, the temperature rose 12° to reach a high of 8° above zero. What was the low temperature of the day?

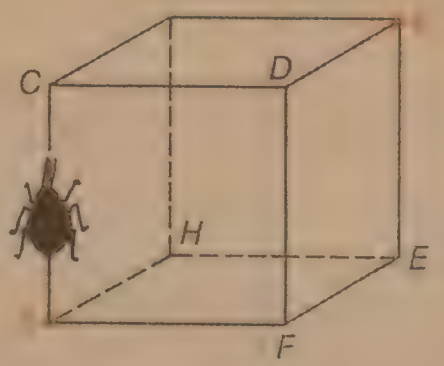
C What number added to $^{-}19$ will give a sum of 7?

think

The symbol *ACDB* describes a "trip" from *A* to *B* along just three edges of the cube. We agree that in a trip no edge is to be traversed twice.

Tell how many different trips there are from *A* to *B* along just

1. three edges of the cube
2. four edges of the cube
3. five edges of the cube
4. seven edges of the cube

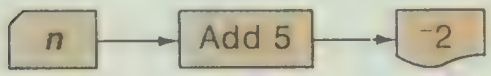


More practice, S-39, Set 51

Investigating the Ideas

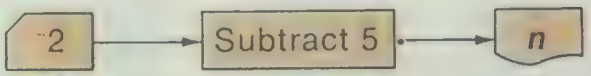
You can think of subtraction of integers as finding a missing addend. If you can find the missing addend in the addition equation below, then you have found the difference for the subtraction equation. Study the flow charts and equations.

ADDITION FLOW CHART



$n + 5 = {}^{-}2$

SUBTRACTION FLOW CHART



$^{-}2 - 5 = n$

Can you find the number for n in either flow chart?	Write five more subtraction equations and then solve them by finding missing addends.

Discussing the Ideas

1. Explain how you found the differences in the subtraction equations that you wrote for the Investigation?

2. Carol checked her first subtraction equation this way:

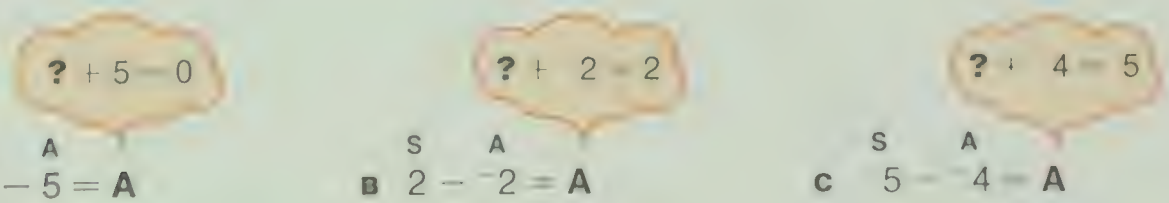
   S A A A A S
   $4 - {}^{-}2 = 6$ because $6 + {}^{-}2 = 4$

   Check the rest of her problems. Did Carol make any mistakes? Can you correct the mistakes?

Carol

1. $4 - {}^{-}2 = n,\ n = 6$
2. $^{-}5 - {}^{-}1 = n,\ n = {}^{-}4$
3. $^{-}7 - 2 = n,\ n = 5$
4. $3 - 9 = n,\ n = {}^{-}6$
5. $1 - {}^{-}3 = n,\ n = {}^{-}2$

3. Explain how to find each difference. The equation in the clouds may help you.



A S A
$0 - 5 = A$

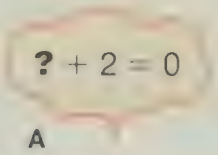
B S A
$2 - {}^{-}2 = A$

C S A
$5 - {}^{-}4 = A$

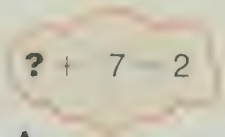
4. Find the differences. Explain your reasoning.

A	B	C	D	E	F	G
$5 - 3$	$^{-}5 - 6$	$^{-}3 - 4$	$9 - 6$	$4 - {}^{-}1$	$5 - {}^{-}8$	$^{-}2 - {}^{-}3$

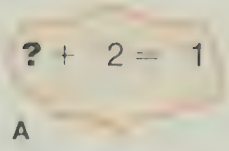
1. Find the differences. Think about missing addends.



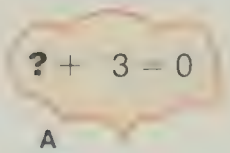
S A
A $0 - 2 = A$



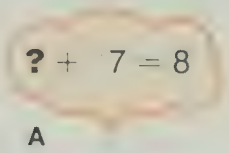
S A
C $2 - {}^{-}7 = A$



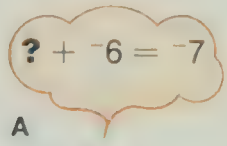
S A
E ${}^{-}1 - {}^{-}2 = A$



S A
B $0 - {}^{-}3 = A$



S A
D $8 - {}^{-}7 = A$



S A
F ${}^{-}7 - {}^{-}6 = A$

2. Solve the equations.

S A A
A $2 - {}^{-}4 = n$
B $3 - {}^{-}2 = n$
C $^{-}5 - {}^{-}2 = n$
D $^{-}4 - {}^{-}3 = n$
E $^{-}1 - {}^{-}1 = n$
F $^{-}4 - 5 = n$

S A A
G $4 - {}^{-}2 = n$
H $^{-}7 - {}^{-}4 = n$
I $^{-}2 - {}^{-}3 = n$
J $^{-}1 - {}^{-}5 = n$
K $^{-}8 - {}^{-}7 = n$
L $^{-}4 - {}^{-}3 = n$

S A A
M $^{-}5 - {}^{-}1 = n$
N $^{-}3 - {}^{-}2 = n$
O $8 - {}^{-}7 = n$
P $^{-}9 - {}^{-}4 = n$
Q $14 - {}^{-}6 = n$
R $9 - {}^{-}5 = n$

S A A
S $1 - 3 = n$
T $4 - 9 = n$
U $^{-}13 - {}^{-}5 = n$
V $^{-}11 - {}^{-}4 = n$
W $74 - {}^{-}3 = n$
X $^{-}16 - {}^{-}14 = n$

3. Find the differences.

A $^{-}3 - 7$
B $3 - 7$
C $^{-}8 - 2$
D $^{-}8 - {}^{-}2$
E $^{-}2 - 8$
F $2 - 8$
G $^{-}2 - {}^{-}8$
H $8 - {}^{-}2$
I $^{-}5 - 3$
J $5 - {}^{-}3$
K $^{-}3 - {}^{-}8$
L $7 - {}^{-}3$
M $3 - 5$
N $^{-}6 - 3$
O $^{-}2 - {}^{-}1$
P $^{-}8 - 12$
Q $15 - {}^{-}3$
R $^{-}6 - 9$
S $^{-}8 - 7$
T $^{-}2 - {}^{-}3$

★ **4.** Solve the equations. It may help you to think about sums and addends.

A $x - 3 = {}^{-}5$
B $x - {}^{-}3 = {}^{-}5$
C $3 - x = {}^{-}5$
D $3 - x = 5$
E $y = 3 - {}^{-}10$
F $y - 3 = {}^{-}10$
G $3 + y = {}^{-}10$
H $3 - y = {}^{-}10$
I $5 - x = 10$
J $c - 6 = {}^{-}6$
K $d - {}^{-}7 = {}^{-}7$
L $^{-}7 - d = 1$
M $^{-}5 - c = 5$
N $y + y = {}^{-}8$
O $^{-}8 - y = y$
P $^{-}10 - y = y$

More practice, page S-30, Set 52

Basic Principles for Multiplication of Integers

Discussing the Ideas

The basic principles for multiplication of integers are suggested by the basic principles for multiplication of whole numbers.

1. What is the product in each part?

   A $6 \cdot 1$ **D** $^{-}8 \cdot 1$

   B $^{-}2 \cdot 1$ **E** $19 \cdot 1$

   C $^{-}10 \cdot 1$ **F** $^{-}23 \cdot 1$

One principle

If you choose an integer and multiply by 1, the product is the integer that you choose.

In general, for any integer a,

$$a \cdot 1 = a$$

2.

Commutative principle

When finding the product of two integers, you can change the order of the factors and the product is the same.

In general, for integers a and b,

$$a \cdot b = b \cdot a$$

Associative principle

When finding the product of three integers, you can change the grouping of the factors and the product is the same.

In general, for integers a, b, and c,

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Since you can change both the **order** and the **grouping** of the factors, you can rearrange factors in any way that is convenient.

A Using the integers $^{-}3$ and $^{-}4$, give an example of the **commutative principle** for multiplication.

B Using the integers 13, 7, and $^{-}5$, give an example of the **associative principle** for multiplication.

C Is the product $(^{-}6 \cdot 4) \cdot ^{-}9$ the same as $(4 \cdot ^{-}9) \cdot ^{-}6$? Why?

3. How would you complete this equation to illustrate the **distributive principle**?

   $2 \cdot (3 + ^{-}4) = ?$

Distributive principle

For all integers a, b, and c,

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$

1. Find the integer for ***a***. Then find the integer for ***b***.

   A $^{-}7 \cdot 1 = a \rightarrow 1 \cdot {}^{-}7 = b$

   B $^{-}12 \cdot 1 = a \rightarrow 1 \cdot {}^{-}12 = b$

   C $5 \cdot 0 = a \rightarrow 0 \cdot 5 = b$

   D $^{-}5 \cdot 0 = a \rightarrow 0 \cdot {}^{-}5 = b$

2. Study the examples. $^{-}3 + {}^{-}3 = 2 \cdot {}^{-}3 = {}^{-}6$

   $^{-}5 + 5 + {}^{-}5 = 1 \cdot {}^{-}5 = 15$

   Find the integer for ***a***. Then find the integer for ***b***.

   A $^{-}3 + {}^{-}3 = a \rightarrow 2 \cdot {}^{-}3 = b$

   B $^{-}4 + {}^{-}4 = a \rightarrow 2 \cdot {}^{-}4 = b$

   C $^{-}5 + {}^{-}5 = a \rightarrow 2 \cdot {}^{-}5 = b$

   D $^{-}7 + {}^{-}7 = a \rightarrow 2 \cdot {}^{-}7 = b$

   E $^{-}58 + {}^{-}58 = a \rightarrow 2 \cdot {}^{-}58 = b$

   F $^{-}4 + {}^{-}4 + {}^{-}4 = a \rightarrow 3 \cdot {}^{-}4 = b$

   G $^{-}5 + {}^{-}5 + {}^{-}5 + {}^{-}5 = a \rightarrow 4 \cdot {}^{-}5 = b$

   H $^{-}8 + {}^{-}8 + {}^{-}8 + {}^{-}8 = a \rightarrow 4 \cdot {}^{-}8 = b$

3. Find the integer for ***a***. Then find the integer for ***b***.

   A $2 \cdot {}^{-}3 = a \rightarrow {}^{-}3 \cdot 2 = b$

   B $2 \cdot {}^{-}5 = a \rightarrow {}^{-}5 \cdot 2 = b$

   C $3 \cdot {}^{-}7 = a \rightarrow {}^{-}7 \cdot 3 = b$

   D $4 \cdot {}^{-}6 = a \rightarrow {}^{-}6 \cdot 4 = b$

4. Copy the equations and write the correct number for each ▒. Each equation can be completed by thinking about the equation above it and a basic principle or multiplication fact.

   A

   $2 + {}^{-}2 = ▒$

   $3 \cdot (2 + {}^{-}2) = ▒$

   $(3 \cdot 2) + (3 \cdot {}^{-}2) = ▒$

   $6 + 3 \cdot {}^{-}2 = ▒$

   $3 \cdot {}^{-}2 = ▒$

   This number adds to 6 to give zero. Therefore, $(3 \cdot 2)$ must be the integer

   B

   $2 + {}^{-}2 = ▒$

   $^{-}3 \cdot (2 + {}^{-}2) = ▒$

   $({}^{-}3 \cdot 2) + ({}^{-}3 \cdot {}^{-}2) = ▒$

   $^{-}6 + {}^{-}3 \cdot {}^{-}2 = ▒$

   $^{-}3 \cdot {}^{-}2 = ▒$

   This number adds to 6 to give zero. Therefore, $(^{-}3 \cdot {}^{-}2)$ must be the integer

   C

   $4 + {}^{-}4 = ▒$

   $^{-}5 \cdot (4 + {}^{-}4) = ▒$

   $({}^{-}5 \cdot 4) + ({}^{-}5 \cdot {}^{-}4) = ▒$

   $^{-}20 + {}^{-}5 \cdot {}^{-}4 = ▒$

   $^{-}5 \cdot {}^{-}4 = ▒$

   This number adds to $^{-}20$ to give zero. Therefore, $(^{-}5 \cdot {}^{-}4)$ must be the integer

5. Use the correct word (**positive** or **negative**) to complete each statement.

   A The product of a positive integer and a negative integer is a ? integer.

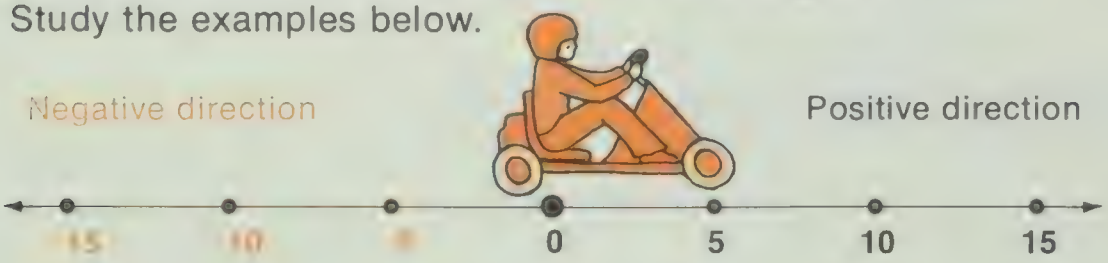
   B The product of a negative integer and a negative integer is a ? integer.

think

A first number is multiplied by a second, and the product is 1 602. When the first number is multiplied by a number one larger than the second, the product is 1 743. Determine the first and the second number.

Investigating the Ideas

Sometimes it may be helpful to think about a physical example that illustrates multiplication of integers. Study the examples below.



You are now at zero	→	Travel 5 km/h to the **right**	→	3 hours **from now**	→	You will be at ▮
5	×	3	=	▮		
You are now at zero	→	Travelled 5 km/h to the **right**	→	3 hours **ago**	→	You were at ▮
5	×	$^{-}3$	=	▮		
You are now at zero	→	Travelled 5 km/h to the **left**	→	3 hours **ago**	→	You were at ▮
$^{-}5$	×	$^{-}3$	=	▮		

Can you give the missing number in each equation?

Discussing the Ideas

1. Explain how you can use the number line above to help you write an equation if the speed was 7 km/h to the right or to the left and time was either 2 hours from now or 2 hours ago.

2. What multiplication equation does this word problem suggest?
   A man is now at zero and riding a bicycle to the left at 12 km/h. Where will he be 2 hours from now?

3. Find the products. Think about the number line if necessary.
   A $6 \cdot {}^{-}4$ **B** $3 \cdot 4$ **C** ${}^{-}6 \cdot {}^{-}2$ **D** ${}^{-}7 \cdot 5$

1. Write a multiplication equation for each part.
Think about the number line.

A You are now at zero, walking 4 km/h to the right.
Where were you 3 hours ago?

B You are now at zero walking 2 km/h to the left.
Where were you 4 hours ago?

C You are now at zero walking 3 km/h to the left.
Where will you be 3 hours from now?

D You are now at zero and running 6 km/h to the right.
Where will you be 1 hour from now?

2. Find the product **P** in each equation.

A $4 \cdot {}^{-}5 = P$	**D** $1 \cdot {}^{-}5 = P$	**G** ${}^{-}2 \cdot {}^{-}5 = P$	**J** ${}^{-}5 \cdot {}^{-}5 = P$
B $3 \cdot {}^{-}5 = P$	**E** $0 \cdot {}^{-}5 = P$	**H** ${}^{-}3 \cdot {}^{-}5 = P$	**K** ${}^{-}6 \cdot {}^{-}5 = P$
C $2 \cdot {}^{-}5 = P$	**F** ${}^{-}1 \cdot {}^{-}5 = P$	**I** ${}^{-}4 \cdot {}^{-}5 = P$	**L** ${}^{-}7 \cdot {}^{-}5 = P$

3. Find the products.

A $2 \cdot {}^{-}4$	**G** ${}^{-}8 \cdot 0$	**M** ${}^{-}5 \cdot 0$	**S** $13 \cdot {}^{-}13$
B ${}^{-}4 \cdot 3$	**H** $12 \cdot {}^{-}3$	**N** ${}^{-}7 \cdot 6$	**T** ${}^{-}13 \cdot {}^{-}13$
C ${}^{-}4 \cdot {}^{-}3$	**I** ${}^{-}3 \cdot 12$	**O** ${}^{-}7 \cdot {}^{-}6$	**U** $10 \cdot {}^{-}27$
D ${}^{-}1 \cdot 1$	**J** ${}^{-}3 \cdot {}^{-}12$	**P** ${}^{-}12 \cdot 13$	**V** ${}^{-}100 \cdot 38$
E ${}^{-}1 \cdot {}^{-}1$	**K** ${}^{-}4 \cdot 8$	**Q** ${}^{-}12 \cdot {}^{-}13$	**W** ${}^{-}100 \cdot {}^{-}38$
F ${}^{-}2 \cdot {}^{-}4$	**L** ${}^{-}4 \cdot {}^{-}8$	**R** $14 \cdot {}^{-}12$	**X** $468 \cdot {}^{-}1000$

4. Find the products.

A $(2 \cdot 3) \cdot {}^{-}2$	**D** $({}^{-}5 \cdot {}^{-}2) \cdot 2$	**G** ${}^{-}2\,({}^{-}5 \cdot {}^{-}3)$	**J** $5 \cdot (3 \cdot 2)$
B $(4 \cdot {}^{-}4) \cdot 2$	**E** $(8 \cdot {}^{-}1) \cdot 1$	**H** ${}^{-}6 \cdot (2 \cdot {}^{-}3)$	**K** ${}^{-}1 \cdot ({}^{-}1 \cdot {}^{-}1)$
C $(1 \cdot {}^{-}1) \cdot {}^{-}3$	**F** $(3 \cdot {}^{-}3) \cdot {}^{-}3$	**I** ${}^{-}2 \cdot ({}^{-}3 \cdot {}^{-}4)$	**L** $({}^{-}1 \cdot 1) \cdot ({}^{-}1 \cdot 1)$

5. Solve the equations.

A ${}^{-}7 \cdot 8 = n$	**D** $8 \cdot r = 32$	**G** ${}^{-}4 \cdot 13 = m$	**J** $12 \cdot w = 108$
B $6 \cdot {}^{-}5 = s$	**E** $z \cdot {}^{-}5 = 15$	**H** ${}^{-}11 \cdot q = 110$	**K** $3 \cdot (x + 1) = {}^{-}12$
C ${}^{-}9 \cdot {}^{-}7 = t$	**F** ${}^{-}3 \cdot k = 24$	**I** ${}^{-}7 \cdot b = {}^{-}49$	**L** ${}^{-}5 \cdot (y - {}^{-}2) = 35$

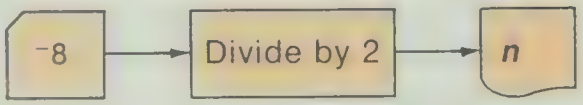
More practice, page S-30, Set 53

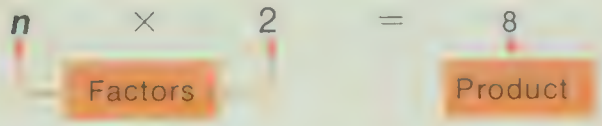
Investigating the Ideas

You can find the quotient of two integers by thinking about missing factors. If you can find the missing factor in a multiplication equation, then you will have found the quotient for a division equation. Study the flow charts and equations.

MULTIPLICATION FLOW CHART

DIVISION FLOW CHART





Can you find the number for *n* in either flow chart?

Write five more division equations and then solve them by finding missing factors.

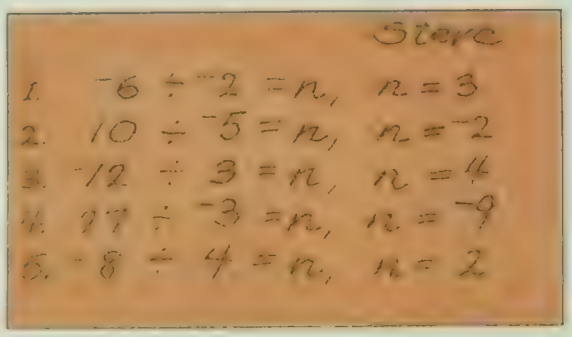
Discussing the Ideas

1. Explain how you found the quotients for the division equations that you wrote for the Investigation?

2. Steve checked his first division problem like this:

   P F F F F P
   $-6 \div -2 = 3$ because $3 \cdot -2 = -6$

   Check the rest of his problems. Did Steve make any mistakes? Can you correct his mistakes?



3. What is the missing factor in each equation?

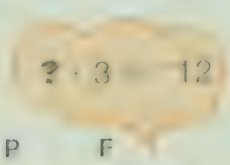
   F F P
   A $-2 \cdot x = -8$

   F F P
   B $n \cdot 7 = -21$

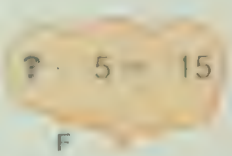
   F F P
   C $r \cdot -4 = 32$

   F F P
   D $3 \cdot t = 24$

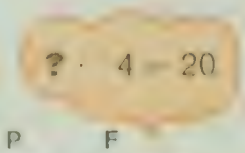
4. Explain how to find the quotients.



P F
A $-12 \div 3 = F$



P F
B $15 \div -5 = F$



P F
C $20 \div 4 = F$

1. Find the missing factors.

	F	F	P		F	F	P		F	F	P		F	F	P
A	$^{-}2 \cdot$	$a =$	$^{-}8$	**F**	$r \cdot$	$4 =$	0	**K**	$3 \cdot$	$n =$	$^{-}18$	**P**	$n \cdot$	$20 =$	80
B	$8 \cdot$	$b =$	$^{-}16$	**G**	$u \cdot$	$5 =$	$^{-}5$	**L**	$s \cdot$	$5 =$	$^{-}30$	**Q**	$n \cdot$	$^{-}20 =$	80
C	$^{-}3 \cdot$	$x =$	3	**H**	$m \cdot$	$3 =$	$^{-}9$	**M**	$2 \cdot$	$y =$	$^{-}10$	**R**	$n \cdot$	$20 =$	$^{-}80$
D	$3 \cdot$	$x =$	12	**I**	$^{-}3 \cdot$	$y =$	$^{-}3$	**N**	$r \cdot$	$^{-}2 =$	$^{-}8$	**S**	$n \cdot$	$11 =$	$^{-}55$
E	$c \cdot$	$^{-}2 =$	6	**J**	$t \cdot$	$^{-}1 =$	8	**O**	$n \cdot$	$^{-}3 =$	12	**T**	$^{-}12 \cdot$	$y =$	36

2. Find the quotients. Exercise 1 may help you.

A	$^{-}9 \div 3 = m$	**F**	$^{-}8 \div {}^{-}2 = a$	**K**	$36 \div {}^{-}12 = y$	**P**	$^{-}30 \div 5 = s$
B	$12 \div 3 = x$	**G**	$80 \div {}^{-}20 = n$	**L**	$12 \div {}^{-}3 = n$	**Q**	$^{-}5 \div 5 = u$
C	$^{-}18 \div 3 = n$	**H**	$^{-}10 \div 2 = y$	**M**	$^{-}80 \div 20 = n$	**R**	$0 \div {}^{-}4 = r$
D	$^{-}8 \div {}^{-}2 = a$	**I**	$8 \div {}^{-}1 = t$	**N**	$^{-}55 \div 11 = n$	**S**	$^{-}3 \div {}^{-}3 = y$
E	$6 \div {}^{-}2 = c$	**J**	$^{-}18 \div 3 = n$	**O**	$3 \div {}^{-}3 = x$	**T**	$^{-}16 \div 8 = b$

3. Find the quotients. Think about missing factors.

A	$^{-}35 \div 7 = n$	**G**	$12 \div 4 = n$	**M**	$12 \div {}^{-}4 = n$
B	$^{-}12 \div 4 = n$	**H**	$^{-}12 \div {}^{-}4 = n$	**N**	$8 \div {}^{-}8 = n$
C	$^{-}8 \div {}^{-}8 = n$	**I**	$0 \div {}^{-}6 = n$	**O**	$20 \div {}^{-}5 = n$
D	$28 \div {}^{-}7 = n$	**J**	$^{-}28 \div 7 = n$	**P**	$^{-}28 \div {}^{-}7 = n$
E	$^{-}16 \div {}^{-}1 = n$	**K**	$^{-}16 \div 1 = n$	**Q**	$^{-}45 \div 15 = n$
F	$^{-}45 \div {}^{-}15 = n$	**L**	$^{-}2448 \div {}^{-}16 = n$	**R**	$(^{-}8 \cdot 7) \div {}^{-}8 = n$

4. Sue is thinking of a number. If you multiply Sue's number by itself and then add it to that product, you get 20. Give two possibilities for the number Sue might be thinking about.

think

1. Copy teach exercise and insert any of the symbols , , , , or in order to make the sentence true.

EXAMPLE: $2 = (4 + 4) \div 4$

A	$0 = 4 \quad 4 \quad 4$	**C**	$4 = 4 \quad 4 \quad 4$	**E**	$12 = 4 \quad 4 \quad 4$	**G**	$32 = 4 \quad 4 \quad 4$
B	$3 = 4 \quad 4 \quad 4$	**D**	$5 = 4 \quad 4 \quad 4$	**F**	$20 = 4 \quad 4 \quad 4$	**H**	$64 = 4 \quad 4 \quad 4$

2. Copy each exercise and insert any of the symbols $+$, , $-$, , , , . or in order to make the sentence true.

EXAMPLES: $\sqrt{4} = 2$ (Read $\sqrt{4}$ as "the square root of 4.")

$! = 4 \times 3 \times 2 \times 1$ (Read 4! as "4 factorial.")

A	$1 = 4 \quad 4 \quad 4$	**B**	$6 = 4 \quad 4 \quad 4$	**C**	$8 = 4 \quad 4 \quad 4$	**D**	$25 = 4 \quad 4 \quad 4$

More practice, page S-31, Set S4

Investigating the Ideas

Ann and Brian played eight games of cards and kept a record of their point scores for each game. The negative integers indicate scores "in the hole" or "going set."

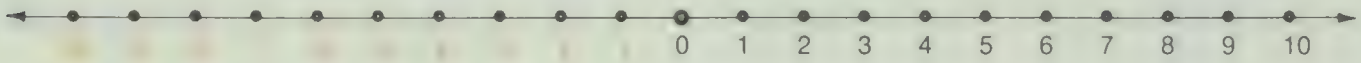
Games	1	2	3	4	5	6	7	8
Brian's scores	7			0			6	
Ann's scores		2	6		6		8	

Can you tell who had the best score for each game?

For each game, tell how many points the loser needed in order to tie the winner.

Discussing the Ideas

1. On the number line below, the integers are matched with points so that the point for the larger of two numbers is to the right of the point for the smaller number.



   A Since 1 is to the right of $^-10$, which is the larger number?
   B Since $^-4$ is to the left of 3, which is the larger number?
   C Since $^-6$ is to the right of $^-10$, which is the larger number?

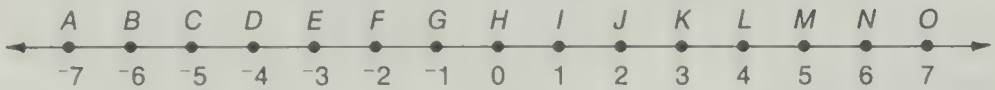
2. Use **greater than** or **less than** to complete each sentence.
   A Any positive integer is ? zero.
   B Any negative integer is ? zero.
   C Any positive integer is ? any negative integer.
   D Any negative integer is ? any positive integer.
   E Any integer is ? any other integer that is to the left of it on the number line.
   F Any integer is ? any other integer that is to the right of it on the number line.

3. We say that 4 **is less than** 7 and write this as $4 < 7$ because we must add a **positive** number to 4 in order to get 7.
   $4 + 3 = 7$; 3 is a positive number.
   We also say that 7 **is greater than** 4 and write $7 > 4$.
   What addition equation would show that the following inequalities are true?
   A $^-1 < 1$ B $^-5 < 0$ C $^-6 < 7$ D $^-7 < 4$

1. Give the correct symbol ($<$ or $>$) for each ◯.
   - **A** Since $3 = 2 + 1$, we know that 3 ◯ 2.
   - **B** Since $^{-}3 = {}^{-}4 + 1$, we know that $^{-}3$ ◯ $^{-}4$.
   - **C** Since $^{-}5 = {}^{-}6 + 1$, we know that $^{-}5$ ◯ $^{-}6$.

2. Give the correct symbol ($<$ or $>$) for each ◯.
   - **A** 7 ◯ $^{-}3$
   - **B** $^{-}8$ ◯ $^{-}4$
   - **C** 100 ◯ $^{-}1$
   - **D** $^{-}12$ ◯ $^{-}8$
   - **E** 0 ◯ 2
   - **F** 0 ◯ $^{-}2$
   - **G** $^{-}99$ ◯ $^{-}100$
   - **H** $^{-}53$ ◯ $^{-}68$

3. Write the letters for the points in each set.



   - **A** The set of points for numbers greater than $^{-}4$ and less than zero
   - **B** The set of points for numbers less than or equal to $^{-}2$
   - **C** The set of points for numbers greater than 3 or less than $^{-}5$

4. The example shows the graph of the set of integers greater than $^{-}4$ and less than 4. We say that we have graphed all possible integers for n in the inequality $^{-}4 < n < 4$.

   Draw number lines and graph each set.
   - **A** Integers greater than 0 and less than 5
   - **B** Integers greater than $^{-}2$ and less than 3
   - **C** Integers greater than $^{-}6$ and less than 6
   - **D** Integers less than 0 and greater than $^{-}6$
   - **E** Integers less than 1 and greater than $^{-}1$
   - **F** Integers greater than $^{-}8$ and less than $^{-}2$
   - **G** Integers less than $^{-}5$ and greater than $^{-}9$
   - **H** Integers for n in the inequality $^{-}2 < n < 5$
   - **I** Integers for n in the inequality $^{-}6 < n < {}^{-}1$

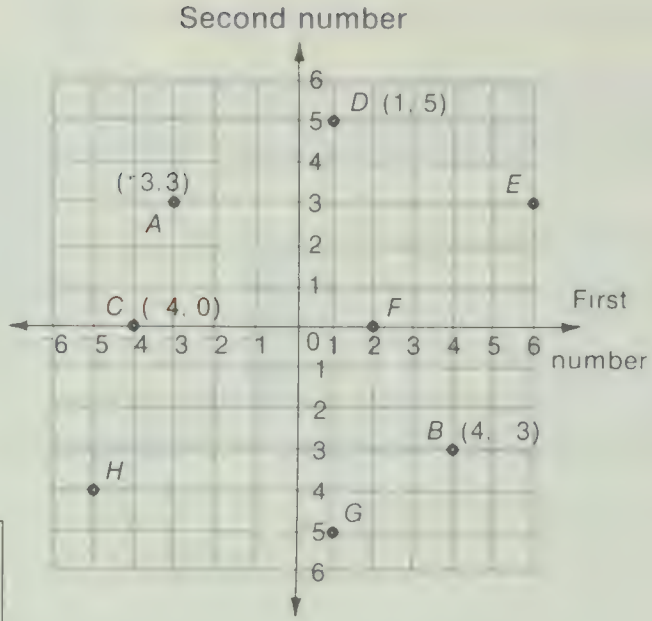
think

Arrange the digits 1 through 7 in the hexagons, so that the sum of the digits is the same number in any three hexagons in a row.

5. Let $A = \{^{-}2, {}^{-}1, 0, 1, 2\}$, $B = \{^{-}5, {}^{-}4, {}^{-}3, {}^{-}2, {}^{-}1, 0\}$, and $C = \{0, 1, 2, 3, 4, 5\}$. Graph each of the following sets on a number line.
   - **A** $A \cup B$
   - **B** $A \cup C$
   - **C** $A \cap B$
   - **D** $A \cap C$
   - **E** $B \cup C$
   - **F** $B \cap (A \cup C)$
   - **G** $(A \cup B) \cup C$
   - **H** $(A \cap B) \cap C$

Investigating the Ideas

You can use two intersecting number lines to show the graph of any pair of integers. The pair of integers that describe the location of a point are the **co-ordinates** of the point. The point of the pair is called the **graph of the integer pair**. The graphs of some integer pairs are shown on the co-ordinate grid.



Can you give the missing co-ordinates in the table below?

Point	A	B	C	D	E	F	G	H
Coordinates	(⁻3, 3)	(4, ⁻3)	(⁻4, 0)	(1, 5)	(,)	(,)	(,)	(,)

Discussing the Ideas

1. The two intersecting number lines are called the **co-ordinate axes** and each number line is an **axis**.
   The point of intersection of the co-ordinate axes is called the **origin**.
   What are the co-ordinates of the origin?

2. Give the co-ordinates of a point whose location can be described as
   - **A** 3 units to the right of the origin and 4 units down
   - **B** 5 units to the left of the origin and 6 units up
   - **C** 2 units to left of the origin
   - **D** 4 units to the left of the origin and 5 units down
   - **E** 3 units down from the origin
   - **F** 6 units to the right of the origin and 6 units up

3. Name some integer pairs so that the numbers in each pair are opposites of each other. The pair (⁻3, 3) is an example.
   What can you say about the graph of all such pairs of opposites?

4. List some integer pairs such that the second number is one more than the first number. Include some negative integers.

1. For each exercise, use graph paper and draw the co-ordinate axes. Mark the points for integers from -5 through 5 on each axis. Then graph the following sets of integer pairs.
   - **A** $\{(4, 2), (4, -2), (4, 0), (-3, 4), (-3, -4), (-3, 0), (-2, 2), (2, -2), (-2, -2), (2, 2)\}$
   - **B** Graph the set of integer pairs whose second number is -2; that is, $(4, -2)$, $(3, -2)$, and so on.
   - **C** Graph the set of integer pairs whose first number is -3.
   - **D** Graph the set of integer pairs in which the first and second number add to give zero.
   - **E** Graph the set of integer pairs in which the first number is one more than the second number.
   - **F** Graph the set of integer pairs in which the second number is 0.

2. Draw the co-ordinate axes with points for integers from -10 through 10 on each axis. Then graph the set of integer pairs in which
   - **A** the first number is equal to the second.
   - **B** the first number is greater than the second.
   - **C** the second number is greater than the first.
   - **D** the second number is two times the first number.
   - **E** the second number is the first number squared.
   - **F** the sum of the first and second number in each pair is 4.
   - **G** twice the second number is three times the first.

3. Draw the graph of each of the two number pairs in each part. Then give the co-ordinates of the point that is halfway between the two points.
   - **A** $(4, -4)$ and $(4, 2)$
   - **B** $(-5, 6)$ and $(-5, 0)$
   - **C** $(1, 1)$ and $(5, 3)$
   - **D** $(-5, 3)$ and $(5, -3)$
   - **E** $(-4, -1)$ and $(2, -5)$
   - **F** $(-1, -1)$ and $(5, -5)$

think

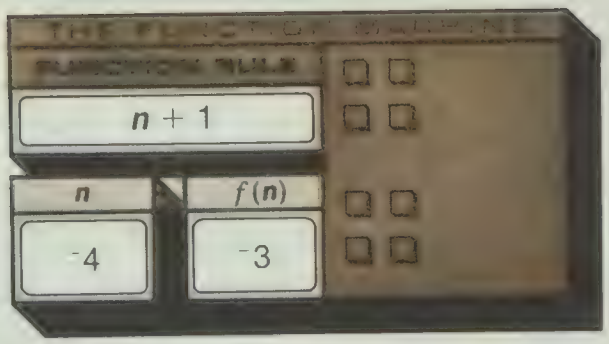
Ben argued that his \$5.00 a week allowance was not enough. His father refused to raise the amount.

Ben offered the following "bargain." He asked his father to give him 1¢ for the first week, 2¢ for the second week, 4¢ for the following week, 8¢ the following week, and so on.

Should Ben's father accept the bargain? Explain your answer.

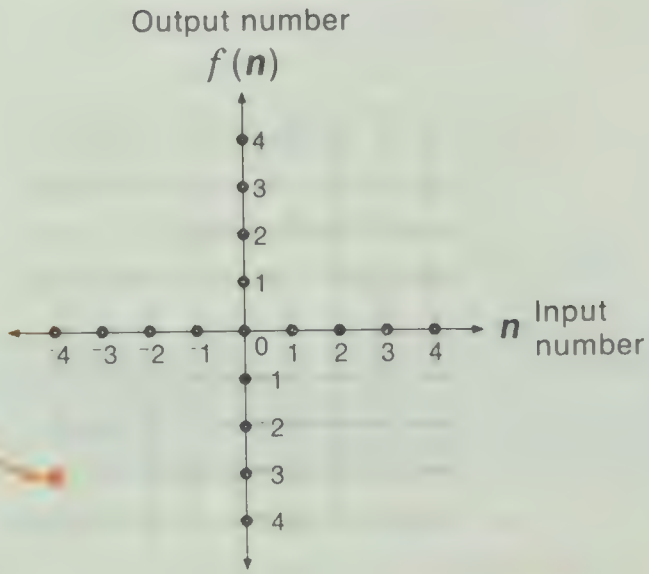
Investigating the Ideas

For each function rule and set of input numbers, you can find a set of number pairs. In each pair, the first number is the input number n and the second number is the output number $f(n)$. You can draw graphs for these sets of number pairs.



Function Rule $n + 1$		Integer pairs
n	$f(n)$	↓
-4	-3	(-4, -3)
-3	[redacted]	(-3, [redacted])
-2	[redacted]	(-2, [redacted])
-1	[redacted]	(-1, [redacted])
0	[redacted]	(0, [redacted])
1	[redacted]	(1, [redacted])
2	[redacted]	(2, [redacted])

Graph of the integer pair



Can you find the missing numbers in the function table, the integer pairs, and then graph each pair on a grid?

Discussing the Ideas

1. Do the points for the integer pairs above form a pattern on your grid? Describe the pattern if there is one.

2. **A** Complete the function table for the rule $n + 2$. Then give the integer pairs.
   B Graph the integer pairs.
   C How does the graph for this function rule compare with the graph in the Investigation?

Function Rule $n + 2$		Integer pairs
n	$f(n)$	↓
-3	-1	(-3, [redacted])
-2	[redacted]	([redacted], [redacted])
-1	[redacted]	([redacted], [redacted])
0	[redacted]	([redacted], [redacted])
1	[redacted]	([redacted], [redacted])
2	[redacted]	([redacted], [redacted])

Copy and complete each function table. Then graph the integer pairs for each function.

1.

	n	$f(n)$
	-4	-7
A	-3	
B	-2	
C	-1	
D	0	
E	1	
F	2	
G	3	
H	4	

2.

	n	$f(n)$
	-4	7
A	-3	
B	-2	
C	-1	
D	0	
E	1	
F	2	
G	3	
H	4	

3.

	n	$f(n)$
	-4	4
A	-3	
B	-2	
C	-1	
D	0	
E	1	
F	2	
G	3	
H	4	

4.

	n	$f(n)$
	-4	16
	-3	9
A	-2	
B	-1	
C	0	
D	1	
	2	4
E	3	
F	4	

think

Find the number pairs for each exercise. Part A is done for you.

		×		Products
A	0	×		-25
B	0	×		-49
C	-8	×		16
D	8	×		16
E	-4	×		0
F	2	×		-15
G	1	×		-20
H	-20	×		100

		÷		Quotients
A	-10	÷		1
B	-9	÷		8
C	-2	÷		1
D	-4	÷		-2
E	-20	÷		1
F	8	÷		-5
G	-40	÷		7
H	-8	÷		0

REVIEWING THE IDEAS

1. Give the opposite of each integer.

A 5	C 27	E $^-365$	G $^-5806$
B $^-3$	D $^-16$	F 273	H 0

2. For each exercise, give the basic principle illustrated.

A $^-28 \cdot 1 = ^-28$

B $^-7 \cdot 1 = 1 \cdot ^-7$

C $^-12 + 12 = 0$

D $3 \cdot (8 + ^-8) = (3 \cdot 8) + (3 \cdot 8)$

E $^-7 \cdot (5 \cdot ^-2) = (^-7 \cdot 5) \cdot ^-2$

F $^-68 + 0 = ^-68$

G $^-8 + 6 = 6 + ^-8$

A Opposites	E One
B Zero	F Commutative (×)
C Commutative (+)	G Associative (×)
D Associative (+)	H Distributive

3. Find the sums.

A $9 + ^-9$	D $^-3 + ^-8$	G $^-4 + 7$
B $10 + ^-8$	E $^-8 + 5$	H $^-17 + 17$
C $^-7 + ^-5$	F $3 + ^-9$	I $^-27 + 0$

4. Find the missing addends.

A $0 + n = ^-3$	D $2 + n = ^-1$
B $n + 3 = ^-2$	E $8 + n = 3$
C $^-6 + n = ^-10$	F $^-6 + n = ^-5$

5. Find the differences.

A $0 - 3$	D $^-4 - 3$	G $5 - ^-1$
B $^-1 - 3$	E $0 - ^-1$	H $^-5 - ^-3$
C $^-2 - 3$	F $1 - ^-1$	I $^-6 - ^-10$

6. Find the products.

A $9 \cdot 8$	D $5 \cdot ^-5$	G $^-86 \cdot ^-10$
B $^-9 \cdot 4$	E $^-11 \cdot 11$	H $^-7 \cdot 20$
C $7 \cdot ^-12$	F $^-8 \cdot ^-7$	I $^-1 \cdot ^-13$

7. Find the missing factors.

A $^-2 \cdot x = ^-6$	D $^-1 \cdot p = 1$
B $7 \cdot y = ^-35$	E $^-9 \cdot b = 45$
C $^-3 \cdot n = 27$	F $17 \cdot z = ^-17$

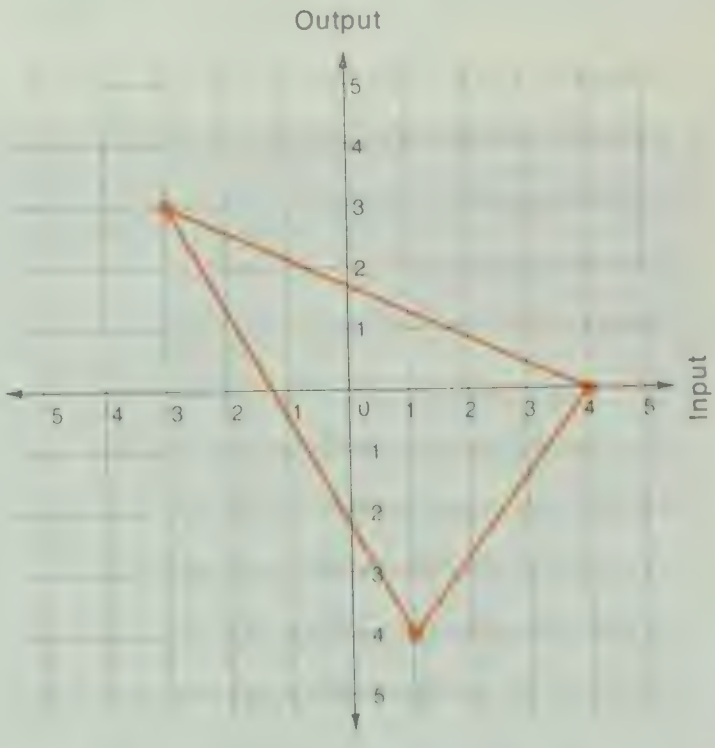
8. Find the quotients.

A $8 \div ^-2$	G $^-72 \div ^-9$
B $^-16 \div ^-4$	H $100 \div 20$
C $^-20 \div 5$	I $^-48 \div ^-6$
D $42 \div 7$	J $54 \div 6$
E $^-12 \div ^-4$	K $^-32 \div 8$
F $0 \div ^-2$	L $15 \div ^-15$

9. Give the correct symbol ($<$ or $>$) each ▒.

A $^-1$ ▒ $^-2$	F $^-21$ ▒ $^-25$
B 7 ▒ 7	G 2 ▒ 0
C 10 ▒ $^-2$	H 100 ▒ $^-1000$
D $^-2$ ▒ $^-3$	I 176 ▒ 716
E 0 ▒ $^-5$	J $^-49$ ▒ $^-50$

10. Give the co-ordinates of each vertex of the triangle.



11. A Make a function table for the rule $n + (^-4)$. Use the integers from $^-5$ through 5 as input numbers.

B Draw a graph for part A.

TEST YOURSELF

1. Give the opposite of each integer.
   A 6 B $^-9$ C 0

2. Complete the sentence:
   The sum of any integer and its opposite is ? .

3. Solve each equation. Think about the basic principles.
   A $8 + {}^-8 = n$
   B $^-23 + 18 = n + {}^-23$
   C $^-9 \cdot n = {}^-9$
   D $^-5 \cdot ({}^-4 + n) = 0$
   E $({}^-1 + 8) + {}^-3 = 1 + (n + {}^-3)$

4. Find the sums.
   A $11 + {}^-6$ B $^-6 + {}^-8$ C $^-9 + 9$

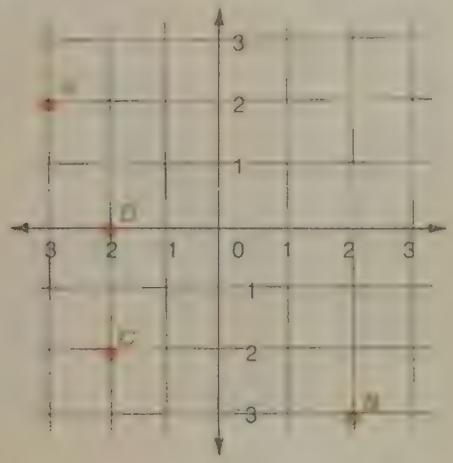
5. Find the differences.
   A $9 - {}^-5$ B $^-3 - 1$ C $^-7 - {}^-2$

6. Find the products.
   A $2 \cdot {}^-4$ B $^-6 \cdot {}^-3$ C $^-1 \cdot 1$

7. Find the quotients.
   A $^-9 \div 3$ B $^-24 \div {}^-3$ C $14 \div {}^-7$

8. Give the correct symbol ($<$ or $>$) for each ▒.
   A 3 ▒ $^-5$ B $^-9$ ▒ $^-7$

9. Give the co-ordinates of points *A*, *B*, *C*, and *D* on the graph.



10. Complete the function table.

Function Rule: $n - 2$

	n	$f(n)$
	3	1
A	2	▒
B	1	▒
C	0	▒
D	$^-1$	▒

RESEARCH PROJECTS

A There is no real number which when multiplied by itself will give a product of $^-1$. Because of this, mathematicians have invented the number i so that $i \cdot i = {}^-1$. The number i is called a **complex number**. Read about complex numbers. Find out how to add and multiply such numbers. (See *Realm of Algebra* by Issac Asimov; Boston: Houghton-Mifflin Company, 1961, pp. 151–161 available from Thomas Nelson & Sons (Canada) Ltd.)

B How long has man used negative numbers? How were they invented and what kind of symbols have been used to denote negative numbers? Read about negative numbers in an encyclopedia or in a mathematics history book. Report your findings to the class. (See *History of Mathematics* by D. E. Smith; New York: Dover Publications, 1953, Vol. II, pp. 257–260 available from General Publishing Co., Ltd.)

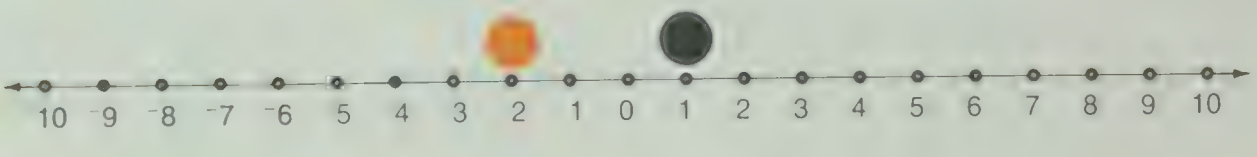
THE NUMBER LINE GAME

This is a game for two people.

Make a spinner like the one shown.

Draw a number line with points for the integers from $^{-}10$ through 10.

Each player also needs a counter or marker.



The players take turns spinning the spinner.

Starting at zero, each player moves his counter to the **right** the number of units on the spinner if the arrow lands on a positive integer. The player moves his counter to the left if the arrow lands on a negative number.

The player who reaches 10 or $^{-}10$ first, wins the game.

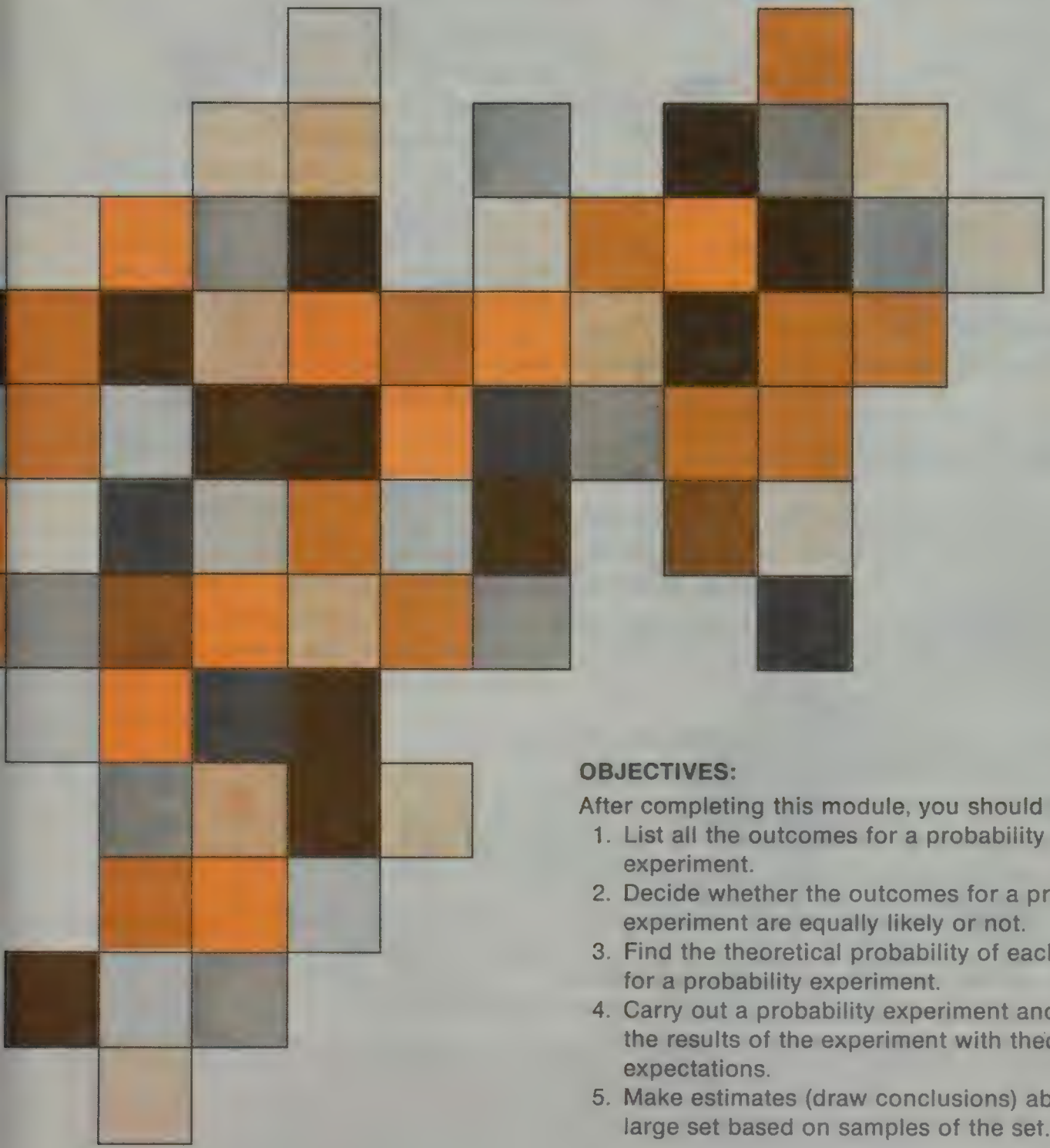
MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

1. A $^{-}6$; B 9; C 0 **2.** zero **3.** A 0; B 18; C 1; D 4; E 8; **4.** A 5; B $^{-}14$; C 0 **5.** A 14; B $^{-}4$; C $^{-}5$ **6.** A $^{-}8$; B 18; C $^{-}1$ **7.** A $^{-}3$; B 8; C $^{-}2$ **8.** A $>$; B $<$ **9.** $A:(^{-}3)$, $B:(2,^{-}3)$, $C:(^{-}2,^{-}2)$, $D:(^{-}2,0)$ **10.** A 0; B $^{-}1$; C $^{-}2$; D $^{-}3$

UNIT D: *Integers, Probability, and Statistics*

MODULE 2: *Experiments in Probability*



OBJECTIVES:

After completing this module, you should be able to:

1. List all the outcomes for a probability experiment.
2. Decide whether the outcomes for a probability experiment are equally likely or not.
3. Find the theoretical probability of each outcome for a probability experiment.
4. Carry out a probability experiment and compare the results of the experiment with theoretical expectations.
5. Make estimates (draw conclusions) about a large set based on samples of the set.

Equally Likely Outcomes

Investigating the Ideas

Try one of these experiments.

A Toss a penny 100 times
Keep a tally of heads and tails.

H	𝍸
T	𝍸 ‖

B Toss a die 100 times.
Keep a tally of the number of dots that come up on each face.

1	2	3	4	5	6
‖	|	‖|	|		‖

Does your tally show that each result occurred about the same number of times?

Discussing the Ideas

1. When each of the possible results or **outcomes** for an experiment has the same chance of occurring, the outcomes are said to be **equally likely outcomes**.
   - **A** Do you think that the outcomes for the coin experiment are equally likely? Give your reasons.
   - **B** Do you think that the outcomes for tossing a die are equally likely? Give your reasons.

2. Jack thought that since he did not get *exactly* 50 tails and 50 heads in 100 tosses, this proved that outcomes were not equally likely. What do you think?

3. Combine the results of your experiment with those of your classmates who carried out the same experiment. Discuss what the combined results show.

4. If the arrow on each spinner is spun and the region in which it stops is tallied, which spinner do you think would have equally likely outcomes?

Spinner A

Spinner B

For each experiment described below, give the following:

- **A** List all of the possible outcomes.
- **B** Tell whether the outcomes seem to be equally likely or not.
- **C** If one outcome is more likely than another, tell which one it is.

EXPERIMENT 1: Spin the arrow on the spinner. Record the number of the region in which the arrow stops.

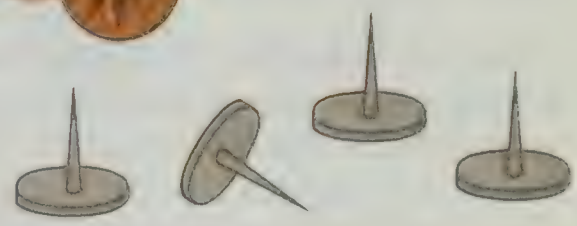
EXPERIMENT 2: Five faces of a cube have a letter **A** on them. The sixth face has the letter **B** on it. The cube is tossed and the letter on the top face is recorded.

EXPERIMENT 3: Spin the arrow on the spinner. Record the number of the region in which the arrow stops.

EXPERIMENT 4: Six red marbles and four blue marbles are placed in a box. Without looking, one marble is drawn from the box, its color recorded, and then replaced into the box.

EXPERIMENT 5: Four pennies are tossed at the same time. The number of heads and tails that come up are recorded.

★EXPERIMENT 6: Toss four thumb tacks. Record the number of tacks that land with their points up and on their side.



think

Use four 7's and any of the mathematical operation symbols of +, −, ×, ÷, and parentheses to write a numeral for 56.

Investigating the Ideas

Mark the faces of a small cube with the letters **A**, **B**, **C**, **D**, **E**, and **F**.

Mark the faces of a second cube with the numerals 1, 2, 3, 4, 5, and 6.

Every time you toss the two cubes, a letter and a numeral on the top faces is an outcome.

For example, the outcome shown of an **E** and a 4 turning up could be written as (E, 4)

How many different outcomes can you get?

Discussing the Ideas

1. **A** How many different outcomes are there that have **E** as part of the outcome?
   B How many different letters can come up on the one cube?
   C How many different numerals can come up on the other cube?

2. Can you explain how to find all the possible outcomes without actually tossing the cubes?

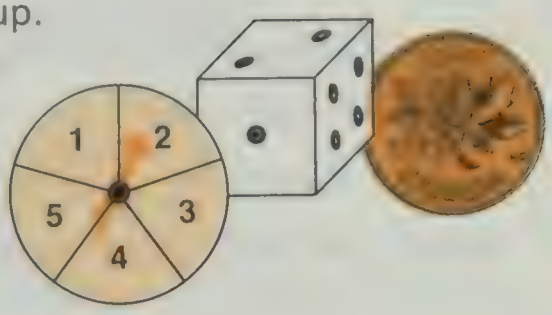
3. The set of all possible outcomes for an experiment is called a **sample space** for the experiment.
   How many outcomes are in the sample space for the Investigation?

4. A sample space for the experiment of tossing a single coin can be designated by the set $\{H, T\}$ where the H stands for heads and the T stands for tails. What is a sample space for an experiment of tossing a die?

5. A penny and a dime are tossed. A table similar to the one shown can help to show the different outcomes. The sample space for the experiment can be written as $\{(H, H), (H, T), (T, H), (T, T)\}$.
   A How many outcomes are in the sample space?
   B What does the symbol (H, T) mean?
   C What does (T, H) mean?

PENNY	DIME
H	*H*
H	*T*
T	*H*
T	*T*

1. List all the possible outcomes for each experiment.
   - **A** Toss a penny. Record which side comes up.
   - **B** Toss a die. Record the number of dots on the top face.
   - **C** Spin the arrow on a dial. Record the number of the region in which the arrow stops.

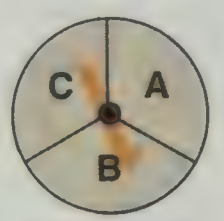


2. If a penny, a nickel, and a dime are tossed at the same time, you might want to make a table similar to the one at the right in order to find the sample space. Complete the table to show all the possible outcomes.

Penny	Nickel	Dime
H	*H*	*T*
⋮	⋮	⋮

3. One of the outcomes for spinning the arrows on the two dials like those at the right is (2, **B**). List all the other outcomes that could occur.

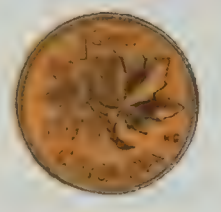
First dial

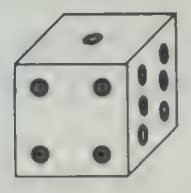


Second dial

4. An experiment consists of tossing a penny and a die at the same time. Complete the sample space.

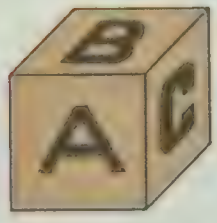
   Sample space: $\{(H, 1), (T, 1), (H, 2), \ldots\}$





5. An experiment consists of shuffling a deck of 52 playing cards and then turning up cards until an ace is turned up. The outcome is the number of cards that must be turned up until the first ace is reached (including the ace).
   - **A** The smallest outcome is obviously 1 and occurs when an ace is the first card turned up. What is the largest possible outcome?
   - **B** On the average, how many cards do you think must be turned up before you reach the first ace? Try the experiment.

Investigating the Ideas



Label 3 faces of a cube with the letter **A**;

Label 2 faces of the cube with the letter **B**.

Label 1 face of the cube with the letter **C**.

Guess how many times each one of the three letters would come up in 60 tosses of the cube.

Can you carry out the experiment of 60 tosses and compare your results with your guess?

Discussing the Ideas

1. Karen explained her guess of "30 **A**'s" for the experiment this way: "Since there are 3 *chances* in 6 of an **A** coming up, then about $\frac{3}{6}$ or $\frac{1}{2}$ of the 60 tosses should be **A**'s."
   How could she estimate the number of times the other letters should appear?

2. Since there are *3 chances in 6* that an **A** will appear on top, we say that the **probability** of getting an **A** is $\frac{3}{6}$ or $\frac{1}{2}$.
   - **A** There are *2 chances in 6* of getting a **B**, what is the **probability** of getting a **B**?
   - **B** What are the chances of getting a **C**?
   - **C** What is the probability of getting a **C**?

3. **A** What are the chances of getting a head when tossing a penny?
   - **B** What is the probability of getting a head?
   - **C** What are the chances of getting a tail?
   - **D** What is the probability of getting a tail?

4. **Equally likely outcomes have equal probabilities.**
   - **A** Since each number is equally likely for this spinner, what is the probability for each number?
   - **B** What is the sum of the probabilities for all three outcomes?

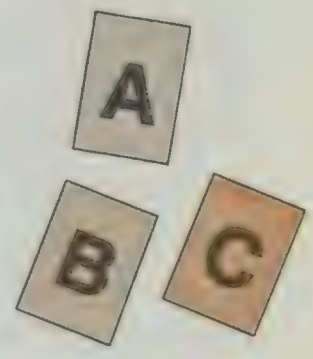
1. Two colored marbles and one black marble are placed in a box. Suppose that a blindfolded person is to draw a marble from the box.

   A There are ||| *chances* in 3 of drawing a colored marble.
   B What is the **probability** of drawing a colored marble.
   C There are ||| chances in 3 of drawing a black marble.
   D What is the probability of drawing a black marble?

2. A box contains 9 blocks all the same size. Four blocks are red, three are black, and two are white. Suppose that the blocks are mixed together and a blindfolded person selects one of the blocks from the box. What is the probability of selecting

   A a red block? B a black block? C a white block?

3. Three cards with the letters **A**, **B**, or **C** printed on one side are turned face down on a table and then their positions are thoroughly mixed. Supply the correct information to complete each sentence.



   A The probability of turning up the card with letter **A** is ? .
   B The probability of turning up a card with either **A** or **B** on it is ? .
   C If an outcome is certain to happen, then the probability of that outcome is 1. The probability of turning up a card with either **A**, **B**, or **C** on it is ? .
   D If an outcome is impossible or can never happen, then the probability of that event is zero. If 1 of the 3 cards is to be turned up, then the probability that it has the letter **Z** on it is ? .

4. A spinner is constructed as shown at the right. When the arrow (paper clip) is spun, it is equally likely to stop in any of the five regions.

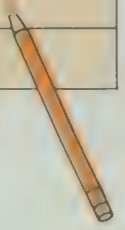
   A What is the probability that the arrow will stop on an odd-numbered region?
   B What is the probability that the arrow will stop on an even-numbered region?
   C What is the probability that the arrow will stop on a region whose number is less than 5?

PROBABILITY EXPERIMENTS

In Exercises 1 through 9, try to carry out as many of the probability experiments as possible. Then compare your **experimental results** with the results that you would *theoretically expect* based on your knowledge of probability.

1. Toss two pennies 100 times and make a tally of the results.

Outcomes	Tally
2*H*	
1*H* and 1*T*	
2*T*	



A What is the ratio of the number of times two heads appear to the number of tosses?

B What is the ratio of the number of times a head and a tail appear to the number of tosses?

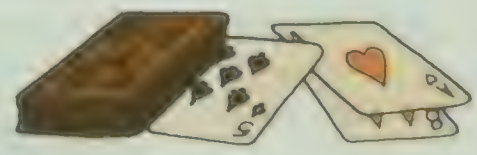
C In 100 tosses, what is the number of times that you would **expect** two tails to appear?

2. Put three black marbles and seven white marbles (all the same size) into a small sack. Without looking, draw a marble from the sack, record its color, and then replace it in the sack. Draw and replace the marbles 50 times.

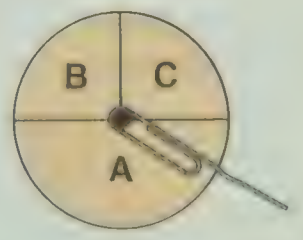
A The probability of drawing a black marble from the bag is $\frac{3}{10}$. Did you draw a black marble about $\frac{3}{10}$ of the time?

B What is the probability of drawing a white marble from the bag? Does your experimentation agree with this?

3. There are four aces in an ordinary deck of 52 playing cards. Thus, if one card is drawn from the deck, the probability that it is an ace is $\frac{4}{52}$ or $\frac{1}{13}$. Shuffle a deck of playing cards and draw a card from the deck. Repeat the experiment 50 times. Keep a record of the number of aces drawn. Did you draw an ace about $\frac{1}{13}$ of the time?



4. Make a spinner like the one shown. The probability of the arrow stopping in region A is $\frac{1}{2}$. The probability of the arrow stopping in either region B or C is $\frac{1}{4}$. Keep a tally of the outcomes for 100 spins. Find each of these ratios:



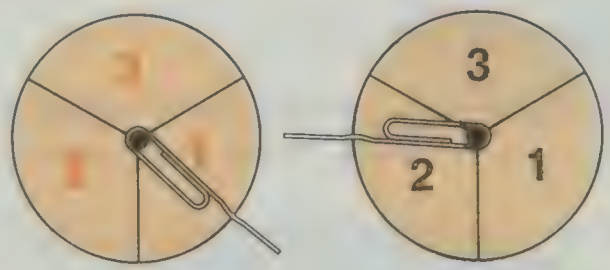
A $\frac{\text{Number of A outcomes}}{100}$ **B** $\frac{\text{Number of B outcomes}}{100}$ **C** $\frac{\text{Number of C outcomes}}{100}$

Outcomes	Probability	Tally
3*H*	$\frac{1}{8}$	
2*H*, 1*T*	$\frac{3}{8}$	
1*H*, 2*T*	$\frac{3}{8}$	
3*T*	$\frac{1}{8}$	

5. Toss three pennies at a time 48 times. Keep a tally of your results.
   - **A** Write a fraction that compares the number of times each outcome occurred to the number of tosses. Compare your result with the probability shown in the table.
   - **B** Write a fraction that compares the actual number of times that two heads and a tail appear to the number of tosses.

6. The probability of getting a sum of 7 when two dice are tossed is $\frac{1}{6}$. Toss a pair of dice 60 times and give a fraction that compares the number of times you got a sum of 7 with the number of tosses.

7. Make two spinners like those shown. Spin both arrows at the same time for 60 times. Keep a tally of the sums that you get.
   - **A** Which sum do you think is most likely to occur?
   - **B** What is the experimental probability of getting that sum?

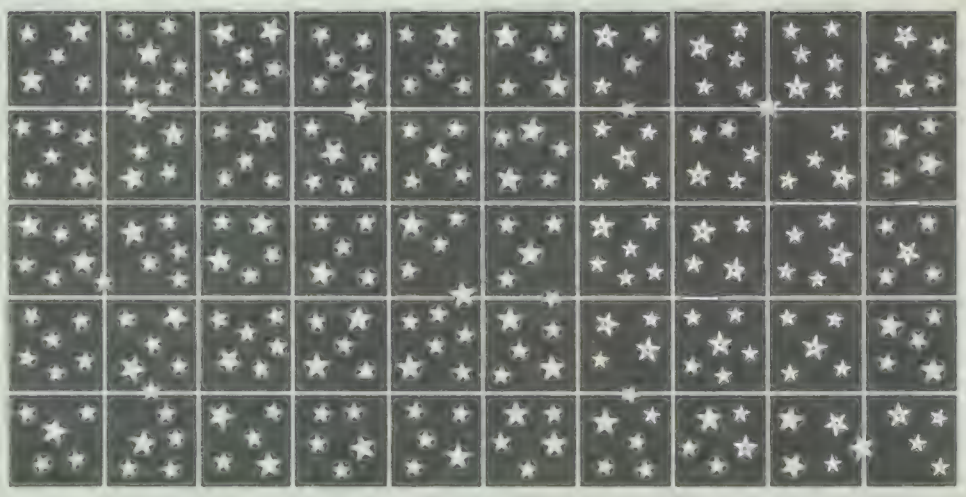


8. Drop a paper drinking cup on the floor 100 times. Keep a tally of the number of times it lands right side up, upside down, and on its side.
   - **A** Determine the experimental probability of the cup landing in each position.
   - **B** Compare your results with those of your classmates who performed the experiment.

9. Make a grid composed of 3-centimetre squares. Drop a penny on the grid from a height of about 30 centimetres. Determine the experimental probability of the coin landing **completely inside** one of the squares on the grid based on 100 drops of the coin.

Investigating the Ideas

Suppose an astronomer made this photograph of a small portion of the sky and placed a 5 by 10 centimetre grid over the photograph.



Can you estimate the number of stars shown in the photograph?

Discussing the Ideas

1. What was your estimate of the number of stars in the photograph? Explain how you made your estimate.

2. A Look at one of the centimetre squares on the photograph. How many stars are in one square?
   B The stars that are in one of the centimetre squares is a **sample** of all the stars in the photograph. How could you use the number of stars in only one of the squares to help you make an estimate of the total number of stars?

3. A Find a centimetre square on the photograph that has as few stars as you can. How many are there?
   B Find a centimetre square that you think has the most stars on it. How many are there?
   C How can these samples help you make an estimate?

4. A If someone estimated that there were 100 stars, how would you convince him that his estimate was too small?
   B If someone estimated 1000 stars, how would you convince him that his estimate was too large?

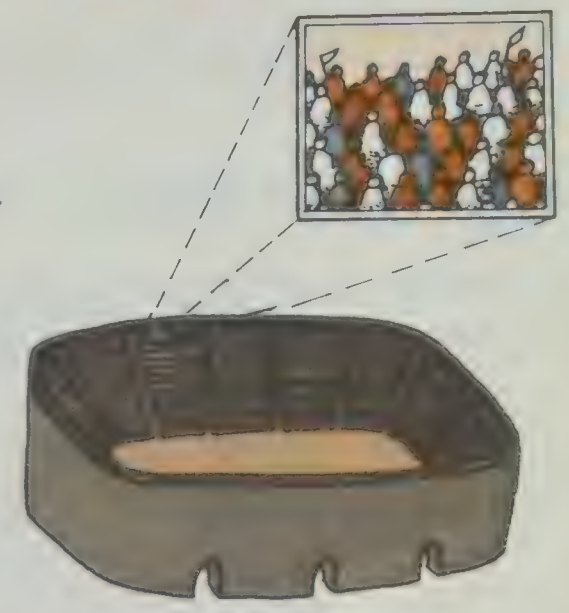
5. Choose as a sample the stars that are in one of the columns of centimetre squares. Estimate the total number of stars. How does this estimate compare with your previous estimates?

1. In order to estimate the number of beans in a plastic bag, Larry counted out a pile of 100 beans. Then he made other piles, without counting, that were about the same size as the pile of 100 beans. Altogether he had 16 piles. What was his estimate of the number of beans?

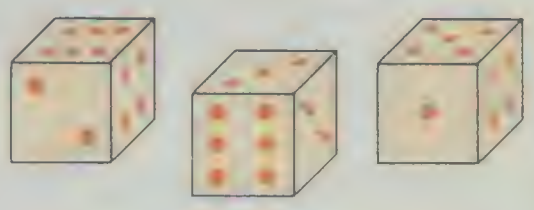
2. Marie used another method of estimating the number of beans. She counted the number of beans that would fill one mug and found that there were 254 beans. Then she emptied the beans from the plastic bag by mugfuls and found that there were $5\frac{1}{2}$ mugs of beans in the bag. What was Marie's estimate for the number of beans?

3. Jack found that in a normal walking gait, the length of his step was about 60 centimetres. Estimate the number of steps that Jack would have to take to walk a distance of one kilometre.

4. A photograph of a group of 64 spectators at a football game showed 20 women and 44 men. Estimate the number of women at the game if the attendance was 58 560 people.



5. Three dice were tossed and the sum of the top faces was recorded. In 216 tosses of the three dice, a sum of 14 was obtained 15 times. Based on this sample, about how many times would a sum of 14 occur in 1000 trials?



think

Trace square *ABCD* and point *X* on another piece of paper and cut out the square region. Make one straight cut through point *X* so that the square region is divided into two pieces that are the same size and shape.

Investigating the Ideas

? Can you choose one of the surveys below and carry it out?

A List four popular television programs. Find out which is the most popular program by surveying ten students.	**B** List the school subjects that your classmates are taking. Find the favorite subject of ten of your classmates.
C Find out what part of the students in your room wear glasses or contact lenses.	**D** Find the number of brothers and sisters for ten of your classmates.

Discussing the Ideas

1. When you conduct a survey or questionnaire of only ten students, you are using a **sample** of all the students. Do you think that the results you get depend upon which students you choose for your sample?

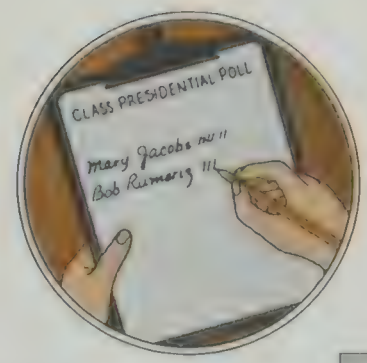
2. If you want to determine the most popular television program, should your sample include both boys and girls? Why?

3. Suppose you found out that about $\frac{1}{10}$ of the students in your room wear glasses or contact lenses. How could you use this information to estimate the number of students in your **school** that wear glasses or contact lenses?

4. Larry carried out investigation D.
   - **A** How would you interpret the results of his sample?
   - **B** What was the total number of brothers and sisters of the ten students sampled?
   - **C** What was the average number of brothers and sisters for the ten students?

Number of brothers or sisters	Student tally
0	|
1	|||
2	||||
3	|
4	|

1. **A** Based upon this poll, who should win the class presidential election?
   B If 100 students vote, estimate the number of votes Mary and Bob will each receive.



2. In a sample of 25 beads from a box of 1000 red and black beads, 15 were red and 10 were black.
   A Estimate the number of red beads in the box.
   B Estimate the number of black beads in the box.

red	𝍸 𝍸 𝍸
black	𝍸 𝍸

3. From previous samples taken, a company expects to find that 2 out of every 1000 parts manufactured will not meet manufacturing standards.
   A What is the probability that a given part will not meet manufacturing standards?
   B If the company manufactures 500 000 parts per year, about how many of these parts will not meet the standards?

4. A sample of voters polled in a province showed these percentages for each of three candidates. If there are 2 124 000 voters in the province, estimate the number of votes each of the candidates might expect to receive.

Candidate	Percentage
A	46%
B	34%
C	20%

5. Using a sample of 200 people in a certain city, it was found that 72% of the people had a television set in their homes. If there are 40 000 households in the city, about how many television sets are there in the city?

6. A light bulb manufacturer expects, from experience with many shipments (samples) of light bulbs, that about $1\frac{1}{2}\%$ of the bulbs shipped will arrive at their destination either defective or broken. About how many defective or broken bulbs would there be in a shipment of 10 000 bulbs?

REVIEWING THE IDEAS

1. Which spinner is made to have equally likely outcomes?

A

B

2. Use the letters *H* (heads) and *T* (tails) to construct a sample space for an experiment of tossing a penny and a nickel.

3. A If you toss a penny 50 times, about how many heads would you expect to turn up?
   B What is the probability of getting a head?

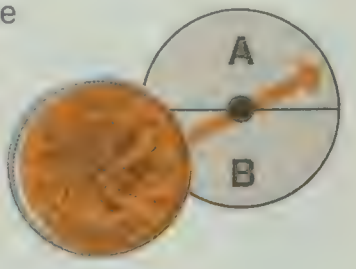
4. What is the probability of getting a head and a tail if two coins are tossed?

5. What is the probability of getting a number less than 5 if a die is tossed?

6. A dial is divided into five equal sized regions and the arrow is equally likely to stop on any of the regions. (Assume that the arrow does not stop on the lines between the regions.)

   A What is the probability that the arrow will stop on 2?
   B What is the probability that the arrow will stop on an even-numbered region?
   C What is the probability that the arrow will stop on an odd-numbered region?

7. An experiment consists of tossing a penny and spinning the arrow on a spinner, and then recording the outcomes.
   A List all possible outcomes in the sample space for the experiment.
   B What is the probability of getting the outcomes (tails, B) for the experiment?



8. Refer to the spinner at the right.

   A What is the probability of each outcome?
   B If the arrow is spun 60 times, about how many times would you expect it to stop on 3?

9. Forty students were asked to name their favorite sport.

Football	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸
Basketball	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸
Baseball	𝍸 𝍸 𝍸 𝍸 𝍸
Hockey	𝍸 𝍸 𝍸 𝍸 𝍸
Tennis	𝍸 𝍸
Golf	𝍸

   A On the basis of this sample, about how many students out of 200 would choose football?
   B About how many out of 200 would choose basketball?
   C About how many out of 200 would choose hockey?

10. In a population of 10 000 000 registered voters, a poll shows that the leading candidate in an election can expect to get 45% of the votes. About how many votes is this?

TEST YOURSELF

1. If a spinner like the one shown is used in a probability experiment, what is the sample space for the experiment?

2. Are the outcomes in the experiment for Exercise 1 equally likely outcomes?

3. Give the probability of each outcome in the sample space for the experiment in Exercise 1.

4. Which outcome is more likely to occur when two pennies are tossed?
   - [A] Two heads
   - [B] A head and a tail

5. What is the probability of getting a 7 when you toss one die?

6. Donna and Karen tossed a penny 220 times. Heads came up 117 times. How many more or less times did heads come up than they would expect from probability theory?

7. Three black marbles and two white marbles are placed in a sack. The marbles are to be drawn from the sack without looking.
   - A If a single marble is drawn, what is the probability that it will be black?
   - B If a single marble is drawn, what is the probability that it will be white?

8. The sum of the probabilities of all the outcomes in a sample space is ?.

9. A sample of cars stopping at a toll booth of a bridge showed that 1 out of every 3 cars had only one person in the car. If 24 000 cars cross the bridge each day, about how many cars have only one person?

10. Using a sample of 300 families in a certain city, it was found that 69 of them go camping during the summer. If there are 21 000 families in the city, how many go camping?

RESEARCH PROJECTS

A Find the probability of getting certain combinations of cards in bridge or poker. (See *Mathematics* by David Bergamini and the Editors of Life; New York: Time, Inc., 1963, pp. 126–147 available from GLC Educational Materials and Services, Ltd.)

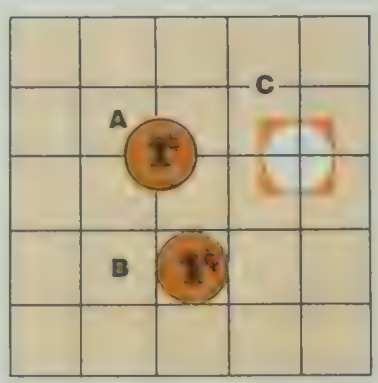
B Determine the possible outcomes of the sample space for the experiment of throwing three dice. Which sum or sums are most likely to occur when three dice are thrown? Can you find the probability of each outcome?

C What is the probability that two persons will have the same birthday (the same day of the month, not necessarily the same year)? What is the fewest number of people needed in order that the probability of two people having the same birthday is $\frac{1}{2}$? (See *Probability: A First Course* by Frederick Mosteller, Robert E. Rourke, and George B. Thomas: Don Mills, Ontario Addison-Wesley (Canada) Ltd., 1970.)

Finding a Value for π by Probability

The number π is the ratio of the circumference of any circle to its diameter and it about 3.14159 . . . . The method below uses a probability experiment to find an approximate value for π.

A Draw a grid of squares on a large sheet of paper, making the squares 1.8 centimetres on each side (the same as the diameter of a penny). If you drop a penny on the grid, it will either cover an intersection point on the grid as in Example A, or it will miss as in Example B, but the outcomes are not equally likely.



B Drop a penny on the grid 100 times and keep a record of hits and misses on the intersection points.

Hits	𝍸 𝍸 ||
Misses	𝍸 |

C Multiply the number of hits in 100 tosses by 4 and then divide this answer by the number of tosses (100). Your answer should be a rough approximation for π ($\pi = 3.14159$ . . .).

Figure C may suggest why you get an approximation for π as a result of this experiment. If the centre of the coin falls inside the white circle, the coin will touch the intersection point. If the centre falls inside the colored portion of the square, it will not touch the intersection point. Find the ratio of the area of the circle to the area of the square.

TEST YOURSELF Answers

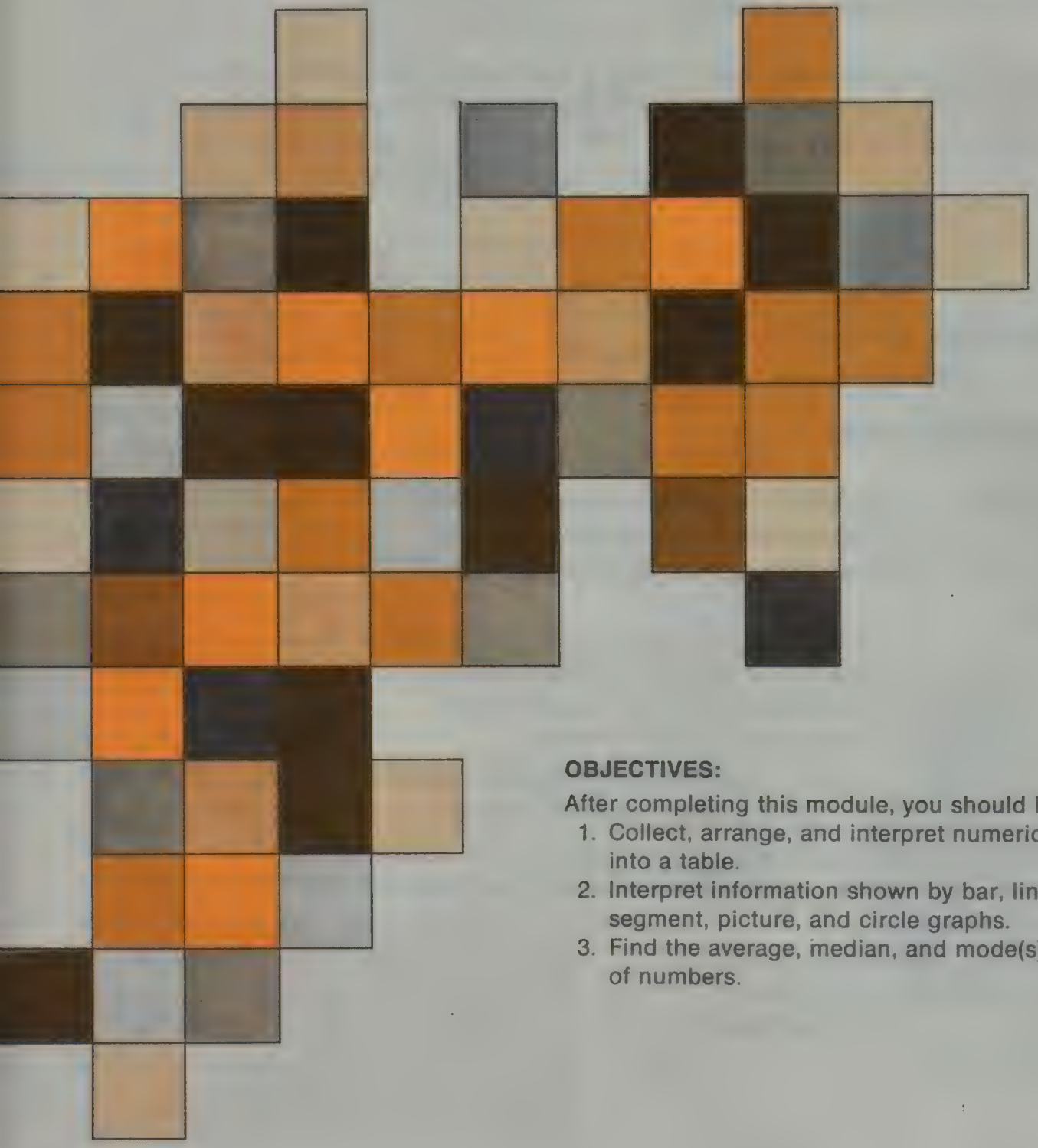
1. {A, B, C} **2.** No **3.** A: $\frac{1}{4}$; B: $\frac{1}{4}$; C: $\frac{1}{2}$ **4.** B **5.** $\frac{1}{6}$

6. Heads came up 7 more times **7.** A $\frac{3}{5}$; B $\frac{2}{5}$ **8.** 1

9. 8 000 **10.** 4830

UNIT D: *Integers, Probability, and Statistics*

MODULE 3: *Statistics*



OBJECTIVES:

After completing this module, you should be able to:

1. Collect, arrange, and interpret numerical data into a table.
2. Interpret information shown by bar, line-segment, picture, and circle graphs.
3. Find the average, median, and mode(s) of a set of numbers.

Investigating the Ideas

You may often find it necessary to use many different sources in order to gather information about a problem. Consider the following questions:

1. What is the population of each of the five largest cities in your province?

2. What are the record high and low temperatures for your city or town and for four other cities?

Can you find some information about the questions above?	Record your information.	

Discussing the Ideas

1. The science of analyzing numerical information is called **statistics**. Could two persons find different information when answering the questions above? Explain.

2. Facts about an object or a topic are called **data**. Judy copied the following data from a reference book in order to answer question 2 above.

   Calgary 36°C, 45°C London 41°C, 33°C Montreal 36°C, -34°C
   St. John's 34°C, -29°C Victoria 35°C, -19°C

   What are some ways in which Judy could have arranged the data in order to make it easier to read and interpret?

3. Frank arranged his material for question 1 in a table. Would you have arranged it differently? In what way?

CITY	POPULATION
"A"	452 524
"B"	750 879
"C"	275 425
"D"	383 818
"E"	540 025

4. After finding information for the Investigation questions above, arrange the data in a manner so that it would be easy for others to understand. What are some things that you should consider when arranging your data?

1. The melting point temperatures of some metals in °C are as follows: gold 1063°C, iron 1535°C, aluminum 649°C, copper 1083°C, lead 328°C, nickel 1453°C, tungsten 3410°C, silver 960°C. Arrange the data in a table.
   - **A** Which metal has the highest melting point?
   - **B** Which metal has the lowest melting point?
   - **C** Which metal has a melting point temperature about twice as high as lead?
   - **D** Which two metals have the closest melting-point temperatures?

2. Annual sales in dollars for some large international companies in 1972 were Exxon Corporation 22 billion, Royal Dutch Shell 14 billion, Unilever 8.9 billion, Volkswagenwerk 5.4 billion, General Motors 28.3 billion, Hitachi 4.4 billion and Nestle 4.1 billion. Arrange the data in a table.
   - **A** Which company had the largest amount of sales?
   - **B** Which company had about half of the amount of sales as Unilever?
   - **C** Which company had about four times as many sales as Volkswagenwerk?
   - **D** Which company had sales that were about seven billion less than the sales of the Exxon Corporation?

3. Find the population of the five largest cities in the world. Arrange this information in a table.

4. Collect and arrange information about these topics.
   - **A** The names and heights of the six tallest buildings in the world.
   - **B** The names and areas of the five largest fresh water lakes.
   - **C** The names and heights of the five highest mountains in the world.

5. Find the names of all the major league baseball players who have hit more than 50 home runs in one season, the number of home runs that they hit, and year in which they were hit.

think

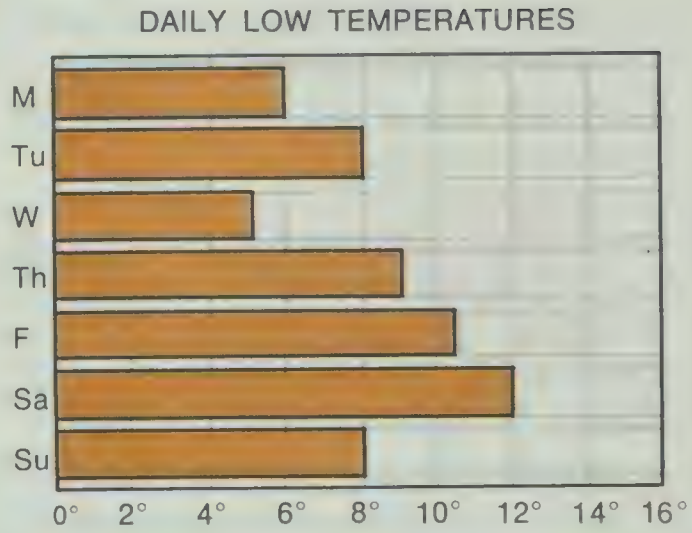
Is this a **magic square**?

4356	3267	8712
9801	5445	1089
2178	7623	6534

Investigating the Ideas

Using the information in the table, a student made a bar graph to show the daily low temperatures for each day in a certain week.

HIGH AND LOW TEMPERATURES (°C)		
Day	Low	High
Monday	6°	20°
Tuesday	8°	22°
Wednesday	5°	18°
Thursday	9°	19°
Friday	10°	22°
Saturday	12°	24°
Sunday	8°	21°



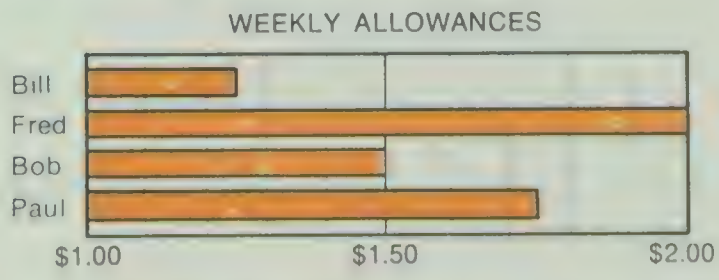
Can you make a bar graph that will show the daily high temperatures for the week?

Discussing the Ideas

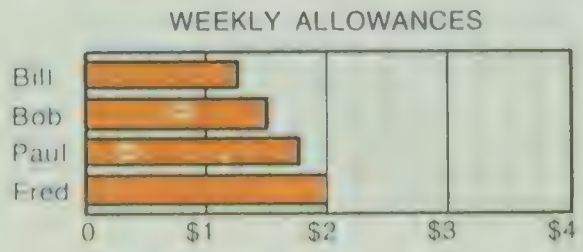
1. If you wanted to compare daily low temperatures, which would you find easier to use, the table of data or the bar graph? Explain your answer.

2. Does the bar graph show the data as accurately as the table?

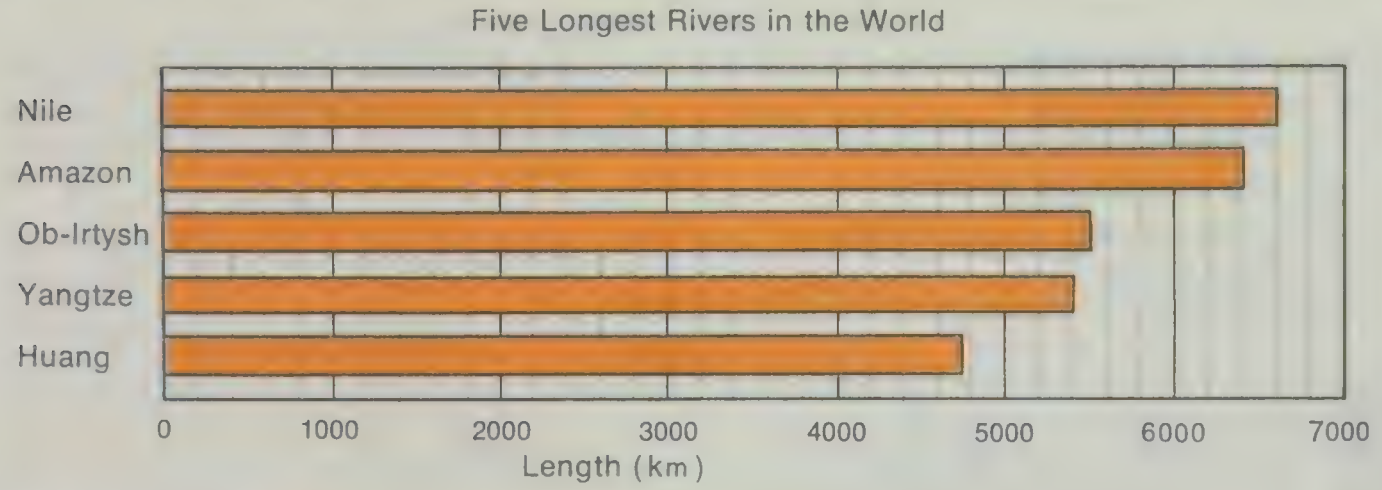
3. Bill drew this bar graph to show how his weekly allowance compared with three of his friends. Estimate each boy's allowance.



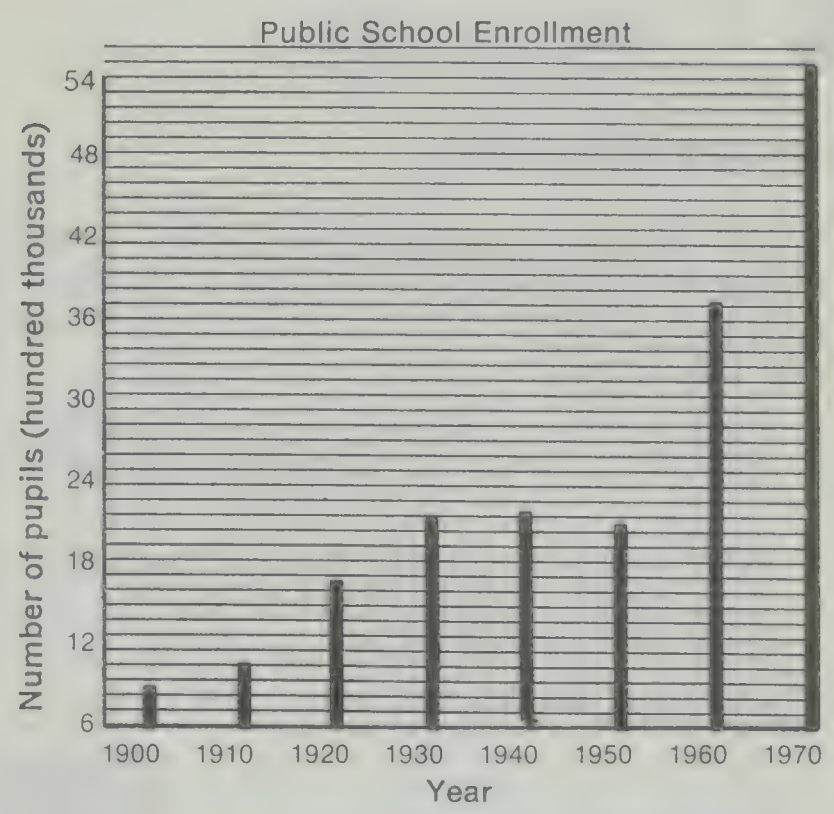
4. Bill's father said that Bill's graph was misleading. He drew a graph like this one. Which graph do you think gives the information best? Why?



1. Estimate the length of each river to the nearest hundred kilometres.



2. The bar graph at the right shows public school enrollment in Canada.
   - **A** Give the approximate school enrollment for each year shown on the graph.
   - **B** How many more pupils were enrolled in 1970 than in 1900?
   - **C** In which 10-year period did school enrollment grow most rapidly? Give the increase.
   - **D** During which years did it decrease?



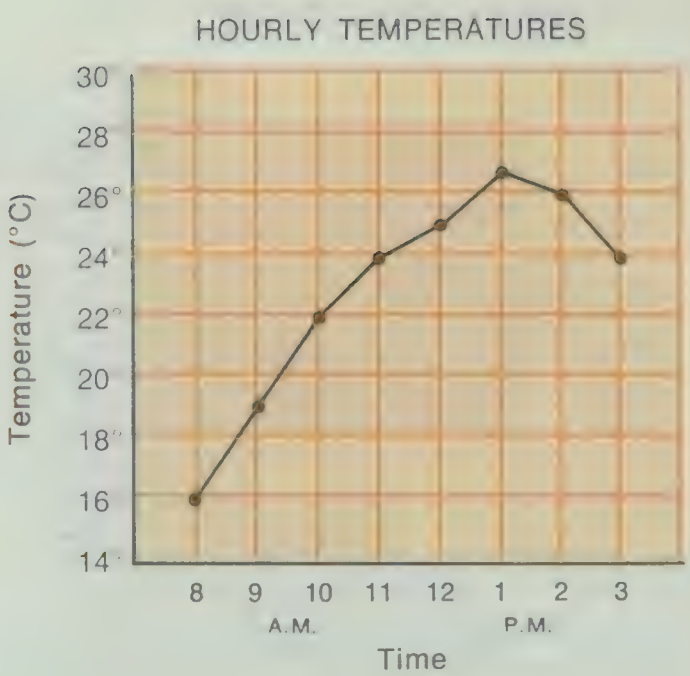
3. Construct a bar graph giving the approximate population of Canada by decades from 1900 to 1970.

Year	Approximate population (in 100 000)
1900	54
1910	72
1920	87
1930	104
1940	115
1950	141
1960	183
1970	217

4. Using the graph that you constructed for Exercise 3, make an estimate of the population of Canada for 1980 and 2000.

Investigating the Ideas

Pam kept a record of hourly temperatures for a certain school day. Then she made a graph like the one at the right in order to show her data.



Can you find the hourly temperatures for one day and draw a graph like this one to show your data?

Discussing the Ideas

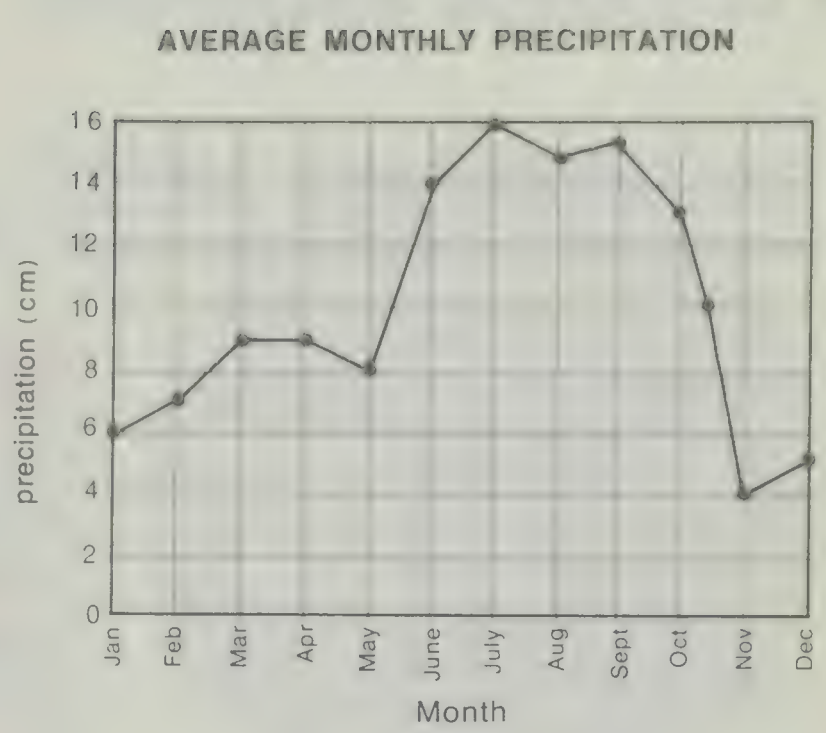
1. Graphs like the one Pam made are **line-segment graphs**. Points are marked corresponding to the time and temperatures, and then successive points are joined with line segments.
   - A Read the temperature for each hour shown on the graph.
   - B Can you estimate the temperature at 10:30 using the graph?

2. A What is the difference between the highest and the lowest temperatures shown on the graph?
   - B How can this difference help you to decide what scale that you should use on the temperature axis?

3. How would you construct a line-segment graph for the temperatures shown in the table?

Hour	Temperature (°C)
8:00	-10°
9:00	-4°
10:00	-1°
11:00	0°
12:00	2°
1:00	4°
2:00	1°
3:00	-2°

Refer to the precipitation graph for Exercises 1, 2, and 3.

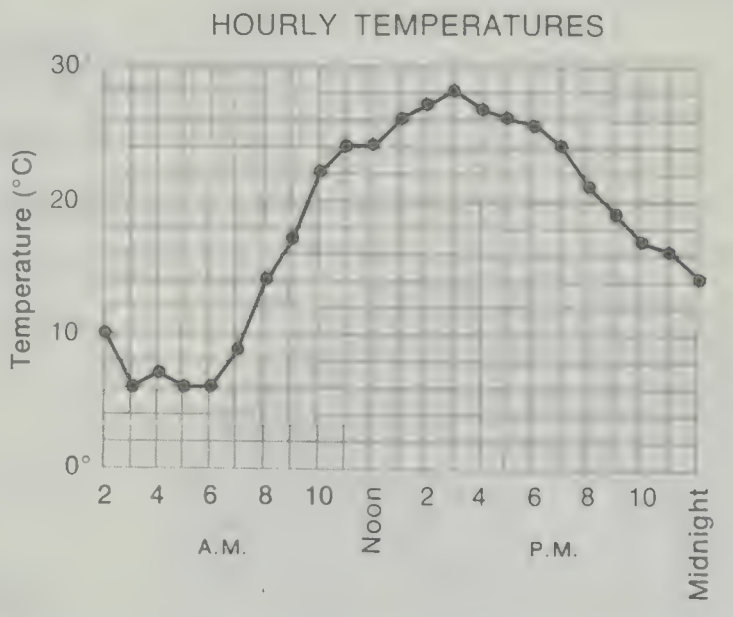


1. Give the approximate average rainfall for each month to the nearest centimetre.

2. In which months is the average rainfall
   - A less than 6 centimetres ?
   - B more than 12 centimetres ?

3. Find the approximate annual rainfall for the area graphed.

4. Refer to the hourly temperature graph at the right.
   - A What is the highest temperature on the graph? When did it occur?
   - B What is the lowest temperature? When did it occur?
   - C How much did the temperature rise between 6:00 A.M. and noon?



5. The average daily high temperatures for the month of July for a certain locality is shown in the table. Construct a line-segment graph to show the data.

Year	Average daily high (°C)	Year	Average daily high (°C)
1965	29°	1970	33°
1966	26°	1971	31°
1967	30°	1972	28°
1968	30°	1973	29°
1969	26°	1974	32°

PICTURE GRAPHS

Picture graphs are often found in newspapers, magazines, and reference books. They can help you to interpret statistical data quickly, but only show *approximate* data.

1. The pictograph at the right shows the approximate number of people that speak each of the languages listed.

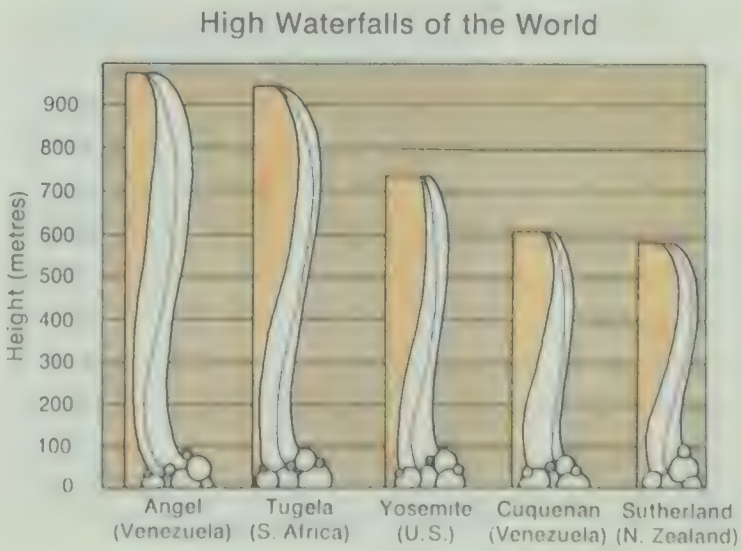
- **A** How many people does each symbol in the graph represent?
- **B** Estimate the number of people (to the nearest ten million) who speak each language.

2. The pictograph at the right shows automobile sales during 5-year periods from 1955 through 1970.

- **A** In which year was there the greatest number of sales?
- **B** In which year was there the least number of sales?
- **C** Estimate the number of sales (to the nearest million) for each year.

3. The graph at the right combines some features of a bar graph with some of a picture graph.

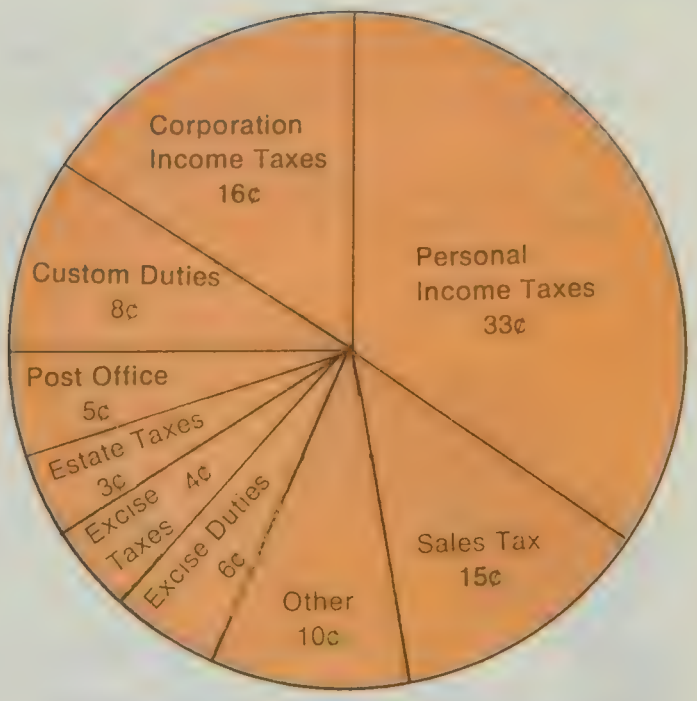
- **A** Estimate the height of each waterfall to the nearest hundred metres.
- **B** How much higher is Angel Falls than Yosemite Falls?
- **C** How much higher is Tugela Falls than Cuquenán Falls?



Circle Graphs

Circle graphs are often used to show how a given quantity is divided into fractional parts.

The graph at the right shows the sources and part of each dollar of government income. For example, 16% of the income (16¢ out of each dollar) comes from corporation income taxes.

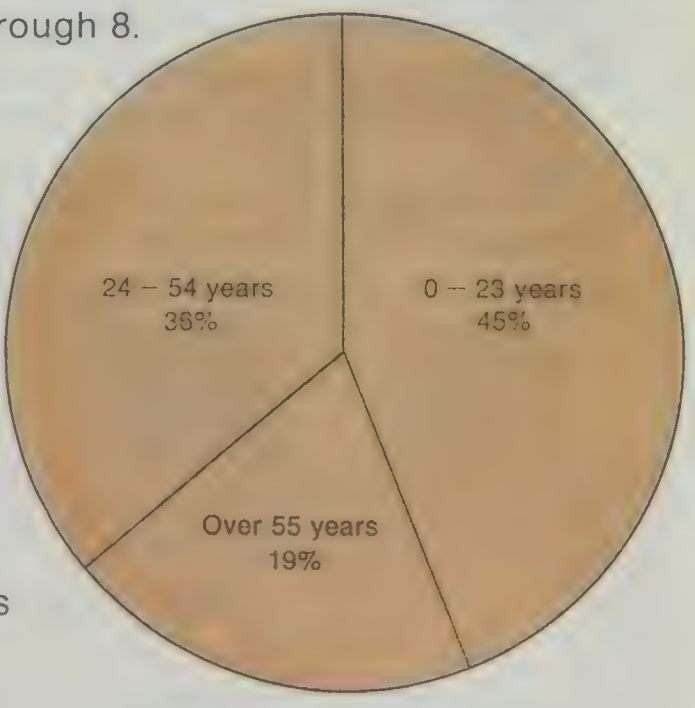


Government Income 1971–1972

1. From where does the government receive the largest part of its income?
2. What item accounts for one-third of government income?
3. The estimated total government income for 1971–1972 was $14 226 600 000. If 33¢ of each dollar came from individual income taxes, what is the total amount of income taxes?
4. What is the amount of income received from sales taxes? from other excise taxes?

Use the population graph for Exercises 5 through 8.

5. In what age range is the largest portion of the population?
6. The population of Canada in 1971 was approximately 21 569 000 people.
   - **A** About how many people were over 55 years old?
   - **B** About how many people were under 24 years old?
7. About 9% of the population is between the ages of 15 and 19. How many persons is that?
8. About $6\frac{1}{2}$% of the population is over 70 years old. About how many persons are over 70 years old?



Population Distribution by Age (1971)

Investigating the Ideas

There are 54 words in this particular newspaper clipping made up of 277 letters.

The average number of letters in each word is about 5.1 (to the nearest tenth).

$$
\begin{array}{r}
5.12 \\
54\overline{)277.00} \\
\underline{270} \\
70 \\
\underline{54} \\
160 \\
\underline{108} \\
52
\end{array}
$$

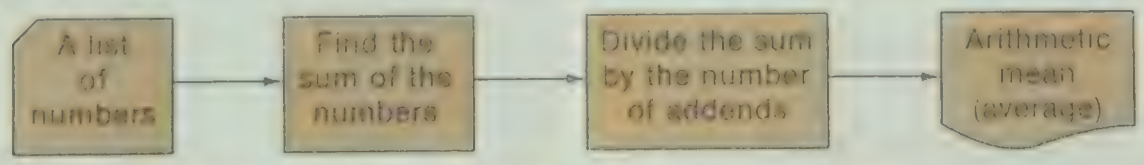
> The prime rate is the minimum interest a bank charges its top corporate customers. Floating primes are adjusted more or less automatically in line with other money market rates, while fixed rates are adjusted administratively.
>
> Short-term money market rates have been firming in recent weeks, paving the way for these prime rate boosts.

Can you select a newspaper clipping containing less than 100 words and find the average length of each word?

Discussing the Ideas

1. A What was the average number of letters per word of your newspaper clipping?
   B Compare the average that you found with that of another classmate. Are the averages about the same?

2. The **average** of a set of numbers is often called the **arithmetic mean** of the set. The flow chart below may help you review how to compute arithmetic means.



   Use the flow chart to help you find the arithmetic mean of the numbers in this set:

   {35, 47, 83, 69, 71}

3. In a long list of numbers, the smallest number is 43 and the largest number is 129. Without any other information, what could you say about the size of the arithmetic mean of this set of numbers?

1. Find the arithmetic mean of each set of numbers to the nearest tenth.
   - **A** {235, 279, 303, 369, 346}
   - **B** {1, 4, 6, 10, 15, 20, 26, 39, 46}
   - **C** {16.4, 23.9, 30.1}
   - **D** {1.2, 0.8, 2.4, 3.7, 1.6, 3.8}

2. What is the arithmetic mean of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$?

3. Bob found that the ages of six of his teachers at school were 26, 29, 59, 38, 48, and 34. What is the arithmetic mean of their ages?

4. Larry scored 83, 78, 92, and 87 on four mathematics tests. What was his average score?

5. A person has an average salary of $193 per week. What is his yearly salary?

6. Beth has 21¢, Mary has 34¢, Diane has 15¢, and Lisa has 44¢. If they put their money together and divide it equally among themselves, how much would each girl have?

7. Four starting players on a professional football team weigh 92, 100, 89, and 95 kilograms. What is the average weight of the men?

8. The arithmetic mean of a list of 32 numbers is 66.4. What is the sum of all the numbers in this list?

★ 9. Suppose the average income for four people is $100 000 per year. Three of the men earn more than $10 000 per year. What is the maximum amount of income the fourth man can earn?

think

The answers to a short True–False quiz are shown for three students. If one of the students got all three of the problems correct, and no student missed all three, find each student's score.

Arnold	Barbara	Chris
1. True	1. True	1. false
2. False	2. True	2. true
3. True	3. False	3. false

More practice, page S-31, Set 55

Investigating the Ideas

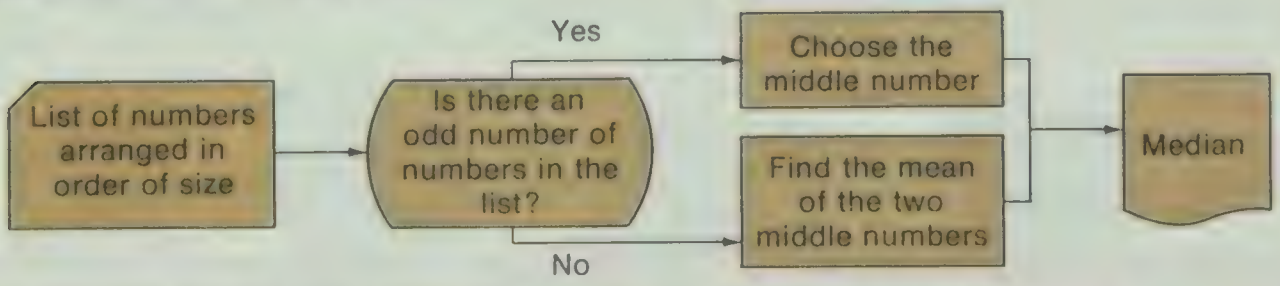
Beth A.	82	Elaine H.	80	Paul P.	77
Art B.	72	Ruth H.	80	Pat R.	86
Jim B.	69	Robert J.	83	Jerry S.	92
Ginny D.	92	Sarah J.	85	Ralph S.	84
Marie D.	96	Max K.	95	Lisa T.	78
Allen E.	88	William L.	93	Diane V.	98
Barbara F.	90	Margaret M.	81	Virginia W.	75
Dave F.	89	Susan N.	76	Jackie W.	87
Ellen G.	79	Eric O.	94	Frank Y.	90

Can you find the student that made this statement?
"Just as many students made lower scores than mine as made higher scores than mine."

Discussing the Ideas

1. The *middle* score in the list above is called the **median** score. How can you arrange the data so that the middle score is easily found?

2. When there is an even number of numbers in a list, there is no single middle number. The flow chart below shows how the median is found in any case.


List of numbers arranged in order of size
Is there an odd number of numbers in the list?
Yes
No
Choose the middle number
Find the mean of the two middle numbers
Median

Use the flow chart to determine the median of each set of numbers.

- **A** {14, 15, 17, 21, 29}
- **B** {22, 24, 25, 27, 31, 33, 34, 34, 37, 41}
- **C** {83, 86, 92, 90, 93, 93, 93, 89, 90, 97, 94, 86}

3. Do you think that the **mean** and the **median** of a set of numbers are usually about the same? Give some examples.

1. Find the median for each set of numbers.
   - **A** {27, 38, 46, 49, 52}
   - **B** {12, 12, 18, 26}
   - **C** {4, 7, 7, 8, 9, 13, 13, 14, 15}
   - **D** {274, 329, 351, 376, 382, 390}
   - **E** {19, 12, 36, 27, 41, 13, 33}
   - **F** {9, 2, 5, 3, 8, 7, 4, 6, 1, 10}

2. On a certain test, six students made these scores:
   88, 82, 94, 91, 81, 92
   What was the median score?

3. Barry had the median score of 81 in his class of 33 students. No one else in the class had a score of 81. How many students in the class had scores that were higher than Barry's score?

4. The low temperatures for each day during a week were 8°, 4°, 16°, 3°, 4°, and 15°. Find the arithmetic mean and the median of this set of temperatures.

5. The heights of a group of students are given in the table at the right. What is the median height?

Heights of Students in Metres				
1.48	1.63	1.45	1.80	1.59
1.55	1.50	1.57	1.51	1.52
1.58	1.53	1.55	1.48	1.76
1.50	1.63	1.70	1.52	1.62
1.60	1.65	1.69	1.75	1.56

★ 6. What is the median of this set if the arithmetic mean is 57?

{23, 37, ||||, 76, 100}
↑ median

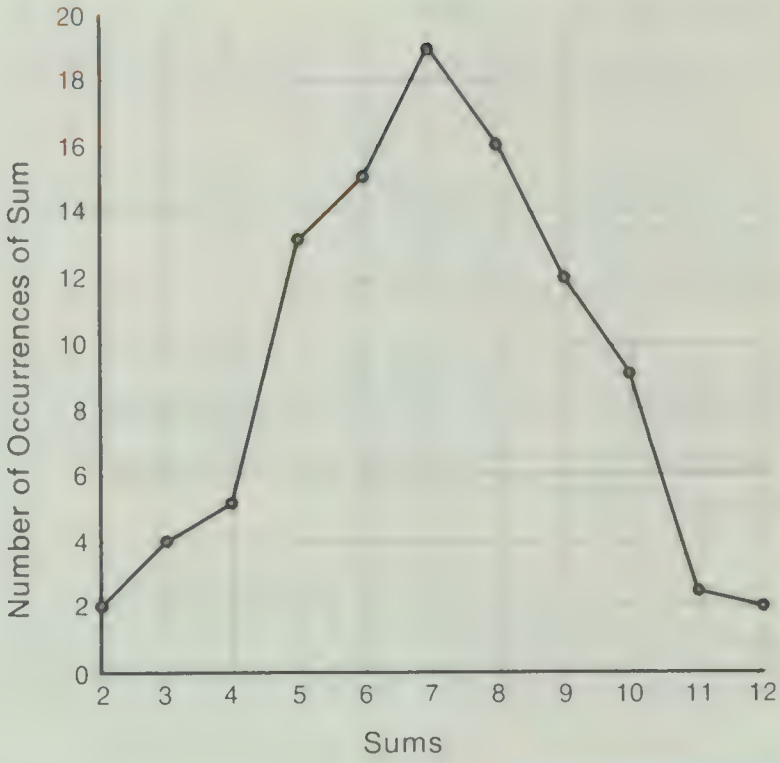
think

A traveller wishes to stay at an inn for several days. He has no money, but he has a valuable gold chain. The innkeeper agrees to let him stay on the condition that each day he pay one link of his chain. The traveller must pay every day; that is, at the end of one day, the innkeeper must have one link, at the end of two days, two links, and so forth. The traveller must cut his chain in order to make these payments. If the chain has seven links and the traveller stays for seven days, show how by cutting only one link in all he can pay for his lodging every day.

More practice, page S-32, Set 56

Investigating the Ideas

Keith and Brian worked together on a probability experiment. They tossed two dice 100 times and kept a tally of the different sums on the tosses. Then they made a graph to show the number of times each sum occurred.



Can you perform the same experiment with a classmate and draw a graph to show your results?

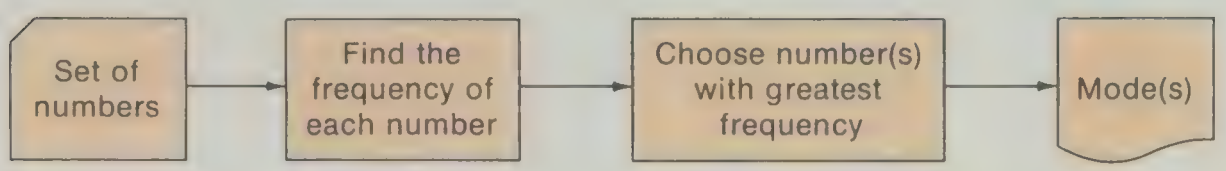
Discussing the Ideas

1. A Is the shape of your graph somewhat like the one shown in the Investigation?
   B Which sum occurred most often in your experiment?
   C Is it the same number as the one Keith and Brian found?

2. The number of times a number occurs in a set of data is called the **frequency** of the number. The graph that you made is called a **frequency distribution graph**. What is the frequency of each sum in the graph in the Investigation?

3. In a set of numerical data, the number or numbers with the largest frequency is called **mode** or **modes** of the data. The mode for the data in the Investigation is 7 because a sum of 7 occurred most frequently. What was the mode(s) for your experiment?

1. Find the mode(s) of each set of numbers. The flow chart may help you.



A

TEST SCORES		
62	79	85
68	80	87
72	80	88
73	81	88
75	83	88
77	84	88
78	85	92
78	85	94

B

STUDENT HEIGHTS (metres)				
1.58	1.60	1.62	1.64	1.66
1.59	1.60	1.62	1.64	1.68
1.59	1.61	1.63	1.64	1.70
1.60	1.61	1.63	1.65	1.71
1.60	1.62	1.63	1.65	1.72

C

NUMBER OF BROTHERS AND SISTERS				
0	1	1	2	3
0	1	1	2	4
0	1	2	2	4
0	1	2	2	5
1	1	2	3	7

2. Jane and Diane tossed four coins at a time and recorded the number of heads and tails for each toss.

Draw a frequency distribution chart for their experiment.

OUTCOME	FREQUENCY
4 heads	5
3 heads, 1 tail	24
2 heads, 2 tails	37
1 head, 3 tails	27
4 tails	7

3. The tally at the right shows the frequency of the last digits of one column of numbers from a telephone directory. What is the mode?

LAST DIGIT	TALLY
0	𝍸 ||
1	𝍸 ||
2	𝍸 |
3	𝍸 𝍸 |
4	𝍸 𝍸 𝍸 ||||
5	𝍸 𝍸 ||||
6	𝍸 𝍸 𝍸 |
7	𝍸 𝍸 |||
8	𝍸 𝍸 |||
9	𝍸 𝍸

4. Using a telephone directory, choose 100 telephone numbers and tally the last digit of the numbers.

- **A** What is the mode(s) of the last digits?
- **D** Make a frequency distribution graph of the last digits.

More practice, page S-37, Set 57

REVIEWING THE IDEAS

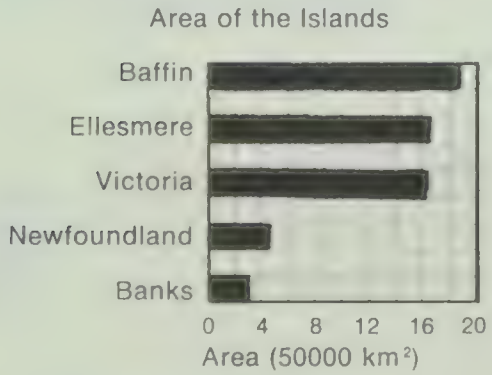
1. Use the table to answer the questions.

MOUNTAIN	HEIGHT (m)
Everest	8848
McKinley	6194
Popocatepetl	5452
Whitney	4418
Pikes Peak	4301

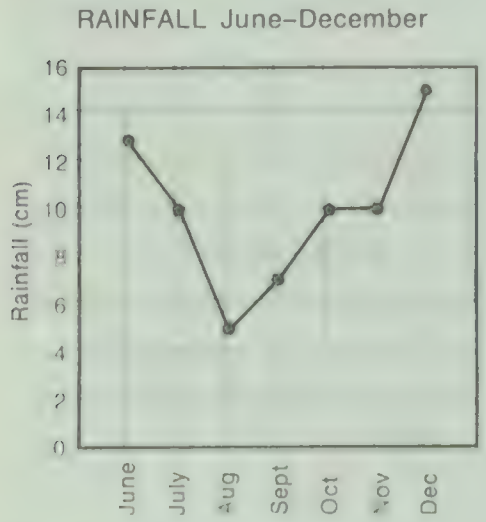
   A How much higher is Everest than McKinley?

   B Which mountain is closest to half the height of Everest?

2. Estimate the area of each island to the nearest 5000 square kilometres.

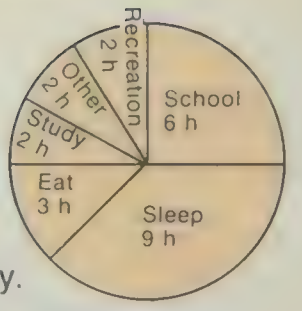


3. Use the line-segment graph to find the amount of rainfall for each month.



4. What was the average rainfall for the June-December period in Exercise 3?

5. Karen made this circle graph to show how she spent her time during a 24-hour day.



   A What fractional part of the day was spent at school?

   B What part of the day was spent sleeping?

   C What part of the day was spent eating and sleeping?

6. Carmen checked the price of a certain article in four supermarkets. The chart shows how the prices compared. To the nearest cent, what was the average cost of the article?

Price (c)
65
69
73
79

7. Find the median of each list of numbers.

   A 54, 14, 29, 38, 68

   B 74, 79, 88, 100, 68, 86

8. What is the mode of the test scores in this table?

Score	Frequency	Score	Frequency
75	1	86	7
76	0	87	7
77	1	88	5
78	3	89	6
79	4	90	4
80	3	91	3
81	5	92	2
82	6	93	1
83	7	94	2
84	9	95	1
85	8	96	1

TEST YOURSELF

Use the data in the table below for Exercises 1 through 5.

LAKE	AREA (km^2)
Superior	82 410
Huron	59 600
Michigan	58 020
Erie	25 740
Ontario	19 530

1. Which lake has the largest area?

2. How much larger is Lake Michigan than Lake Erie?

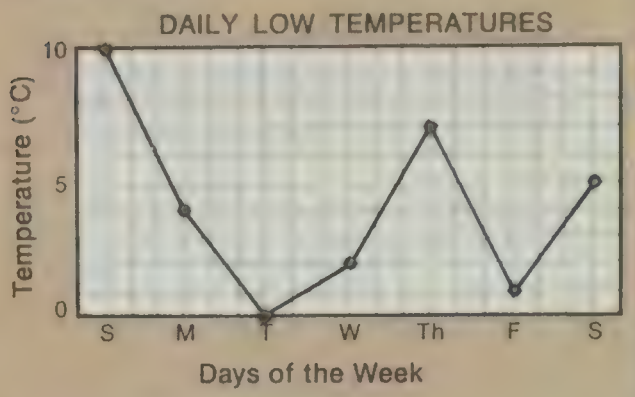
3. What is the total area of the five lakes?

4. What is the arithmetic mean of the areas?

5. What is the median area?

6. TRUE or FALSE:
   The arithmetic mean of any set of numbers is the same as the median of the set of numbers.

7. The line-segment graph shows the daily low temperatures for a week.



   A. What was the lowest temperature?
   B. On which day did the highest of the daily lows occur?

8. The mean of a list of 20 numbers is 16.4. What is the sum of all the numbers in the list?

9. The ages of ten students are 12, 12, 13, 13, 13, 14, 14, 14, 14, 15. What is the mode of their ages?

10. On a circle graph, the complete circular region represents one dollar. What part of the region would be used to represent 20 cents?

RESEARCH PROJECTS

A. The letter "e" is said to be the most frequently used letter in the English language. Choose a paragraph from a newspaper or magazine and make a frequency distribution chart or a graph of each of the letters appearing in the paragraph. What do you think is the order of the letters in the alphabet if arranged according to their frequency of use?

B. Find a book that has the decimal representation of π to several hundred decimal places. (The book *The Lore of Large Numbers* by Philip J. Davis; New Mathematical Library, Yale, 1961, contains 4000 decimal places for π.) Make a tally of the digits appearing in the decimal. Then make a frequency distribution graph to show your results. Does one digit occur more often than any others or are there several modes?

CUMULATIVE REVIEW

1. Give the opposite of each integer.

   A 6 C $^{-}29$ E 99 G 1

   B $^{-}4$ D 46 F $^{-}99$ H 0

2. Find the sums.

   A $8 + {}^{-}3$ F $^{-}7 + 2$

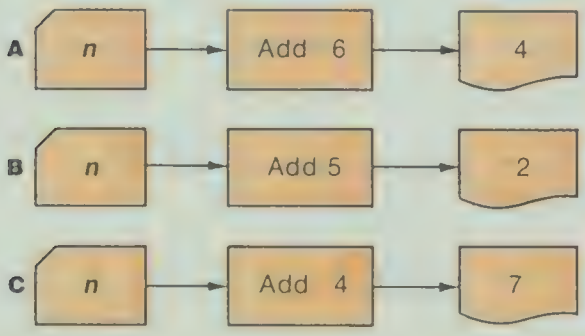
   B $^{-}2 + 9$ G $^{-}7 + 9$

   C $^{-}4 + {}^{-}5$ H $^{-}6 + 0$

   D $9 + 8$ I $^{-}9 + {}^{-}2$

   E $6 + {}^{-}1$ J $^{-}10 + {}^{-}1$

3. Give the input number for each flow chart.



4. Find the differences.

   A $0 - 8$ E $^{-}5 - 6$

   B $3 - 7$ F $9 - {}^{-}3$

   C $^{-}1 - 2$ G $^{-}4 - {}^{-}5$

   D $8 - {}^{-}1$ H $^{-}3 - {}^{-}3$

5. Complete each sentence with the words **positive** or **negative**.

   A The product of two positive integers is a ___?___ integer.

   B The product of two negative integers is a ___?___ integer.

   C The product of a positive integer and a negative integer is a ___?___ integer.

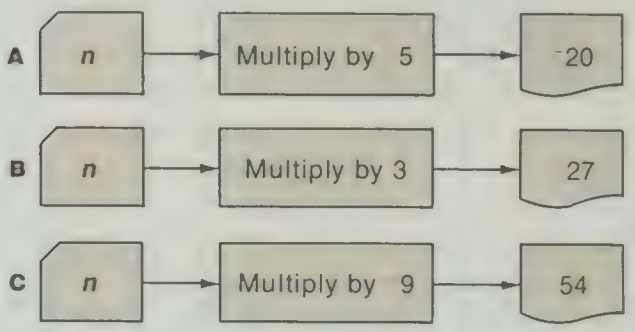
6. Find the products.

   A $8 \cdot 3$ D $3 \cdot 16$

   B $^{-}5 \cdot 2$ E $^{-}8 \cdot {}^{-}9$

   C $^{-}2 \cdot {}^{-}7$ F $^{-}1 \cdot {}^{-}1$

7. What is the input number for each flow chart?



8. Find the quotients.

   A $9 \div {}^{-}3$ D $2 \div {}^{-}2$

   B $^{-}18 \div {}^{-}6$ E $^{-}12 \div 3$

   C $^{-}20 \div 4$ F $24 \div {}^{-}8$

9. Give the correct symbol ($<$ or $>$) for each ▨.

   A $^{-}9$ ▨ 10 C 8 ▨ $^{-}8$

   B $^{-}6$ ▨ $^{-}8$ D $^{-}100$ ▨ $^{-}1$

10. A probability experiment consists of spinning two spinners like those below and recording the number pairs on the spinners.

First spinner

Second spinner

   A List the number pairs in the sample space.

   B What is the probability of getting the pair (3, 2)?

11. Two red marbles, 3 yellow marbles, and 4 white marbles are placed in a box and mixed. Without looking, a student draws a marble from the box. What is the probability that the marble drawn will be

    A red? B yellow? C white?

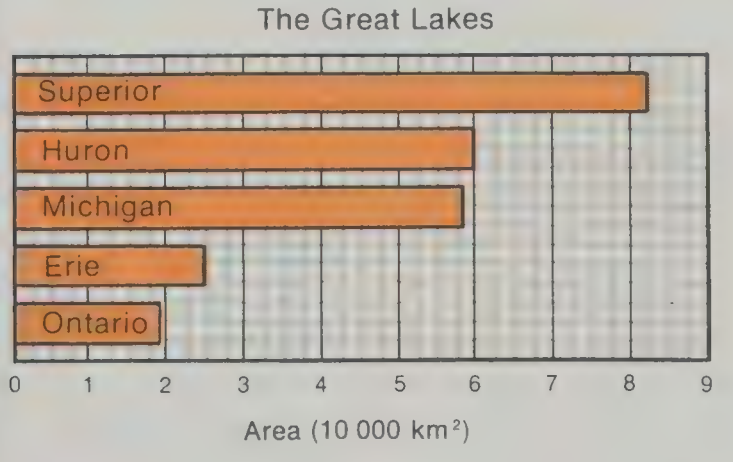
12. If one of the whole numbers from one through ten is selected at random, what is the probability that it will be larger than seven?

13. Mary Ann is going to toss a single die 1000 times as a probability experiment. Which number is the best estimate for the number of times the face with three dots should come up in the 1000 tosses?

[A] 120 [B] 170 [C] 230 [D] 300

14. A political candidate expects to get 34% of the votes of the registered voters in a certain city. If there are 108 000 registered voters in the city, how many votes can the candidate expect? (Round your answer to the nearest thousand.)

15. Use the bar graph to estimate the area of each lake to the nearest thousand square kilometres.



16. On a circle graph, the complete circular region represents one dollar. What fractional part of the region would represent $12\frac{1}{2}$ cents?

17. The price per share of a certain kind of stock for one week is given in the table below.

M	T	W	T	F
$35\frac{1}{2}$	34	$33\frac{1}{2}$	$34\frac{3}{4}$	36

Construct a line-segment graph to illustrate this information.

18. A Estimate the average (arithmetic mean) of the numbers in this set.
{28, 43, 71, 36, 58, 49, 67}

B Compute the average of the set of numbers. Round your answer to the nearest whole number.

19. John made the following scores on seven mathematics tests:
86, 94, 91, 85, 80, 96, 90

A What was his median score?

B What was his average score to the nearest whole number?

C Which is higher, his median score or his average score?

20. Karen had a bowling average of 124 pins per game. Then she bowled three more games and scored 119, 136, and 108.

A Did she bowl as much as her average in the last three games?

B How much more or less than her average did she bowl in the last three games?

21. On a mathematics test more students made a score of 82 than any other score. What is the number 82 called for this set of test scores?

THE UNLUCKY 13 GAME

There is an interesting counting game that can be challenging to everyone. This game is played with 13 counters laid in a row.

Two persons play against each other. Each one must, at his turn, take one, two, or three counters. The player who is forced to take the last counter loses the game.

Try to answer the two questions below to help you to understand the game and to develop some strategies.

1. If there are five counters left and it is your turn to play, will you win or will you lose?
2. If there are seven counters left and it is your turn to play, how many should you take in order to win the game?

Play the game with someone. Can you develop some winning strategies?

MATHEMATICAL RECREATIONS

TEST YOURSELF Answers

1. Superior
2. 32 280 km^2
3. 245 300 km^2
4. 49 060 km^2
5. 58 020 km^2
6. False
7. A 0 , B Sunday
8. 328
9. 14
10. $\frac{1}{5}$ or 20%

Supplementary Exercises

SET 1

1. Let $X = \{a, b, c, d, e\}$, $Y = \{a, c, t\}$, and $Z = \{b, a, d\}$. List the elements in each of the following sets.

A $X \cup Y$ **C** $X \cup Z$ **E** $Y \cup Z$ **G** $X \cup \phi$

B $X \cap Y$ **D** $X \cap Z$ **F** $Y \cap Z$ **H** $X \cap \phi$

2. Let $R = \{0, 2, 4, 6, 8\}$, $S = \{1, 3, 5, 7, 9\}$, and $T = \{0, 3, 6, 9\}$. List the elements in each of the following sets. Consider the sets in the parentheses first.

A $R \cup S$ **E** $S \cup T$ **I** $S \cup \phi$ **M** $(R \cap S) \cup T$

B $R \cap S$ **F** $S \cap T$ **J** $S \cap \phi$ **N** $R \cap (S \cup T)$

C $R \cup T$ **G** $R \cup R$ **K** $(R \cup S) \cup T$ **O** $(R \cup S) \cap (R \cup T)$

D $R \cap T$ **H** $R \cap R$ **L** $(R \cup S) \cap T$ **P** $(R \cup S) \cap (R \cap T)$

SET 2

1. Simplify each of the following by using 10 with an exponent.

A $10 \cdot 10 \cdot 10 \cdot 10$ **C** $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

B $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ **D** $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

2. Give the usual numeral for each power of ten.

A 10^3 **C** 10^1 **E** 10^7 **G** 10^6 **I** 10^{15} **K** 10^{12}

B 10^5 **D** 10^9 **F** 10^{10} **H** 10^0 **J** 10^{23} **L** 10^8

3. Find the products.

A $10^2 \cdot 10^2$ **C** $10^5 \cdot 10^1$ **E** $10^4 \cdot 10^4$ **G** $10^6 \cdot 10^0$

B $10^3 \cdot 10^5$ **D** $10^3 \cdot 10^3$ **F** $10^3 \cdot 10^5$ **H** $10^0 \cdot 10^0$

4. Find the quotients.

A $10^3 \div 10^1$ **C** $10^5 \div 10^3$ **E** $10^6 \div 10^1$ **G** $10^3 \div 10^0$

B $10^4 \div 10^2$ **D** $10^3 \div 10^3$ **F** $10^5 \div 10^4$ **H** $10^7 \div 10^5$

SET 3

1. Write the usual numeral for each.

A $3 \cdot 10^2$	**C** $6 \cdot 10^4$	**E** $4 \cdot 10^7$	**G** $5 \cdot 10^0$	**I** $8 \cdot 10^1$
B $8 \cdot 10^5$	**D** $9 \cdot 10^6$	**F** $2 \cdot 10^{10}$	**H** $6 \cdot 10^9$	**J** $4 \cdot 10^{13}$

2. Express each numeral in scientific notation.

A 600	**C** 4000	**E** 500 000	**G** 30 000 000
B 50	**D** 80 000	**F** 9 000 000	**H** 50 000 000 000 000

3. Use scientific notation to represent each number name.

A forty	**E** five million	**I** two hundred thousand
B three hundred	**F** eighty million	**J** four hundred million
C twenty thousand	**G** six billion	**K** nine hundred billion
D nine thousand	**H** thirty billion	**L** five trillion

SET 4

1. Give the correct symbol ($<$, $=$, or $>$) for each ▨.

A 35 ▨ 39	**E** 7146 ▨ 7064	**I** 58 755 ▨ 59 855
B 79 ▨ 97	**F** 3909 ▨ 3990	**J** 638 710 ▨ 638 075
C 349 ▨ 294	**G** 8838 ▨ 8783	**K** 5 400 500 ▨ 5 450 000
D 678 ▨ 687	**H** 1010 ▨ 1100	**L** 100 100 100 ▨ 101 000 000

2. Write the numeral for each number indicated.

A one less than one hundred	**G** 1000 less than 100 000
B 10 less than one hundred	**H** 901 less than 100 000
C 100 less than one thousand	**I** 1111 less than 10 000
D one less than one thousand	**J** 100 000 less than one million
E 1000 less than 10 000	**K** 10 001 less than one million
F 99 less than 10 000	**L** 100 001 less than one million

SET 5

1. Find the sums. Use base-eight numerals.

A $4_{(8)} + 2_{(8)}$	**B** $14_{(8)} + 2_{(8)}$	**C** $27_{(8)} + 1_{(8)}$	**D** $36_{(8)} + 2_{(8)}$	**E** $14_{(8)} + 6_{(8)}$	**F** $32_{(8)} + 15_{(8)}$
G $54_{(8)} + 14_{(8)}$	**H** $27_{(8)} + 43_{(8)}$	**I** $15_{(8)} + 25_{(8)}$	**J** $73_{(8)} + 12_{(8)}$	**K** $55_{(8)} + 23_{(8)}$	**L** $67_{(8)} + 14_{(8)}$
M $132_{(8)} + 205_{(8)}$	**N** $426_{(8)} + 133_{(8)}$	**O** $265_{(8)} + 241_{(8)}$	**P** $377_{(8)} + 301_{(8)}$	**Q** $555_{(8)} + 123_{(8)}$	**R** $547_{(8)} + 251_{(8)}$

2. Find the differences. Use base-eight numerals.

A $7_{(8)} - 3_{(8)}$	**B** $10_{(8)} - 1_{(8)}$	**C** $15_{(8)} - 4_{(8)}$	**D** $14_{(8)} - 5_{(8)}$	**E** $26_{(8)} - 12_{(8)}$	**F** $47_{(8)} - 35_{(8)}$
G $63_{(8)} - 5_{(8)}$	**H** $50_{(8)} + 7_{(8)}$	**I** $32_{(8)} - 15_{(8)}$	**J** $70_{(8)} - 54_{(8)}$	**K** $126_{(8)} - 35_{(8)}$	**L** $101_{(8)} - 46_{(8)}$

SET 6

1. Find the products. Use base-eight numerals.

A $4_{(8)} \times 3_{(8)}$	**B** $10_{(8)} \times 5_{(8)}$	**C** $23_{(8)} \times 2_{(8)}$	**D** $14_{(8)} \times 5_{(8)}$	**E** $34_{(8)} \times 2_{(8)}$	**F** $13_{(8)} \cdot 5_{(8)}$
G $40_{(8)} \times 2_{(8)}$	**H** $15_{(8)} \times 6_{(8)}$	**I** $31_{(8)} \times 4_{(8)}$	**J** $54_{(8)} \times 5_{(8)}$	**K** $103_{(8)} \times 4_{(8)}$	**L** $215_{(8)} \times 3_{(8)}$

2. Find the quotients. Use base-eight numerals.

A $6_{(8)} \div 2_{(8)}$	**C** $14_{(8)} \div 4_{(8)}$	**E** $43_{(8)} \div 5_{(8)}$	**G** $104_{(8)} \div 4_{(8)}$
B $10_{(8)} \div 2_{(8)}$	**D** $30_{(8)} \div 4_{(8)}$	**F** $61_{(8)} \div 7_{(8)}$	**H** $137_{(8)} \div 5_{(8)}$

SET 7

1. Tell what number each expression represents.

A $(3 + 4) \cdot 5$

B $3 + 4 \cdot 5$

C $30 \div 5 + 1$

D $30 \div (5 + 1)$

E $56 - (8 \div 2)$

F $(56 - 8) \div 2$

G $18 + 5 \cdot 6$

H $(18 + 5) \cdot 6$

I $2 \cdot 3 \cdot 3 + 4$

J $(2 \cdot 3) \cdot (3 + 4)$

K $2 \cdot (3 \cdot 3 + 4)$

L $2 \cdot 3 \cdot 3 + 15 \div 5$

M $24 \div 3 + 5 \cdot 2$

N $(24 \div 3 + 5) \cdot 2$

O $24 \div (3 + 5) \cdot 2$

2. Solve the equations.

A $3 \cdot n + 1 = 10$

B $15 - 2 \cdot n = 3$

C $(n \div 5) + 1 = 2$

D $6 \cdot n - 25 = 29$

E $8 \cdot (n - 1) = 40$

F $(32 \div n) \cdot 3 = 24$

G $10 \cdot n + 3 \cdot n = 26$

H $5 \cdot n - 6 \cdot 2 = 8$

I $2 \cdot 5 \cdot 3 + n = 37$

J $12 \cdot n - 12 = 0$

K $24 \div (2 + n) = 3$

L $(25 \div n) + 5 = 10$

M $(n \div 2) - 1 = 49$

N $2 \cdot (n + 3) - 6 = 0$

SET 8

1. Find the sums.

A
$$\begin{array}{r} 25 \\ +69 \\ \hline \end{array}$$

B
$$\begin{array}{r} 87 \\ +43 \\ \hline \end{array}$$

C
$$\begin{array}{r} 456 \\ +\ 99 \\ \hline \end{array}$$

D
$$\begin{array}{r} 732 \\ +861 \\ \hline \end{array}$$

E
$$\begin{array}{r} 2568 \\ +\ 432 \\ \hline \end{array}$$

F
$$\begin{array}{r} 18\,063 \\ +15\,119 \\ \hline \end{array}$$

G
$$\begin{array}{r} 26 \\ 19 \\ +33 \\ \hline \end{array}$$

H
$$\begin{array}{r} 149 \\ 73 \\ +452 \\ \hline \end{array}$$

I
$$\begin{array}{r} 2809 \\ 503 \\ +\ 45 \\ \hline \end{array}$$

J
$$\begin{array}{r} 6829 \\ 5931 \\ 7522 \\ +1099 \\ \hline \end{array}$$

K
$$\begin{array}{r} 28\,176 \\ 44\,360 \\ 50\,905 \\ +11\,111 \\ \hline \end{array}$$

L
$$\begin{array}{r} 10\,068 \\ 1\,149 \\ 658 \\ +\ 5 \\ \hline \end{array}$$

2. Find the sums.

A $58 + 79 + 103$

B $125 + 47 + 1500$

C $135 + 8 + 46$

D $2700 + 34 + 200$

E $149 + 383 + 76 + 168$

F $18\,457 + 10\,790 + 999$

G $4315 + 653 + 27$

H $965 + 966 + 967$

I $47\,973 + 140\,750$

J $488 + 97 + 23\,705$

K $6513 + 8805 + 7009$

L $7900 + 79\,000 + 790\,000$

SET 9

1. Find the differences.

A 53 − 39	**B** 87 − 74	**C** 100 − 76	**D** 235 − 47	**E** 563 − 334	**F** 905 − 833
G 4986 − 1595	**H** 5740 − 2981	**I** 8800 − 5411	**J** 7651 − 5555	**K** 4001 − 3802	**L** 9000 − 547
M 29 795 − 13 686	**N** 61 837 − 8 535	**O** 76 000 − 39 436	**P** 85 000 − 83 952	**Q** 348 966 − 149 865	**R** 743 009 − 356 075

2. Find the differences.

A 71 − 45	**D** 2000 − 1371	**G** 27 126 − 26 139
B 348 − 199	**E** 9001 − 49	**H** 408 970 − 98 790
C 1423 − 746	**F** 10 000 − 950	**I** 900 009 − 101 010

SET 10

1. Find the products.

A 7 · 10	**G** 30 · 10	**M** 20 · 60	**S** 200 · 60
B 100 · 9	**H** 300 · 10	**N** 300 · 50	**T** 500 · 20
C 6 · 1000	**I** 300 · 100	**O** 90 · 40	**U** 300 · 200
D 5 · 10 000	**J** 30 · 1000	**P** 400 · 20	**V** 800 · 400
E 100 000 · 3	**K** 300 · 1000	**Q** 50 · 70	**W** 3000 · 40
F 4 · 1 000 000	**L** 30 · 10 000	**R** 80 · 50	**X** 600 · 5000

2. Find the quotients.

A 240 ÷ 6	**G** 1500 ÷ 300	**M** 60 000 ÷ 3000	**S** 120 000 ÷ 4000
B 800 ÷ 4	**H** 4200 ÷ 600	**N** 40 000 ÷ 8000	**T** 150 000 ÷ 500
C 350 ÷ 5	**I** 4000 ÷ 50	**O** 21 000 ÷ 700	**U** 270 000 ÷ 900
D 600 ÷ 10	**J** 2800 ÷ 40	**P** 32 000 ÷ 400	**V** 480 000 ÷ 80
E 900 ÷ 30	**K** 6300 ÷ 90	**Q** 56 000 ÷ 70	**W** 100 000 ÷ 50
F 1200 ÷ 600	**L** 7200 ÷ 800	**R** 20 000 ÷ 50	**X** 8 100 000 ÷ 90

SET 11

1. Find the products.

A 38×3	**D** 158×6	**G** 853×9	**J** 9638×7
B 53×7	**E** 356×5	**H** 508×4	**K** $23\,604 \times 6$
C 78×9	**F** 740×8	**I** 945×7	**L** $430\,895 \times 3$

2. Find the products.

A 23×17	**D** 68×52	**G** 249×21	**J** 6589×35
B 49×23	**E** 97×86	**H** 809×73	**K** 8359×97
C 63×36	**F** 76×93	**I** 777×46	**L** $42\,895 \times 64$

3. Find the products.

A 217×175	**D** 340×216	**G** 508×749	**J** 1697×806
B 345×444	**E** 459×107	**H** 671×488	**K** 2304×350
C 199×675	**F** 901×901	**I** 606×999	**L** 5893×649

SET 12

1. Find the quotients.

A $3\overline{)84}$	**D** $5\overline{)270}$	**G** $8\overline{)536}$	**J** $3\overline{)885}$	**M** $7\overline{)2527}$
B $7\overline{)224}$	**E** $6\overline{)288}$	**H** $4\overline{)548}$	**K** $6\overline{)2970}$	**N** $9\overline{)6201}$
C $4\overline{)664}$	**F** $3\overline{)561}$	**I** $9\overline{)504}$	**L** $5\overline{)2670}$	**O** $8\overline{)12\,432}$

2. Find the quotients and remainders.

A $5\overline{)83}$	**D** $6\overline{)238}$	**G** $9\overline{)210}$	**J** $4\overline{)8631}$	**M** $6\overline{)5005}$
B $7\overline{)99}$	**E** $8\overline{)310}$	**H** $8\overline{)531}$	**K** $7\overline{)2444}$	**N** $8\overline{)12\,153}$
C $4\overline{)123}$	**F** $3\overline{)199}$	**I** $6\overline{)527}$	**L** $5\overline{)4301}$	**O** $9\overline{)25\,984}$

SET 13

1. Find the quotients and remainders.

A $21\overline{)473}$ **D** $69\overline{)795}$ **G** $81\overline{)6583}$ **J** $48\overline{)19\,870}$

B $17\overline{)538}$ **E** $43\overline{)1095}$ **H** $75\overline{)5738}$ **K** $89\overline{)46\,795}$

C $33\overline{)865}$ **F** $58\overline{)3710}$ **I** $92\overline{)6047}$ **L** $65\overline{)100\,008}$

2. Find the quotients and remainders.

A $191\overline{)578}$ **D** $403\overline{)2590}$ **G** $921\overline{)34\,795}$ **J** $873\overline{)413\,888}$

B $316\overline{)683}$ **E** $278\overline{)1193}$ **H** $748\overline{)15\,000}$ **K** $609\overline{)793\,502}$

C $599\overline{)1246}$ **F** $611\overline{)5034}$ **I** $542\overline{)79\,851}$ **L** $777\overline{)333\,333}$

SET 14

1. Write an equation for each flow chart.

A $n \rightarrow -16 \rightarrow 57$

B $6 \rightarrow \cdot n \rightarrow 96$

C $n \rightarrow -8 \rightarrow 13$

D $n \rightarrow \cdot 2 \rightarrow +3 \rightarrow 25$

E $n \rightarrow \div 5 \rightarrow +5 \rightarrow 15$

F $7 \rightarrow +n \rightarrow \cdot 3 \rightarrow 36$

2. Write an equation for each sentence.

A When a certain number is subtracted from 45, the difference is 31.

B The product of a certain number and 9 is 108.

C When a number is divided by 8, the quotient is 25.

D When 3 is added to 5 times a number, the result is 18.

E When a certain number is divided by 6 and then 9 is subtracted from the quotient, the result is 0.

F When the sum of a number and 8 is multiplied by 3, the result is 60.

SET 15

Write an equation or draw a diagram for each problem.
Then solve the problem.

1. The sum of a certain number and 5 is 27.

2. When 7 is subtracted from the product of a certain number and 3, the result is 35.

3. Tom is 165 centimetres tall. Tom's brother is 18 centimetres shorter than he is. How tall is Tom's brother?

4. A man travelled 371 kilometres in 7 hours. What was his average speed?

5. When 8 is added to a number and then the sum is divided by 5, the result is 10.

6. The perimeter of a triangle is 69 centimetres. If all the sides are the same length, what is the length of each side of the triangle?

7. A commuter train is travelling at 96 kilometres per hour. How far will it travel in 3 hours?

8. Susan is three years more than twice her brother's age. Her brother is 8 years old. How old is Susan?

9. A piece of ribbon was 199 centimetres long. If there is 7 centimetres left over after cutting it into 3 pieces of the same length, how long is each piece?

10. The perimeter of a rectangle is 30 metres. The length is 3 metres longer than the width. Find the length and width of the rectangle.

11. The population of Rockville is 18 less than twice the size of Emmeryton. If Rockville has 2754 people, how many people are in Emmeryton?

12. If the length of a rectangle is 5 times as long as the width and the width is 6 metres, what is the perimeter of the rectangle?

13. Jeremy paid a total of $4.52 for 3 ball point pens. If 26¢ of the total amount was sales tax, how much was each pen?

14. Karen bought several barrettes that cost 79 cents each and a comb for 63 cents. She spent $3.79 in all, how many barrettes did she buy?

SET 16

1. Copy and complete the factor trees.

A $5 \times \square \times \square$; $5 \times \square$; 45

B $2 \times \square \times \square$; $6 \times \square$; 42

C $\square \times \square \times \square \times \square$; $10 \times \square$; 40

D $\square \times \square \times \square \times \square$; $25 \times \square$; 150

E $\square \times \square \times \square \times \square$; $15 \times \square$; 60

F $\square \times \square \times \square$; $\square \times 21$; 105

G $\square \times \square \times \square \times \square$; $\square \times 6$; 54

H $\square \times \square \times \square \times \square$; $\square \times 15$; 90

I $\square \times \square \times \square \times \square$; $14 \times \square$; 210

J $\square \times \square \times \square \times \square \times \square$; $6 \times \square \times \square$; $24 \times \square$; 120

K $\square \times \square \times \square \times \square \times \square \times \square$; $4 \times \square \times \square$; $36 \times \square$; 144

2. Give the prime factorization of each number.

A 8	**E** 27	**I** 35	**M** 51	**Q** 54	**U** 150
B 15	**F** 30	**J** 39	**N** 80	**R** 72	**V** 210
C 18	**G** 31	**K** 40	**O** 90	**S** 96	**W** 500
D 24	**H** 32	**L** 41	**P** 103	**T** 100	**X** 512

SET 17

1. **A** List the factors of 15. **B** List the factors of 9.
   C List the common factors of 15 and 9.
   D What is the greatest common factor (GCF) of 15 and 9?

2. **A** List the factors of 24. **B** List the factors of 40.
   C List the common factors of 24 and 40.
   D What is the greatest common factor (GCF) of 24 and 40?

3. Find the GCF of each pair of numbers.

A 6 and 10	**F** 25 and 20	**K** 10 and 45	**P** 51 and 39
B 15 and 7	**G** 18 and 27	**L** 36 and 45	**Q** 200 and 45
C 20 and 12	**H** 48 and 36	**M** 25 and 40	**R** 80 and 200
D 15 and 18	**I** 31 and 51	**N** 75 and 100	**S** 630 and 240
E 21 and 14	**J** 40 and 15	**O** 24 and 108	**T** 121 and 70

SET 18

1. **A** List the first 8 multiples of 4. **B** List the first 8 multiples of 12.
   C List the common multiples of 4 and 12.
   D What is the least common multiple (LCM) of 4 and 12?

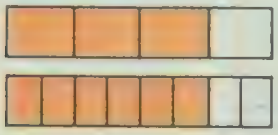
2. **A** List the first 8 multiples of 9. **B** List the first 8 multiples of 15.
   C List the common multiples of 9 and 15.
   D What is the least common multiple (LCM) of 9 and 15?

3. Find the LCM of each group of numbers.

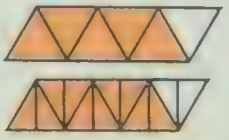
A 3 and 7	**F** 15 and 20	**K** 2, 4, and 5	**P** 3, 8, and 12
B 2 and 11	**G** 10 and 25	**L** 2, 3, and 9	**Q** 3, 12, and 16
C 5 and 15	**H** 6 and 33	**M** 2, 5, and 10	**R** 5, 8, and 16
D 9 and 12	**I** 30 and 45	**N** 3, 5, and 20	**S** 3, 10, and 15
E 8 and 10	**J** 25 and 40	**O** 2, 7, and 8	**T** 25, 30, and 75

SET 19

1. Name a pair of equivalent fractions suggested by the shading in each pair of girues.

A 

B

C 

D

E

F

2. Give the next three fractions for each set of equivalent fractions.

A $\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \ldots\right\}$

B $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \ldots\right\}$

C $\left\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \ldots\right\}$

D $\left\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21}, \ldots\right\}$

E $\left\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \ldots\right\}$

F $\left\{\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \ldots\right\}$

G $\left\{\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \ldots\right\}$

H $\left\{\frac{6}{5}, \frac{12}{10}, \frac{18}{15}, \ldots\right\}$

I $\left\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \ldots\right\}$

J $\left\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \ldots\right\}$

K $\left\{\frac{7}{9}, \frac{14}{18}, \ldots\right\}$

L $\left\{\frac{11}{4}, \frac{22}{8}, \ldots\right\}$

M $\left\{\frac{6}{1}, \frac{12}{2}, \ldots\right\}$

N $\left\{\frac{0}{1}, \frac{0}{2}, \ldots\right\}$

O $\left\{\frac{1}{25}, \frac{2}{50}, \ldots\right\}$

SET 20

1. Give the lowest-terms fraction for each set.

A $\left\{\frac{6}{12}, \frac{10}{20}, \frac{15}{30}\right\}$

B $\left\{\frac{5}{15}, \frac{6}{18}, \frac{10}{30}\right\}$

C $\left\{\frac{6}{10}, \frac{12}{20}, \frac{300}{500}\right\}$

D $\left\{\frac{70}{100}, \frac{100}{1000}, \frac{10\,000}{100\,000}\right\}$

E $\left\{\frac{3}{12}, \frac{5}{20}, \frac{15}{60}\right\}$

F $\left\{\frac{9}{30}, \frac{15}{50}, \frac{27}{90}\right\}$

G $\left\{\frac{100}{60}, \frac{20}{15}, \frac{55}{33}\right\}$

H $\left\{\frac{27}{36}, \frac{36}{48}, \frac{48}{64}\right\}$

I $\left\{\frac{18}{66}, \frac{24}{88}, \frac{33}{121}\right\}$

2. Give the lowest-terms fraction for each given fraction.

A $\frac{6}{10}$	D $\frac{9}{12}$	G $\frac{8}{20}$	J $\frac{14}{27}$	M $\frac{12}{20}$	P $\frac{15}{36}$	S $\frac{28}{42}$	V $\frac{24}{64}$
B $\frac{8}{12}$	E $\frac{6}{15}$	H $\frac{9}{21}$	K $\frac{16}{28}$	N $\frac{20}{32}$	Q $\frac{27}{36}$	T $\frac{36}{45}$	W $\frac{27}{37}$
C $\frac{6}{9}$	F $\frac{13}{18}$	I $\frac{15}{24}$	L $\frac{6}{16}$	O $\frac{21}{35}$	R $\frac{16}{40}$	U $\frac{18}{60}$	X $\frac{12}{200}$

SET 21

1. Give the correct symbol (= or ≠) for each ▨.

A $\frac{2}{3}$ ▨ $\frac{6}{9}$	**E** $\frac{1}{8}$ ▨ $\frac{5}{40}$	**I** $\frac{7}{7}$ ▨ $\frac{7}{1}$	**M** $\frac{1}{6}$ ▨ $\frac{6}{100}$
B $\frac{4}{6}$ ▨ $\frac{10}{15}$	**F** $\frac{3}{1}$ ▨ $\frac{28}{9}$	**J** $\frac{4}{20}$ ▨ $\frac{2}{8}$	**N** $\frac{0}{5}$ ▨ $\frac{0}{10}$
C $\frac{9}{20}$ ▨ $\frac{4}{5}$	**G** $\frac{4}{5}$ ▨ $\frac{36}{45}$	**K** $\frac{18}{6}$ ▨ $\frac{6}{3}$	**O** $\frac{5}{25}$ ▨ $\frac{12}{60}$
D $\frac{3}{6}$ ▨ $\frac{9}{12}$	**H** $\frac{9}{12}$ ▨ $\frac{6}{8}$	**L** $\frac{30}{50}$ ▨ $\frac{9}{15}$	**P** $\frac{49}{64}$ ▨ $\frac{7}{8}$

2. Solve the equations.

A $\frac{2}{3} = \frac{n}{9}$	**E** $\frac{n}{40} = \frac{4}{5}$	**I** $\frac{5}{8} = \frac{n}{40}$	**M** $\frac{35}{100} = \frac{7}{n}$
B $\frac{5}{6} = \frac{10}{n}$	**F** $\frac{6}{6} = \frac{n}{24}$	**J** $\frac{3}{1} = \frac{9}{n}$	**N** $\frac{25}{n} = \frac{1}{3}$
C $\frac{n}{3} = \frac{8}{12}$	**G** $\frac{n}{18} = \frac{10}{15}$	**K** $\frac{n}{20} = \frac{16}{80}$	**O** $\frac{9}{12} = \frac{45}{n}$
D $\frac{6}{7} = \frac{n}{70}$	**H** $\frac{5}{9} = \frac{5}{n}$	**L** $\frac{n}{15} = \frac{24}{90}$	**P** $\frac{n}{9} = \frac{0}{6}$

SET 22

1. Give the correct symbol (< or >) for each ▨.

A $\frac{1}{4}$ ▨ $\frac{1}{5}$	**F** $\frac{2}{3}$ ▨ $\frac{27}{30}$	**K** $\frac{1}{8}$ ▨ $\frac{1}{9}$	**P** $\frac{3}{8}$ ▨ $\frac{5}{16}$
B $\frac{4}{3}$ ▨ $\frac{5}{4}$	**G** $\frac{8}{9}$ ▨ $\frac{9}{10}$	**L** $\frac{15}{16}$ ▨ $\frac{3}{4}$	**Q** $\frac{2}{5}$ ▨ $\frac{15}{40}$
C $\frac{1}{5}$ ▨ $\frac{5}{20}$	**H** $\frac{4}{5}$ ▨ $\frac{8}{15}$	**M** $\frac{3}{10}$ ▨ $\frac{1}{3}$	**R** $\frac{2}{3}$ ▨ $\frac{3}{4}$
D $\frac{3}{4}$ ▨ $\frac{8}{12}$	**I** $\frac{7}{8}$ ▨ $\frac{6}{7}$	**N** $\frac{1}{6}$ ▨ $\frac{3}{20}$	**S** $\frac{5}{6}$ ▨ $\frac{4}{5}$
E $\frac{5}{8}$ ▨ $\frac{15}{32}$	**J** $\frac{6}{5}$ ▨ $\frac{6}{7}$	**O** $\frac{7}{12}$ ▨ $\frac{5}{9}$	**T** $\frac{13}{16}$ ▨ $\frac{9}{12}$

2. Which number in each group is the largest?

A $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$	**D** $\frac{5}{2}, \frac{3}{2}, \frac{7}{2}$	**G** $\frac{5}{4}, \frac{5}{7}, \frac{5}{12}$	**J** $\frac{2}{3}, \frac{3}{3}, \frac{4}{3}$
B $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$	**E** $\frac{4}{9}, \frac{5}{10}, \frac{6}{11}$	**H** $\frac{5}{8}, \frac{3}{4}, \frac{7}{16}$	**K** $\frac{9}{10}, \frac{10}{10}, \frac{10}{11}$
C $\frac{3}{5}, \frac{3}{7}, \frac{3}{8}$	**F** $\frac{7}{8}, \frac{3}{4}, \frac{9}{12}$	**I** $\frac{9}{4}, \frac{7}{3}, \frac{11}{5}$	**L** $\frac{3}{10}, \frac{33}{100}, \frac{1}{3}$

SET 23

1. Find the sums. Give your answers in lowest terms.

A $\frac{2}{3}+\frac{1}{6}$	D $\frac{2}{3}+\frac{2}{9}$	G $\frac{2}{7}+\frac{1}{3}$	J $\frac{5}{8}+\frac{2}{3}$	M $\frac{5}{6}+\frac{1}{5}$
B $\frac{2}{5}+\frac{3}{10}$	E $\frac{3}{4}+\frac{1}{8}$	H $\frac{5}{6}+\frac{2}{9}$	K $\frac{3}{5}+\frac{3}{7}$	N $\frac{4}{27}+\frac{1}{3}$
C $\frac{1}{3}+\frac{1}{2}$	F $\frac{2}{3}+\frac{4}{15}$	I $\frac{3}{4}+\frac{2}{3}$	L $\frac{1}{8}+\frac{4}{5}$	O $\frac{3}{10}+\frac{5}{6}$

2. Find the sums. Give your answers in lowest terms.

A $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$	D $\frac{1}{4}+\frac{3}{8}+\frac{5}{16}$	G $\frac{4}{5}+\frac{2}{3}+\frac{1}{2}$	J $\frac{1}{10}+\frac{1}{25}+\frac{1}{50}$
B $\frac{2}{3}+\frac{1}{2}+\frac{5}{6}$	E $\frac{3}{10}+\frac{1}{4}+\frac{2}{5}$	H $\frac{5}{6}+\frac{1}{3}+\frac{7}{10}$	K $\frac{7}{8}+\frac{1}{3}+\frac{5}{6}$
C $\frac{5}{8}+\frac{3}{4}+\frac{1}{2}$	F $\frac{1}{6}+\frac{5}{9}+\frac{2}{3}$	I $\frac{11}{12}+\frac{7}{8}+\frac{2}{3}$	L $\frac{4}{15}+\frac{1}{3}+\frac{6}{25}$

3. Find the differences. Give your answers in lowest terms.

A $\frac{5}{6}-\frac{1}{2}$	D $\frac{11}{12}-\frac{2}{3}$	G $\frac{17}{15}-\frac{5}{6}$	J $\frac{9}{8}-\frac{4}{5}$	M $\frac{9}{11}-\frac{2}{5}$
B $\frac{3}{4}-\frac{5}{8}$	E $\frac{6}{7}-\frac{2}{3}$	H $\frac{7}{8}-\frac{2}{3}$	K $\frac{15}{16}-\frac{2}{3}$	N $\frac{8}{9}-\frac{5}{6}$
C $\frac{7}{9}-\frac{1}{3}$	F $\frac{4}{5}-\frac{3}{4}$	I $\frac{7}{10}-\frac{1}{4}$	L $\frac{5}{7}-\frac{2}{5}$	O $\frac{11}{15}-\frac{3}{12}$

SET 24

1. Write a mixed numeral for each improper fraction.

A $\frac{4}{3}$	D $\frac{17}{3}$	G $\frac{25}{9}$	J $\frac{10}{4}$	M $\frac{14}{12}$	P $\frac{42}{20}$	S $\frac{75}{30}$	V $\frac{72}{27}$
B $\frac{8}{5}$	E $\frac{13}{2}$	H $\frac{61}{31}$	K $\frac{22}{10}$	N $\frac{35}{25}$	Q $\frac{50}{8}$	T $\frac{49}{21}$	W $\frac{150}{40}$
C $\frac{21}{10}$	F $\frac{32}{7}$	I $\frac{9}{6}$	L $\frac{15}{9}$	O $\frac{30}{12}$	R $\frac{26}{6}$	U $\frac{36}{16}$	X $\frac{210}{150}$

2. Write an improper fraction for each mixed numeral.

A $1\frac{1}{7}$	D $4\frac{1}{10}$	G $8\frac{5}{6}$	J $10\frac{5}{6}$	M $57\frac{1}{2}$	P $12\frac{3}{8}$	S $8\frac{3}{20}$	V $24\frac{1}{3}$
B $1\frac{3}{5}$	E $3\frac{2}{3}$	H $3\frac{5}{12}$	K $25\frac{1}{4}$	N $33\frac{1}{3}$	Q $19\frac{9}{10}$	T $3\frac{15}{16}$	W $1\frac{43}{45}$
C $2\frac{1}{3}$	F $4\frac{1}{2}$	I $16\frac{2}{3}$	L $11\frac{1}{9}$	O $6\frac{4}{15}$	R $3\frac{8}{25}$	U $47\frac{3}{4}$	X $8\frac{13}{19}$

SET 25

1. Find the sums.

A $1\frac{1}{2} + \frac{2}{3}$	**B** $5\frac{1}{3} + 3\frac{1}{4}$	**C** $2\frac{7}{10} + 5\frac{1}{2}$	**D** $8\frac{5}{6} + 1\frac{1}{12}$	**E** $4\frac{2}{9} + 1\frac{5}{6}$	**F** $6\frac{4}{5} + 3\frac{2}{3}$
G $8\frac{1}{6} + 3\frac{3}{4}$	**H** $12\frac{4}{7} + 2\frac{1}{3}$	**I** $23\frac{1}{8} + 7\frac{2}{3}$	**J** $17\frac{5}{6} + 4\frac{3}{5}$	**K** $31\frac{4}{9} + 8\frac{3}{4}$	**L** $65\frac{3}{20} + 13\frac{5}{6}$

2. Find the differences.

A $3\frac{1}{2} - 1\frac{1}{6}$	**B** $8\frac{3}{4} - 5\frac{1}{3}$	**C** $9\frac{1}{2} - 1\frac{7}{8}$	**D** $4\frac{1}{3} - 3\frac{4}{5}$	**E** $12\frac{4}{9} - 8\frac{5}{6}$	**F** $11\frac{3}{10} - 3\frac{3}{4}$
G $27\frac{1}{8} - 19\frac{1}{3}$	**H** $73\frac{1}{3} - 50\frac{5}{7}$	**I** $9\frac{1}{100} - 4\frac{1}{10}$	**J** $33\frac{3}{8} - 13\frac{5}{6}$	**K** $173\frac{5}{8} - 41\frac{4}{5}$	**L** $457\frac{1}{7} - 125\frac{1}{6}$

SET 26

1. Find the products.

A $\frac{1}{2} \cdot \frac{1}{4}$	**D** $\frac{1}{3} \cdot \frac{1}{5}$	**G** $\frac{1}{5} \cdot \frac{1}{6}$	**J** $\frac{1}{4} \cdot \frac{1}{6}$	**M** $\frac{1}{100} \cdot \frac{1}{4}$
B $\frac{1}{4} \cdot \frac{1}{3}$	**E** $\frac{1}{6} \cdot \frac{1}{2}$	**H** $\frac{1}{10} \cdot \frac{1}{7}$	**K** $\frac{1}{2} \cdot \frac{1}{8}$	**N** $\frac{1}{30} \cdot \frac{1}{2}$
C $\frac{1}{2} \cdot \frac{1}{2}$	**F** $\frac{1}{4} \cdot \frac{1}{4}$	**I** $\frac{1}{2} \cdot \frac{1}{5}$	**L** $\frac{1}{10} \cdot \frac{1}{5}$	**O** $\frac{1}{60} \cdot \frac{1}{50}$

2. Find the products. Give your answers in lowest-terms.

A $2 \cdot \frac{1}{3}$	**F** $6 \cdot \frac{1}{4}$	**K** $5 \cdot \frac{1}{5}$	**P** $36 \cdot \frac{1}{16}$
B $4 \cdot \frac{1}{7}$	**G** $8 \cdot \frac{1}{12}$	**L** $32 \cdot \frac{1}{12}$	**Q** $\frac{1}{5}$ of 10
C $6 \cdot \frac{1}{5}$	**H** $10 \cdot \frac{1}{5}$	**M** $0 \cdot \frac{1}{12}$	**R** $\frac{1}{3}$ of 25
D $9 \cdot \frac{1}{2}$	**I** $15 \cdot \frac{1}{9}$	**N** $42 \cdot \frac{1}{9}$	**S** $\frac{1}{4}$ of 30
E $5 \cdot \frac{1}{10}$	**J** $1 \cdot \frac{1}{16}$	**O** $100 \cdot \frac{1}{4}$	**T** $\frac{1}{10}$ of 46

SET 27

1. Find the products. Give your answers in lowest terms.

A $\frac{1}{2} \cdot \frac{3}{5}$	**E** $\frac{5}{9} \cdot \frac{3}{5}$	**I** $\frac{5}{8} \cdot \frac{2}{3}$	**M** $\frac{8}{9} \cdot \frac{15}{16}$	**Q** $\frac{5}{16} \cdot \frac{3}{20}$
B $\frac{3}{4} \cdot \frac{3}{4}$	**F** $\frac{4}{5} \cdot \frac{3}{8}$	**J** $\frac{6}{7} \cdot \frac{3}{20}$	**N** $\frac{6}{6} \cdot \frac{17}{21}$	**R** $\frac{13}{27} \cdot \frac{9}{10}$
C $\frac{5}{6} \cdot \frac{5}{7}$	**G** $\frac{2}{7} \cdot \frac{5}{4}$	**K** $\frac{4}{3} \cdot \frac{3}{8}$	**O** $\frac{9}{10} \cdot \frac{25}{36}$	**S** $\frac{12}{35} \cdot \frac{7}{24}$
D $\frac{3}{10} \cdot \frac{7}{8}$	**H** $\frac{4}{15} \cdot \frac{3}{8}$	**L** $\frac{6}{11} \cdot \frac{7}{8}$	**P** $\frac{24}{25} \cdot \frac{5}{6}$	**T** $\frac{48}{49} \cdot \frac{7}{16}$

2. Find the products. Give your answers in lowest terms.

A $4 \cdot \frac{2}{3}$	**E** $4\frac{1}{2} \cdot 6$	**I** $6\frac{2}{5} \cdot \frac{3}{4}$	**M** $1\frac{1}{2} \cdot 3\frac{1}{3}$	**Q** $6\frac{2}{3} \cdot 1\frac{7}{10}$
B $\frac{5}{6} \cdot 10$	**F** $8 \cdot 7\frac{3}{4}$	**J** $4\frac{3}{10} \cdot \frac{5}{8}$	**N** $2\frac{1}{5} \cdot 1\frac{1}{4}$	**R** $3\frac{3}{5} \cdot 8\frac{1}{3}$
C $1\frac{1}{2} \cdot 7$	**G** $\frac{4}{9} \cdot 1\frac{1}{8}$	**K** $\frac{4}{15} \cdot 2\frac{2}{9}$	**O** $1\frac{2}{3} \cdot 8\frac{1}{2}$	**S** $4\frac{4}{9} \cdot 6\frac{3}{10}$
D $3 \cdot 1\frac{5}{6}$	**H** $3\frac{1}{3} \cdot \frac{2}{5}$	**L** $7\frac{1}{5} \cdot \frac{7}{12}$	**P** $4\frac{1}{5} \cdot 1\frac{5}{6}$	**T** $12\frac{3}{4} \cdot 1\frac{11}{21}$

SET 28

1. Find the products. Think of the distributive principle.

A $8\frac{1}{2}$ $\times 4$	**B** $3\frac{1}{3}$ $\times 6$	**C** $7\frac{1}{10}$ $\times 5$	**D** 18 $\times 1\frac{1}{3}$	**E** 4 $\times 3\frac{1}{8}$
F $15\frac{1}{2}$ $\times 2\frac{1}{3}$	**G** $24\frac{1}{5}$ $\times 10\frac{1}{2}$	**H** $9\frac{4}{5}$ $\times 5\frac{1}{3}$	**I** $35\frac{3}{4}$ $\times 8\frac{1}{5}$	**J** 45 $\times 8\frac{4}{9}$

2. Find the products using the distributive principle.

A $3 \cdot 7\frac{1}{3}$	**E** $5 \cdot 8\frac{1}{15}$	**I** $\frac{1}{12} \cdot 60\frac{1}{2}$	**M** $\frac{1}{2} \cdot 4\frac{3}{5}$	**Q** $\frac{2}{3} \cdot 33\frac{2}{3}$
B $\frac{1}{4} \cdot 8\frac{1}{2}$	**F** $8 \cdot 3\frac{1}{16}$	**J** $\frac{2}{3} \cdot 9\frac{1}{2}$	**N** $\frac{3}{10} \cdot 20\frac{1}{2}$	**R** $18\frac{1}{7} \cdot \frac{1}{6}$
C $20 \cdot 6\frac{1}{4}$	**G** $\frac{1}{6} \cdot 6\frac{2}{7}$	**K** $45 \cdot 2\frac{1}{9}$	**O** $4 \cdot 5\frac{1}{16}$	**S** $150 \cdot 3\frac{1}{10}$
D $21 \cdot 3\frac{1}{3}$	**H** $\frac{1}{2} \cdot 100\frac{1}{5}$	**L** $6 \cdot 8\frac{5}{12}$	**P** $\frac{2}{7} \cdot 35\frac{1}{4}$	**T** $\frac{4}{9} \cdot 90\frac{3}{4}$

SET 29

1. Find the quotients.

A $\frac{1}{3} \div \frac{2}{5}$ **E** $6 \div \frac{9}{10}$ **I** $\frac{7}{8} \div \frac{7}{16}$ **M** $\frac{7}{12} \div 3$ **Q** $\frac{2}{27} \div \frac{10}{27}$

B $\frac{3}{4} \div \frac{3}{4}$ **F** $\frac{3}{4} \div \frac{1}{2}$ **J** $\frac{9}{20} \div \frac{9}{10}$ **N** $\frac{4}{15} \div \frac{4}{5}$ **R** $\frac{4}{25} \div \frac{6}{5}$

C $\frac{2}{3} \div \frac{9}{6}$ **G** $\frac{4}{9} \div \frac{2}{3}$ **K** $4 \div \frac{1}{5}$ **O** $\frac{13}{16} \div \frac{1}{32}$ **S** $\frac{4}{9} \div \frac{8}{27}$

D $10 \div \frac{5}{12}$ **H** $\frac{6}{7} \div \frac{7}{6}$ **L** $\frac{8}{9} \div 4$ **P** $\frac{15}{32} \div \frac{5}{32}$ **T** $\frac{5}{36} \div 15$

2. Find the quotients.

A $1\frac{1}{4} \div \frac{2}{3}$ **E** $6\frac{2}{3} \div 5$ **I** $1 \div 4\frac{5}{6}$ **M** $4\frac{1}{2} \div 18$ **Q** $35 \div 3\frac{3}{4}$

B $2\frac{1}{2} \div 3$ **F** $3\frac{1}{3} \div \frac{3}{10}$ **J** $1\frac{1}{3} \div 1\frac{1}{2}$ **N** $10 \div 8\frac{3}{5}$ **R** $3\frac{1}{3} \div 16\frac{2}{3}$

C $8\frac{2}{5} \div 6$ **G** $3 \div 1\frac{1}{5}$ **K** $4\frac{1}{2} \div 1\frac{7}{8}$ **O** $4\frac{7}{8} \div 4\frac{7}{8}$ **S** $3\frac{9}{10} \div 4\frac{1}{3}$

D $4\frac{1}{6} \div \frac{1}{3}$ **H** $25 \div 8\frac{1}{3}$ **L** $1\frac{5}{6} \div 3\frac{1}{2}$ **P** $9\frac{1}{5} \div 6\frac{3}{5}$ **T** $6\frac{7}{8} \div 9\frac{2}{7}$

SET 30

1. Write each complex fraction as a quotient of two fractional numbers.

A $\dfrac{4}{\frac{3}{5}}$ **B** $\dfrac{\frac{5}{6}}{10}$ **C** $\dfrac{\frac{4}{3}}{\frac{1}{2}}$ **D** $\dfrac{\frac{1}{2}}{3\frac{1}{2}}$ **E** $\dfrac{4\frac{1}{3}}{\frac{9}{10}}$ **F** $\dfrac{21\frac{1}{3}}{8\frac{7}{10}}$

G $\dfrac{1\frac{3}{5}}{6}$ **H** $\dfrac{8}{2\frac{1}{2}}$ **I** $\dfrac{5\frac{1}{3}}{\frac{1}{4}}$ **J** $\dfrac{10\frac{2}{3}}{4\frac{7}{8}}$ **K** $\dfrac{1}{\frac{2}{5}}$ **L** $\dfrac{6\frac{3}{5}}{\frac{1}{100}}$

2. Simplify each complex fraction.

A $\dfrac{\frac{1}{2}}{4}$ **B** $\dfrac{2}{\frac{3}{4}}$ **C** $\dfrac{\frac{1}{8}}{\frac{3}{4}}$ **D** $\dfrac{\frac{5}{6}}{\frac{1}{2}}$ **E** $\dfrac{2}{\frac{4}{9}}$ **F** $\dfrac{\frac{3}{10}}{\frac{5}{6}}$

G $\dfrac{1\frac{1}{2}}{8}$ **H** $\dfrac{1\frac{4}{15}}{\frac{1}{3}}$ **I** $\dfrac{8\frac{1}{6}}{7}$ **J** $\dfrac{9\frac{3}{4}}{\frac{3}{8}}$ **K** $\dfrac{4\frac{1}{3}}{1\frac{2}{3}}$ **L** $\dfrac{7\frac{3}{5}}{3\frac{4}{5}}$

SET 31

1. Find the sums.

A $4.36 + 1.53$

B $23.7 + 16.5$

C $0.65 + 0.39$

D $1.376 + 0.881$

E $1.5 + 2.3 + 9.4$

F $8.71 + 14.3 + 0.9$

G $13.7 + 1.09 + 8$

H $0.9 + 0.16 + 0.027$

I $854.1 + 3.46 + 14.007$

J $4.93 + 0.667 + 12.359$

K $235 + 18.7 + 0.46$

L $6.009 + 21.73 + 4.654$

M $0.83 + 0.965 + 0.3566$

N $5008 + 47.03 + 751.9$

O $90.2 + 9.02 + 0.902$

P $6793 + 4.788 + 54.17$

Q $5432 + 543.2 + 5.432$

R $0.909 + 0.009 + 0.999$

2. Find the differences.

A $3.78 - 1.45$

B $73.9 - 59.3$

C $4.35 - 1.99$

D $0.981 - 0.347$

E $3.25 - 2.8$

F $18.6 - 3.33$

G $21.7 - 6.45$

H $5 - 1.3$

I $0.09 - 0.031$

J $10 - 4.862$

K $45.1 - 0.394$

L $8.03 - 4.009$

M $155 - 79.6$

N $99 - 0.9$

O $10 - 0.91$

P $100 - 98.375$

Q $25 - 0.015$

R $9 - 0.0009$

S $40 - 0.001$

T $1000 - 0.001$

SET 32

1. Find the products.

A 3×0.9

B 0.5×0.7

C 0.03×0.6

D 0.6×0.007

E 0.05×0.09

F 0.02×0.008

G 0.006×0.4

H 0.05×0.0008

2. Find the products.

A 6.5×7

B 0.37×8

C 1.38×0.8

D 43.6×0.5

E 0.358×0.9

F 53×1.7

G 7.3×46

H 15.3×8.2

I 1.27×53

J 85.4×6.5

K 0.357×4.2

L 1.68×0.77

M 935×0.08

N 47.9×0.65

O 0.036×0.05

P 435×0.007

Q 67.3×2.17

R 8.55×4.03

S 0.999×6.75

T 1.078×0.937

SET 33

1. Find the products.

A 10×7.1	F 100×3.1	K 1000×32	P 10×0.0036
B 10×0.38	G 100×8.73	L 1000×8.7	Q 100×0.809
C 10×1.25	H 100×13.9	M 1000×1.39	R 100×0.0023
D 10×0.904	I 100×0.375	N 1000×0.765	S 1000×0.038
E 10×0.003	J 100×0.004	O 1000×0.069	T 1000×0.70004

2. Find the products.

A 0.1×26	F 0.01×412	K 0.001×2567	P 0.1×0.097
B 0.1×4.9	G 0.01×11.9	L 0.001×453.1	Q 0.01×0.033
C 0.1×124	H 0.01×1.635	M 0.001×85.7	R 0.01×0.5503
D 0.1×1.69	I 0.01×97.04	N 0.001×3.76	S 0.001×0.018
E 0.1×0.43	J 0.01×0.67	O 0.001×0.96	T 0.0001×23.45

SET 34

1. Find the quotients.

A $3\overline{)25.59}$	D $31\overline{)1.209}$	G $1.7\overline{)14.62}$	J $0.92\overline{)0.05796}$
B $8\overline{)4.256}$	E $0.6\overline{)3.96}$	H $0.68\overline{)0.884}$	K $3.21\overline{)205.44}$
C $23\overline{)11.04}$	F $0.8\overline{)75.2}$	I $0.53\overline{)0.4717}$	L $0.801\overline{)0.34443}$

2. Find the quotients. Annex zeros when necessary.

A $6\overline{)0.882}$	F $4\overline{)5.1}$	K $9.2\overline{)30.82}$	P $96\overline{)7.2}$
B $0.8\overline{)18.48}$	G $0.8\overline{)2.6}$	L $0.44\overline{)134.2}$	Q $3.2\overline{)3}$
C $5\overline{)25.7}$	H $0.5\overline{)6}$	M $0.09\overline{)36}$	R $0.99\overline{)4554}$
D $0.19\overline{)1.197}$	I $0.12\overline{)6.6}$	N $6.5\overline{)1976}$	S $0.82\overline{)3095.5}$
E $8.1\overline{)2.835}$	J $0.36\overline{)9}$	O $1.6\overline{)7}$	T $0.104\overline{)4.68}$

SET 35

1. Give the number for n. Then give the correct decimal.

A $\frac{2}{5} = \frac{n}{10} = ?$

B $\frac{9}{20} = \frac{n}{100} = ?$

C $\frac{37}{50} = \frac{n}{100} = ?$

D $\frac{12}{25} = \frac{n}{100} = ?$

E $\frac{37}{200} = \frac{n}{1000} = ?$

F $\frac{49}{500} = \frac{n}{1000} = ?$

G $\frac{93}{250} = \frac{n}{1000} = ?$

H $\frac{71}{125} = \frac{n}{1000} = ?$

I $\frac{7}{40} = \frac{n}{1000} = ?$

J $\frac{124}{125} = \frac{n}{1000} = ?$

K $\frac{5}{8} = \frac{n}{1000} = ?$

L $\frac{1}{400} = \frac{n}{10\,000} = ?$

2. Write a decimal for each fraction.

A $\frac{1}{2}$	**E** $\frac{1}{20}$	**I** $\frac{3}{5}$	**M** $\frac{9}{25}$	**Q** $\frac{1}{40}$
B $\frac{1}{4}$	**F** $\frac{1}{25}$	**J** $\frac{3}{4}$	**N** $\frac{37}{50}$	**R** $\frac{1}{200}$
C $\frac{1}{5}$	**G** $\frac{1}{50}$	**K** $\frac{3}{10}$	**O** $\frac{93}{100}$	**S** $\frac{1}{250}$
D $\frac{1}{10}$	**H** $\frac{1}{100}$	**L** $\frac{17}{20}$	**P** $\frac{79}{200}$	**T** $\frac{1}{500}$

3. Find the quotients. Continue dividing until the remainder is zero.

A $\frac{3}{4} \rightarrow 4\overline{)3} = ?$

B $\frac{4}{5} \rightarrow 5\overline{)4} = ?$

C $\frac{1}{8} \rightarrow 8\overline{)1} = ?$

D $\frac{17}{20} \rightarrow 20\overline{)17} = ?$

E $\frac{19}{25} \rightarrow 25\overline{)19} = ?$

F $\frac{11}{40} \rightarrow 40\overline{)11} = ?$

G $\frac{5}{16} \rightarrow 16\overline{)5} = ?$

H $\frac{17}{80} \rightarrow 80\overline{)17} = ?$

I $\frac{9}{16} \rightarrow 16\overline{)9} = ?$

J $\frac{37}{40} \rightarrow 40\overline{)37} = ?$

K $\frac{63}{80} \rightarrow 80\overline{)63} = ?$

L $\frac{5}{32} \rightarrow 32\overline{)5} = ?$

4. Write a decimal for each fraction.

A $\frac{4}{5}$	**E** $\frac{5}{8}$	**I** $\frac{15}{4}$	**M** $\frac{325}{200}$	**Q** $\frac{7}{200}$
B $\frac{7}{20}$	**F** $\frac{3}{16}$	**J** $\frac{93}{50}$	**N** $\frac{26}{80}$	**R** $\frac{15}{16}$
C $\frac{8}{25}$	**G** $\frac{43}{250}$	**K** $\frac{27}{25}$	**O** $\frac{21}{125}$	**S** $\frac{17}{32}$
D $\frac{17}{50}$	**H** $\frac{11}{40}$	**L** $\frac{87}{500}$	**P** $\frac{90}{150}$	**T** $\frac{1}{64}$

SET 36

1. Write a mixed decimal numeral for each number in **tenths**.

A $\frac{2}{3}$ **B** $\frac{3}{8}$ **C** $\frac{1}{6}$ **D** $\frac{11}{15}$ **E** $\frac{7}{40}$ **F** $\frac{65}{75}$

2. Write a mixed decimal numeral for each number in **hundredths**.

A $\frac{5}{8}$ **B** $\frac{1}{3}$ **C** $\frac{7}{16}$ **D** $\frac{7}{12}$ **E** $\frac{1}{15}$ **F** $\frac{3}{40}$

3. Write a mixed decimal numeral for each number in **thousandths**.

A $\frac{1}{6}$ **B** $\frac{7}{11}$ **C** $\frac{11}{12}$ **D** $\frac{27}{80}$ **E** $\frac{4}{15}$ **F** $\frac{1}{600}$

4. Write a fraction for each mixed decimal numeral.

A $0.1\frac{1}{2}$ **B** $0.2\frac{1}{4}$ **C** $0.25\frac{1}{2}$ **D** $0.06\frac{2}{3}$ **E** $0.333\frac{1}{3}$ **F** $0.007\frac{1}{2}$

SET 37

1. Represent each number in scientific notation.

A 600	**G** 72 000	**M** 4 735 000	**S** 1 050 000
B 2000	**H** 10 500	**N** 125 000 000	**T** 948 000 000
C 40 000	**I** 600 000	**O** 5 100 000	**U** 915 000 000
D 8100	**J** 210 000	**P** 9 750 000	**V** 4 069 000 000
E 620	**K** 8 300 000	**Q** 2 603 000	**W** 6 705 000 000
F 4370	**L** 23 000 000	**R** 48 900 000	**X** 400 800 000 000

2. Write each numeral in the usual way.

A 3×10^2	**E** 3.8×10^3	**I** 3.75×10^2	**M** 1.53×10^7
B 9×10^4	**F** 4.7×10^4	**J** 6.03×10^3	**N** 4.006×10^9
C 6×10^5	**G** 1.1×10^6	**K** 9.99×10^5	**O** 3.49×10^8
D 1.2×10^2	**H** 7.09×10^5	**L** 1.106×10^8	**P** 8.3×10^{12}

SET 38

1. Solve the following proportions for n.

A $\frac{2}{3} = \frac{n}{18}$

B $\frac{4}{5} = \frac{20}{n}$

C $\frac{1}{6} = \frac{7}{n}$

D $\frac{2}{n} = \frac{8}{20}$

E $\frac{5}{1} = \frac{n}{17}$

F $\frac{6}{n} = \frac{16}{24}$

G $\frac{6}{15} = \frac{n}{40}$

H $\frac{n}{12} = \frac{10}{15}$

I $3:4 = n:36$

J $5:6 = 20:n$

K $2:n = 10:35$

L $6:1 = n:13$

M $n:8 = 21:28$

N $7:10 = 140:n$

O $n:5 = 320:200$

P $4:3 = 180:n$

3. Write a proportion for each problem and then solve it.

A Baseball player: 3 hits for every 8 times at bat. How many hits for 96 times at bat?

B Car: 6 kilometres driven per litre of gasoline. How many kilometres driven on 37 litres?

C Basketball player: 7 free throws out of every 9 attempts. How many free throws in 135 attempts?

D Picnic: 2 hot dogs per person. How many hot dogs needed to feed 126 people?

E Class: ratio of girls to boys is 7 to 9. If there are 56 girls, how many boys are there?

F Population: ratio of City X to City Y is 6 to 5. If 3000 people live in City Y, how many in City X?

G Scale on map: 1 cm = 35 km. What is the actual distance if the distance on the map is 4.9 cm?

H Recipe: 5 grams salt per litre of water. How many grams of salt needed for 1 kilolitre of water?

SET 39

1. Write a decimal for each percent.

A 17%	**E** 34%	**I** 10%	**M** 100%	**Q** 105%	**U** 700%
B 28%	**F** 40%	**J** 9%	**N** 200%	**R** 5%	**V** 70%
C 85%	**G** 11%	**K** 2%	**O** 20%	**S** 50%	**W** 7%
D 31%	**H** 99%	**L** 1%	**P** 2%	**T** 15%	**X** 0.7%

2. Write a percent for each decimal.

A 0.29	**E** 0.12	**I** 0.6	**M** 0.3	**Q** 0.18	**U** 0.2
B 0.66	**F** 0.08	**J** 1.2	**N** 0.03	**R** 1.8	**V** 10
C 0.13	**G** 0.02	**K** 1	**O** 3	**S** 18	**W** 0.43
D 0.47	**H** 0.01	**L** 2	**P** 30	**T** 1.08	**X** 0.05

3. Write a lowest-terms fraction for each percent.

A 57%	**E** 25%	**I** 5%	**M** 15%	**Q** 80%	**U** 140%
B 50%	**F** 20%	**J** 4%	**N** 16%	**R** 8%	**V** 175%
C 30%	**G** 10%	**K** 2%	**O** 17%	**S** 800%	**W** 44%
D 31%	**H** 6%	**L** 1%	**P** 18%	**T** 88%	**X** 135%

4. Write a percent for each fraction.

A $\frac{37}{100}$	**E** $\frac{7}{10}$	**I** $\frac{1}{5}$	**M** $\frac{3}{5}$	**Q** $\frac{7}{20}$	**U** $\frac{21}{20}$
B $\frac{9}{100}$	**F** $\frac{1}{10}$	**J** $\frac{1}{50}$	**N** $\frac{1}{25}$	**R** $\frac{9}{25}$	**V** $\frac{4}{25}$
C $\frac{7}{100}$	**G** $\frac{1}{2}$	**K** $\frac{3}{4}$	**O** $\frac{7}{50}$	**S** $\frac{13}{50}$	**W** $\frac{7}{5}$
D $\frac{1}{100}$	**H** $\frac{1}{4}$	**L** $\frac{1}{20}$	**P** $\frac{49}{50}$	**T** $\frac{24}{25}$	**X** $\frac{11}{4}$

SET 40

1. Find the percent of each number.

A 45% of 100
B 50% of 64
C 25% of 80
D 20% of 45
E 10% of 340
F 29% of 200
G 43% of 540
H 11% of 950
I 1% of 43
J 5% of 840
K 3% of 90
L 110% of 950
M 400% of 35
N 125% of 240
O 93% of 682
P 81% of 1209

2. Find the products.

A 28% of 450
B 83% of 50
C 300% of 79
D 175% of 48
E 125% of 200
F 12.5% of 200
G 1.25% of 200
H 0.125% of 200
I 43.5% of 400
J 8.7% of 65
K 11.8% of 90
L 4.35% of 340
M $7\frac{1}{2}$% of 400
N $15\frac{1}{10}$% of 360
O $6\frac{1}{4}$% of 200
P $5\frac{3}{4}$% of 600

3. Solve the problems.

A Price of stereo: \$75
Sales tax: 4%
Total price of stereo?

B Earnings: \$135 per week
Deductions: 28%
How much take-home pay?

C Amount of money: \$350
Rate of interest: $6\frac{1}{2}$%
Amount of interest?

D Monthly bill: \$46.10
Service charge: $1\frac{1}{2}$%
What is the total amount of bill?

SET 41

1. Find the number for each ▒.

A 7 is ▒% of 10
B 5 is ▒% of 20
C 38 is ▒% of 100
D 9 is ▒% of 12
E 28 is ▒% of 56
F 16 is ▒% of 20
G 9 is ▒% of 15
H 45 is ▒% of 150
I 18 is ▒% of 90
J 35 is ▒% of 140
K 8 is ▒% of 24
L 25 is ▒% of 40
M 10 is ▒% of 16
N 120 is ▒% of 80
O 8 is ▒% of 200

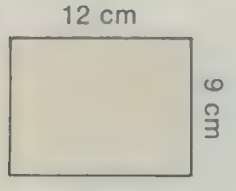
2. Find the percent the first number is of the second.
Round your answers to the nearest tenth of a percent.

A 4, 33
B 9, 26
C 19, 21
D 15, 27
E 18, 13
F 25, 28
G 18, 75
H 98, 150
I 83, 99
J 92, 101

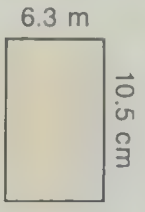
SET 42

1. Find the area of each rectangular region. Use the formula $A = l \cdot w$.

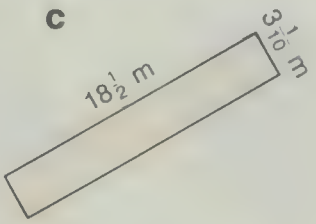
A



B



C

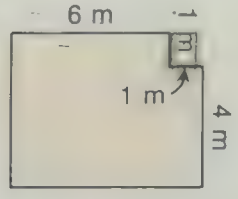


D $l = 600$ mm
$w = 150$ mm

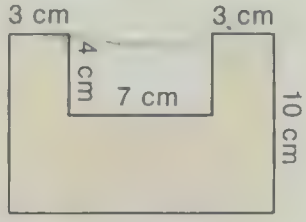
E $l = 8\frac{1}{2}$ km
$w = 4\frac{7}{10}$ km

2. Find the area of each region.

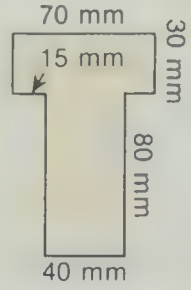
A



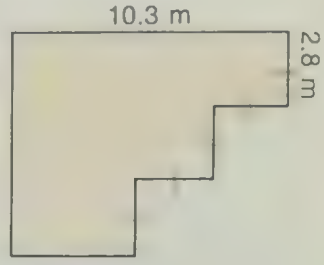
B



C

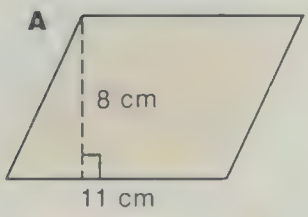


D

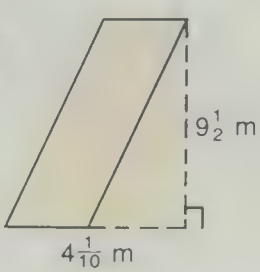


3. Find the area of each parallelogram. Use the formula $A = b \cdot h$.

A



B



C

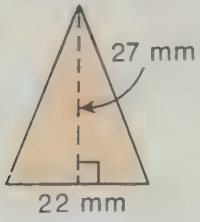
D $b = 18$ m
$h = 3\frac{1}{2}$ m

E $b = 15\frac{3}{10}$ m
$h = 8\frac{1}{5}$ m

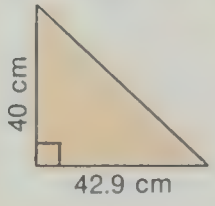
SET 43

1. Find the area of each triangular region. Use the formula $A = \frac{1}{2} \cdot b \cdot h$.

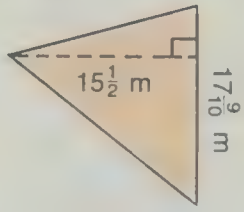
A



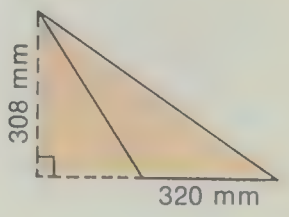
B



C



D



E $b = 101$ cm
$h = 46$ cm

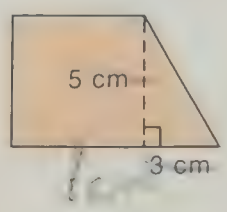
F $b = 3.8$ m
$h = 5.2$ m

G $b = 15\frac{7}{10}$ cm
$h = 3\frac{1}{2}$ cm

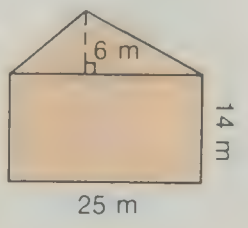
H $b = 450$ mm
$h = 860$ mm

2. Find the area of each region.

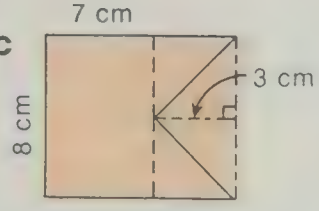
A



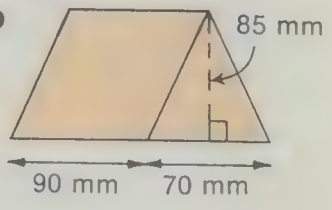
B



C



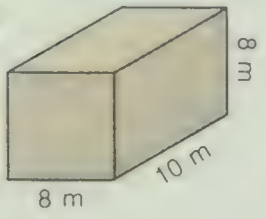
D



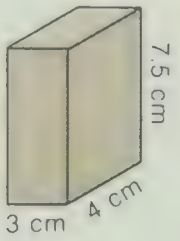
SET 44

1. Find the volume of each rectangular solid. Use the formula $V = l \cdot w \cdot h$.

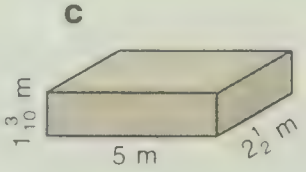
A



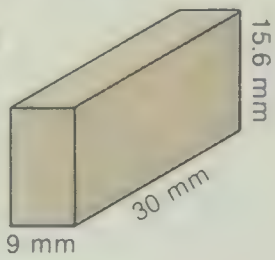
B



C



D



E $l = 9$ cm
$w = 5$ cm
$h = 12$ cm

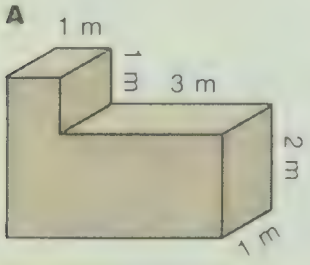
F $l = 40$ m
$w = 40$ m
$h = 25$ m

G $l = 8.3$ cm
$w = 4.6$ cm
$h = 6.23$ cm

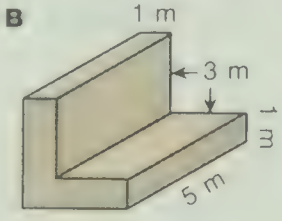
H $l = 8\frac{7}{10}$ m
$w = 10$ m
$h = 4\frac{1}{5}$ m

2. Find the volume of each space figure.

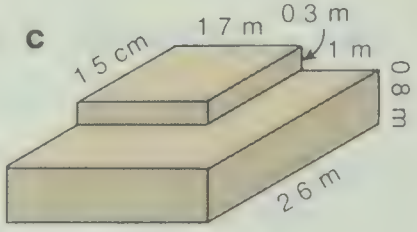
A



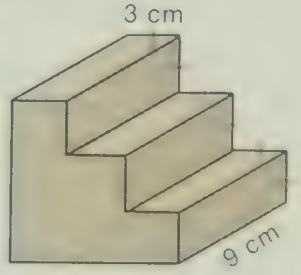
B



C



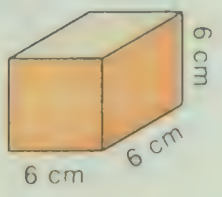
D



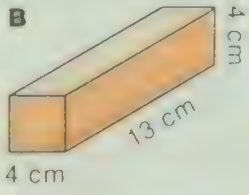
SET 45

1. Find the surface area of each space figure.

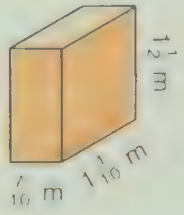
A



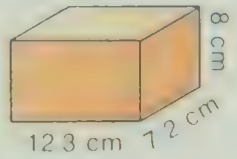
B



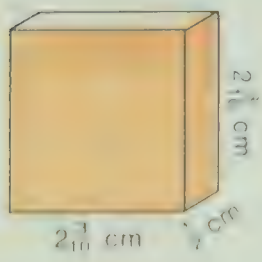
C



D

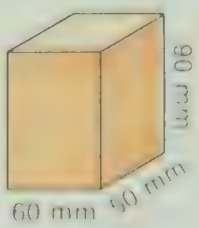


E

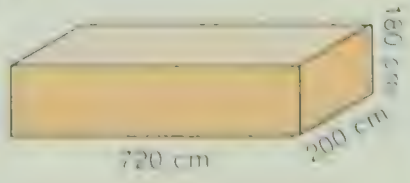


F

G



H

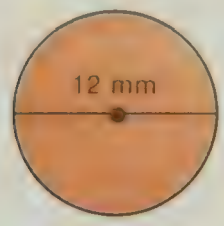


2. Find the surface area of each space figure in Exercise 2 in Set 44 above.

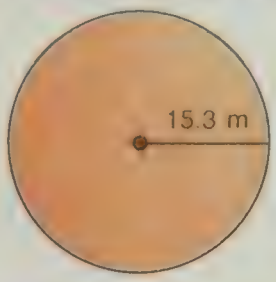
SET 46

1. Find the circumference of each circle. Use the formula $C = \pi \cdot d$ and $\pi \approx 3.14$.

A



B



C $d = 105$ mm

D $r = 45$ cm

E $d = 8.35$ m

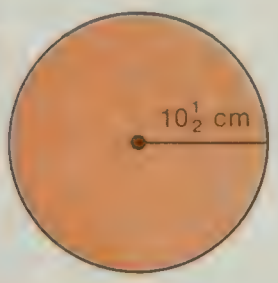
F $r = 166$ mm

G $r = 0.9$ km

H $d = 4.26$ m

2. Use the fact $\pi \approx 3\frac{1}{7}$ to find the circumference of each circle.

A



B

C $r = 5$ cm

D $d = 70$ mm

E $r = 17\frac{1}{2}$ m

F $d = 490$ mm

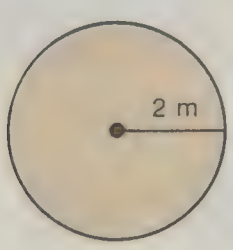
G $d = 4\frac{9}{10}$ m

H $d = 6\frac{3}{10}$ cm

SET 47

1. Find the area of each circular region. Use the formula $A = \pi \cdot r^2$ and $\pi \approx 3.14$.

A



B

C

D $r = 30$ mm

E $d = 1.6$ m

F $r = 41$ cm

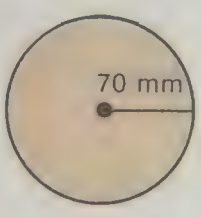
G $d = 210$ mm

2. Find the area of each circular region. Use the fact $\pi \approx 3\frac{1}{7}$.

A

B

C



D $r = 4$ cm

E $d = 20$ m

F $r = \frac{6}{10}$ km

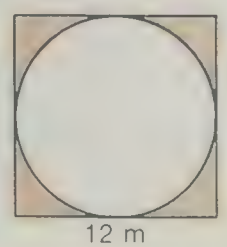
G $r = 5\frac{1}{2}$ m

3. Find the area of the shaded portion of each region.

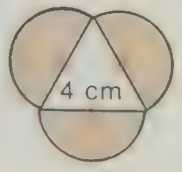
A

B

C

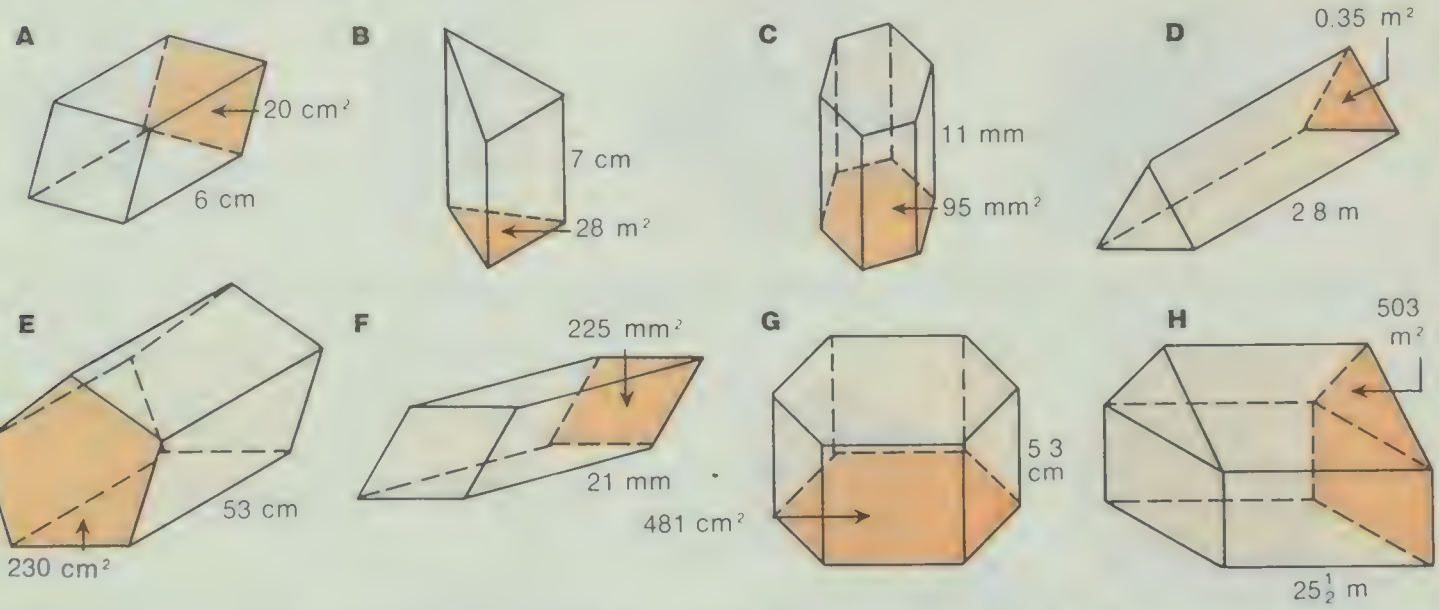


D

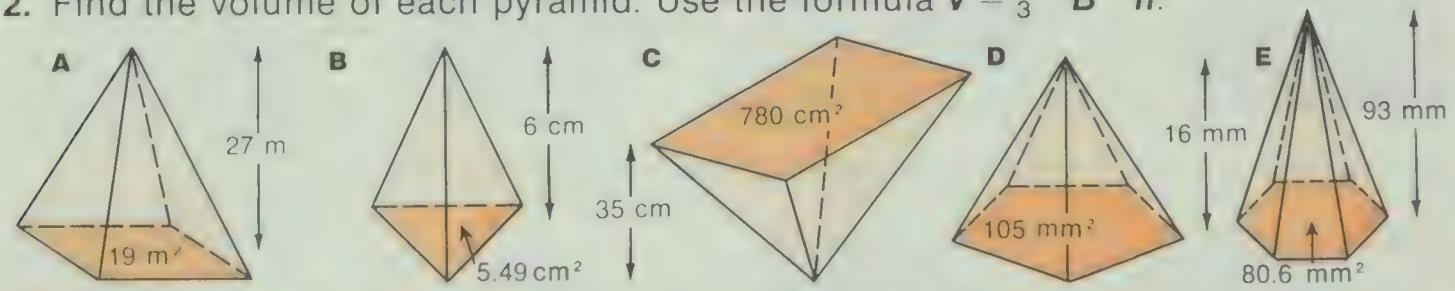


SET 48

1. Find the volume of each prism. Use the formula $V = B \cdot h$.

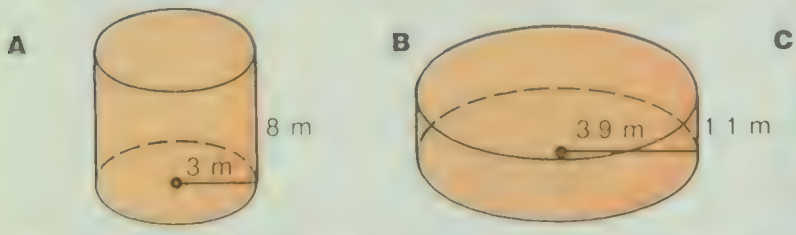


2. Find the volume of each pyramid. Use the formula $V = \frac{1}{3} \cdot B \cdot h$.



SET 49

1. Find the volume of each circular cylinder. Use the formula $V = \pi \cdot r^2 \cdot h$.

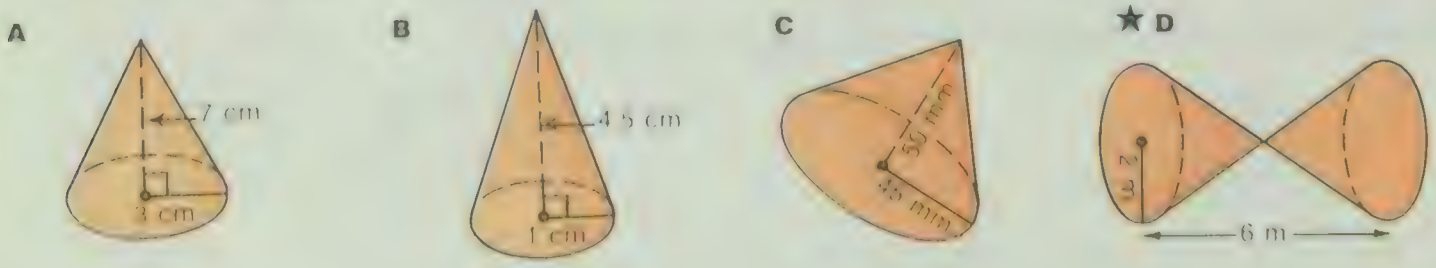


D $r = 50, h = 37$

E $r = 0.1, h = 0.6$

F $r = \frac{7}{10}, h = 1\frac{1}{2}$

2. Find the volume of each circular cone. Use the formula $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$.



SET 50

1. Find the sums.

A $7 + {}^{-}5$

B ${}^{-}8 + {}^{-}6$

C ${}^{-}9 + 3$

D $12 + {}^{-}15$

E ${}^{-}10 + 4$

F $0 + {}^{-}13$

G $21 + {}^{-}16$

H $21 + 16$

I ${}^{-}21 + 16$

J ${}^{-}21 + {}^{-}16$

K $21 + {}^{-}8 + 8$

L ${}^{-}21 + {}^{-}8 + {}^{-}8$

M $75 + {}^{-}53$

N $49 + {}^{-}53$

O $75 + {}^{-}49$

P ${}^{-}75 + 53$

Q ${}^{-}53 + 49$

R $53 + {}^{-}49$

S $125 + {}^{-}99$

T ${}^{-}82 + 18$

2. Find the sums.

A $5 + 5 + 5$

B $5 + 5 + {}^{-}5$

C $5 + {}^{-}5 + {}^{-}5$

D ${}^{-}5 + {}^{-}5 + {}^{-}5$

E $27 + {}^{-}13 + 7$

F $100 + ({}^{-}23 + 43)$

G $(45 + {}^{-}18) + {}^{-}25$

H $35 + {}^{-}15 + 23$

I ${}^{-}35 + {}^{-}15 + 23$

J ${}^{-}35 + 15 + {}^{-}23$

K ${}^{-}35 + {}^{-}15 + {}^{-}23$

L $35 + {}^{-}15 + {}^{-}23$

M $18 + 17 + {}^{-}15 + 23$

N ${}^{-}18 + 17 + 15 + {}^{-}23$

O $185 + {}^{-}37 + {}^{-}8$

P ${}^{-}250 + ({}^{-}126 + 106)$

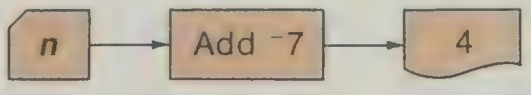
Q $467 + ({}^{-}310 + {}^{-}25)$

R ${}^{-}755 + (825 + {}^{-}57)$

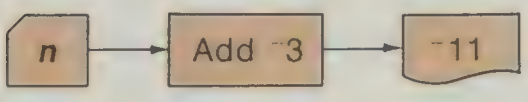
SET 51

1. Find the number n in each flow chart.

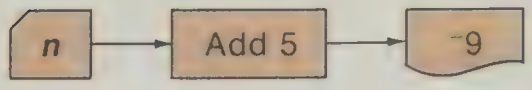
A



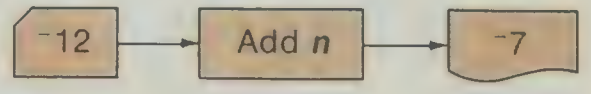
B



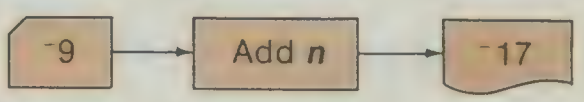
C



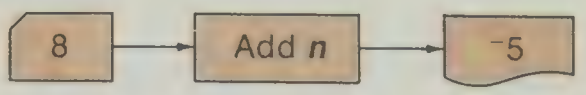
D



E



F



2. Solve the equations.

A $n + {}^{-}5 = 15$

B $n + {}^{-}14 = 0$

C $n + 5 = {}^{-}7$

D $n + 6 = 3$

E $n + 8 = {}^{-}8$

F $n + 6 = 12$

G $n + {}^{-}6 = {}^{-}12$

H $n + 6 = {}^{-}12$

I $n + {}^{-}6 = 12$

J $n + {}^{-}6 = 0$

K $14 + n = 9$

L $18 + n = 23$

M ${}^{-}15 + n = 2$

N $n + {}^{-}25 = {}^{-}28$

O ${}^{-}13 + n = {}^{-}34$

P $n + {}^{-}53 = 26$

Q $79 + n = {}^{-}10$

R $n + 46 = {}^{-}12$

S ${}^{-}55 + n = 100$

T $n + 43 = {}^{-}43$

SET 52

1. Solve the equations. Think about missing addends.

A $0 - 5 = n$	**D** $0 - {}^{-}5 = n$	**G** $^{-}3 - {}^{-}5 = n$	**J** $5 - {}^{-}5 = n$
B $3 - 5 = n$	**E** $3 - {}^{-}5 = n$	**H** $^{-}8 - {}^{-}5 = n$	**K** $^{-}5 - {}^{-}5 = n$
C $8 - 5 = n$	**F** $8 - {}^{-}5 = n$	**I** $5 - 5 = n$	**L** $^{-}5 - 5 = n$

2. Find the differences.

A $15 - 8$	**G** $0 - 27$	**M** $38 - {}^{-}42$	**S** $401 - 189$
B $8 - 15$	**H** $0 - {}^{-}13$	**N** $^{-}56 - 4$	**T** $529 - {}^{-}125$
C $15 - {}^{-}8$	**I** $43 - 77$	**O** $^{-}73 - {}^{-}27$	**U** $^{-}630 - {}^{-}575$
D $^{-}15 - 8$	**J** $24 - {}^{-}24$	**P** $44 - {}^{-}45$	**V** $193 - {}^{-}87$
E $^{-}15 - {}^{-}8$	**K** $^{-}24 - 24$	**Q** $^{-}65 - 3$	**W** $^{-}153 - 153$
F $8 - {}^{-}15$	**L** $^{-}24 - {}^{-}24$	**R** $99 - 199$	**X** $495 - 495$

SET 53

1. Find the products.

A $5 \cdot {}^{-}9$	**E** $^{-}1 \cdot {}^{-}9$	**I** $^{-}10 \cdot {}^{-}43$	**M** $21 \cdot {}^{-}21$	**Q** $^{-}125 \cdot 1000$
B $9 \cdot {}^{-}5$	**F** $^{-}3 \cdot 11$	**J** $10 \cdot {}^{-}67$	**N** $^{-}21 \cdot {}^{-}21$	**R** $^{-}1001 \cdot 1$
C $0 \cdot {}^{-}9$	**G** $5 \cdot {}^{-}18$	**K** $100 \cdot {}^{-}6$	**O** $^{-}100 \cdot 23$	**S** $29 \cdot {}^{-}36$
D $^{-}1 \cdot 9$	**H** $^{-}10 \cdot 17$	**L** $14 \cdot {}^{-}100$	**P** $^{-}100 \cdot {}^{-}56$	**T** $^{-}53 \cdot {}^{-}18$

2. Find the products.

A $2 \cdot 5 \cdot {}^{-}4$	**I** $10 \cdot {}^{-}10 \cdot 10$	**Q** $12 \cdot {}^{-}3 \cdot {}^{-}4$
B $2 \cdot {}^{-}5 \cdot {}^{-}4$	**J** $^{-}10 \cdot {}^{-}10 \cdot {}^{-}10$	**R** $^{-}21 \cdot {}^{-}3 \cdot {}^{-}3$
C $^{-}2 \cdot {}^{-}5 \cdot {}^{-}4$	**K** $^{-}10 \cdot {}^{-}10 \cdot 10$	**S** $^{-}1 \cdot 12 \cdot 0 \cdot {}^{-}8$
D $3 \cdot 15 \cdot {}^{-}1$	**L** $^{-}10 \cdot {}^{-}10 \cdot {}^{-}100$	**T** $4 \cdot 10 \cdot {}^{-}1 \cdot {}^{-}10$
E $8 \cdot {}^{-}6 \cdot {}^{-}1$	**M** $^{-}4 \cdot {}^{-}3 \cdot 8$	**U** $^{-}63 \cdot {}^{-}2 \cdot 1 \cdot {}^{-}1$
F $12 \cdot {}^{-}5 \cdot 2$	**N** $^{-}7 \cdot {}^{-}11 \cdot {}^{-}1$	**V** $^{-}6 \cdot ({}^{-}15 \cdot {}^{-}8 \cdot {}^{-}1)$
G $^{-}1 \cdot 10 \cdot 10$	**O** $5 \cdot {}^{-}25 \cdot 2$	**W** $({}^{-}7 \cdot {}^{-}9) \cdot ({}^{-}1 \cdot {}^{-}5)$
H $^{-}1 \cdot 10 \cdot {}^{-}10$	**P** $^{-}1 \cdot 43 \cdot {}^{-}10$	**X** $^{-}1 \cdot ({}^{-}5 \cdot {}^{-}2) \cdot ({}^{-}4 \cdot 3)$

SET 54

1. Solve the equations.

A $^{-}3 \cdot n = {}^{-}12$	**F** $n \cdot 6 = {}^{-}54$	**K** $83 \cdot n = {}^{-}83$	**P** $4 \cdot {}^{-}3 \cdot n = 60$
B $5 \cdot n = {}^{-}35$	**G** $n \cdot {}^{-}8 = 72$	**L** $^{-}5 \cdot n = {}^{-}65$	**Q** $^{-}1 \cdot 14 \cdot n = 70$
C $n \cdot 6 = {}^{-}24$	**H** $^{-}5 \cdot n = {}^{-}100$	**M** $10 \cdot n = {}^{-}140$	**R** $^{-}2 \cdot {}^{-}5 \cdot n = 190$
D $n \cdot {}^{-}5 = 45$	**I** $5 \cdot n = {}^{-}100$	**N** $n \cdot {}^{-}8 = 120$	**S** $n \cdot {}^{-}3 \cdot 8 = 120$
E $4 \cdot n = 44$	**J** $^{-}5 \cdot n = 100$	**O** $^{-}11 \cdot n = 88$	**T** $n \cdot n = 49$

2. Find the quotients.

A $^{-}21 \div 7$	**F** $^{-}12 \div {}^{-}12$	**K** $^{-}72 \div 6$	**P** $^{-}88 \div {}^{-}11$	**U** $^{-}144 \div {}^{-}6$
B $^{-}27 \div {}^{-}3$	**G** $0 \div {}^{-}9$	**L** $43 \div {}^{-}43$	**Q** $^{-}75 \div {}^{-}25$	**V** $256 \div {}^{-}8$
C $^{-}42 \div {}^{-}6$	**H** $^{-}45 \div {}^{-}3$	**M** $81 \div {}^{-}9$	**R** $90 \div {}^{-}18$	**W** $^{-}128 \div 16$
D $54 \div {}^{-}9$	**I** $^{-}25 \div {}^{-}1$	**N** $60 \div {}^{-}4$	**S** $100 \div {}^{-}10$	**X** $^{-}216 \div 216$
E $^{-}48 \div {}^{-}16$	**J** $63 \div {}^{-}7$	**O** $36 \div {}^{-}9$	**T** $^{-}300 \div 15$	**Y** $^{-}579 \div {}^{-}3$

SET 55

1. Find the average (arithmetic mean) of each set of numbers.

A 23, 26, 29	**F** 125, 99, 132, 120, 124
B 63, 57, 72	**G** 4301, 4078, 3996, 4125
C 87, 82, 69, 98	**H** 209, 205, 199, 241, 238, 216
D 53, 68, 79, 64	**I** 10.4, 9.9, 7.7, 8.2, 9.1, 9.5
E 879, 904, 946, 891	**J** 1.59, 1.52, 1.76, 1.62, 1.56

2. Find the average of each set of number to the nearest whole number.

A 38, 41, 37	**F** 1537, 1302, 1496, 1388
B 50, 49, 61, 55	**G** 1001, 1003, 1004, 999, 1011
C 93, 97, 85, 104	**H** 250, 260, 180, 190, 360, 250
D 208, 210, 212, 214, 215	**I** 697, 588, 634, 596, 619
E 398, 406, 421	**J** 4873, 5370, 5093, 4961

SET 56

1. Find the median of each set of numbers.

   A 27, 29, 35, 37, 37

   B 26, 29, 35, 37

   C 43, 49, 53, 51, 54

   D 397, 405, 399, 401

   E 87, 88, 89, 92, 94, 93

   F 5, 6, 6, 7, 8, 9, 9, 10, 12, 12, 15

   G 43, 45, 48, 50, 53, 54, 54, 57, 58

   H 17, 20, 21, 19, 18, 17, 24, 26, 22

   I 39, 40, 41, 39, 38, 43, 45, 47, 51, 46

   J 3.83, 4.61, 3.52, 4.89, 4.05, 3.99

2. Solve the problems.

   A Test scores:
   79, 88, 93, 87, 100, 86, 94
   What is the median score?

   B Annual incomes:
   $9100, $7200, $10 000, $8500, $9600
   What is the median income?

   C Refer to the chart.
   What is the median height of the girls?
   of the boys?
   of all students?

Student Heights (metres)			
Girls		Boys	
1.50	1.61	1.55	1.67
1.51	1.54	1.60	1.74
1.48	1.64	1.57	1.82
1.56	1.51	1.64	1.77
1.54	1.57	1.73	1.69

SET 57

1. Find the mode(s) of each set of numbers.

A

Die Tossing Outcomes			
6	3	4	6
5	4	3	5
4	2	5	4
5	5	4	3
6	1	3	4

B

Test Scores		
85	93	79
79	91	86
78	73	85
86	80	89
93	86	93

C

Students' Ages			
13	14	14	14
12	15	12	14
14	12	13	13
13	13	12	12
14	14	12	15

2. Refer to the chart to answer parts **A**, **B**, and **C**.

   A Find the mode.

   B Find the median.

   C Find the arithmetic mean.

Men's Masses (kilograms)			
72.6	68.1	93.1	79.4
74.9	79.4	99.8	86.3
70.4	86.3	74.9	90.8
81.7	88.5	83.9	74.9
74.9	79.4	90.8	72.6

Mathematical Symbols

$=$	Is equal to
$\neq$	Is not equal to
$>$	Is greater than
$<$	Is less than
$\leq$	Is less than or equal to
$\approx$	Is approximately
$\cong$	Is congruent to
$\sim$	Is similar to
$\{a, b, c\}$	Set with elements a, b, c
ϕ	Empty set
$\cup$	Union
$\cap$	Intersection
$\overleftrightarrow{AB}$	Line through points A and B
$\overrightarrow{AB}$	Ray AB with endpoint A
$\overline{AB}$	Segment with endpoints A and B
$\angle ABC$	Angle ABC with vertex B
$m\angle ABC$	Measure of angle ABC
$\triangle ABC$	Triangle ABC
$\overset{\frown}{RS}$	Arc with endpoints R and S
$35°\ 14'\ 20''$	Thirty-five *degrees*, fourteen *minutes*, twenty *seconds*
%	Percent
π	Pi

Metric System Prefixes

kilo-	one thousand
hecto-	one hundred
deca-	ten
deci-	one tenth
centi-	one hundredth
milli-	one thousandth

Formulae

$P = a + b + c$	Perimeter of triangle
$P = 2 \cdot l + 2 \cdot w$	Perimeter of rectangle
$A = l \cdot w$	Area of rectangle
$A = b \cdot h$	Area of parallelogram
$A = \frac{1}{2} \cdot b \cdot h$	Area of triangle
$a^2 + b^2 = c^2$	Pythagorean Theorem
$V = l \cdot w \cdot h$	Volume of rectangular solid
$C = \pi \cdot d$	Circumference of circle
$A = \pi \cdot r^2$	Area of circle
$V = \frac{4}{3} \cdot \pi \cdot r^3$	Volume of sphere
$V = \pi \cdot r^2 \cdot h$	Volume of cylinder
$V = \frac{1}{3} \cdot \pi \cdot r^2 h$	Volume of cone
$V = B \cdot h$	Volume of prism (B = base area)
$V = \frac{1}{3} \cdot B \cdot h$	Volume of pyramid

abacus A device used for calculating, usually involving sliding beads or counters along a wire.

acute angle An angle smaller than a right angle.

addend Any one member of a set of numbers to be added. In the equation $7 + 9 = 16$, 7 and 9 are addends.

addition An operation that combines a first number (*addend*) and a second number (*addend*) to give another number (this number is called the *sum* of the two addends).

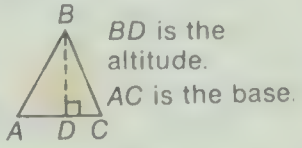
additive inverse Each of two numbers whose sum is zero is said to be the additive inverse of the other.

adjacent angles Two angles with a common vertex, a common side, and no common interior points. In the figure, $\angle 1$ and $\angle 2$ are adjacent angles.

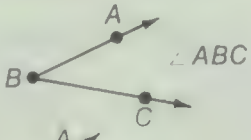
adjacent sides Two sides of a polygon with a common vertex.

algorithm (algorism) A computation procedure. For example, the algorithm for finding the quotient of two numbers.

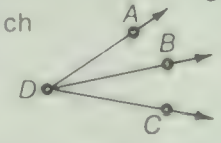
altitude (of a triangle) The segment from any vertex perpendicular to the opposite side. Also, see *height*.



angle. The union of two rays with a common endpoint (vertex).



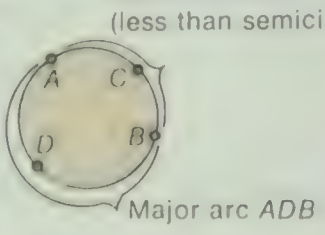
angle bisector The ray which divides an angle into two congruent angles. In the figure $\overrightarrow{DB}$ bisects $\angle ADC$.



approximation A number that is suitably "close" to another number. For example, a decimal approximation for $\frac{2}{9}$ is 0.22. The symbol "$\approx$" denotes "is approximately"; for example $\pi \approx 3.14$

are (pronounced "air") A unit of area. 100 square metres

arc A part of a circle. Note that three points, instead of two, are used to denote a specific arc since AB could denote a *minor arc AB* or a *major arc ADB*



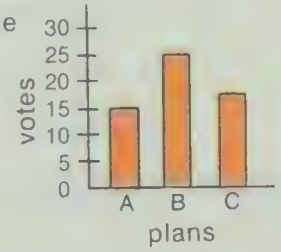
area The measure of a plane region in terms of a chosen unit, usually a square

arithmetic mean The quotient of the sum of numbers in a set divided by the number of numbers in a set. (Often called the "average".)

associative (grouping) principle The principle which states that the sum (or product) of three or more numbers is the same regardless of grouping:
$(a + b) + c = a + (b + c)$
or $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

average See *arithmetic mean*.

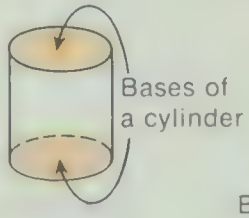
bar graph A diagram made of graduated markings and rectangular bars to show statistical information, usually about specifically stated quantities.

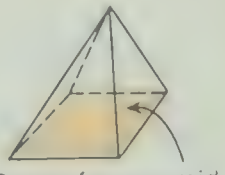


base (of numeration) The term "base" refers to the type of grouping involved in a system of numeration. For example, in base eight: $25_{(8)}$ means 2 eights and 5 or $346_{(8)}$ means 3 sixty-fours, 4 eights and 6.

base (of a polygon) Any side of a polygon may be referred to as a base. See figure for *altitude of a triangle*.

base (of a space figure) See examples below.





bisect To divide into two congruent parts.

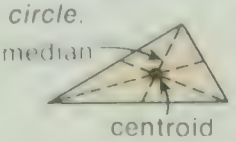
centre A (fixed) point of symmetry for a geometric figure. See the figure for *circle*.

centi- A prefix meaning *one hundredth*

centimetre A unit of length that is $\frac{1}{100}$ of a metre.

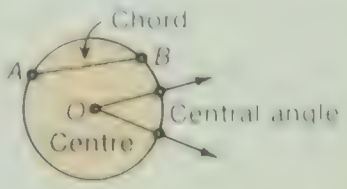
central angle An angle with its vertex at the centre of the circle. See the figure for *circle*.

centroid (of a triangle) The point of intersection of the three medians of a triangle.

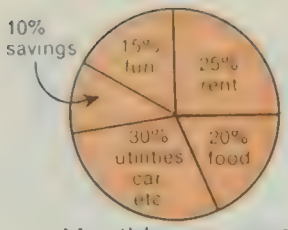


chord A segment with both endpoints on a circle. See the figure for *circle*.

circle The set of all points in a plane which are a specified distance from a fixed point called the *centre*.



circle graph A diagram used to show statistical information, particularly to show how a given quantity is divided into fractional parts.

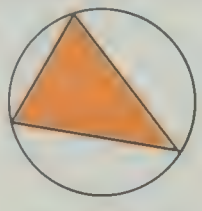


Monthly expenses

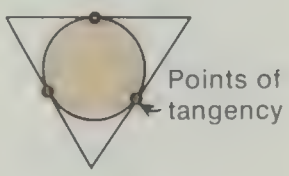
circumcentre The point of intersection of the perpendicular bisectors of the sides of a triangle. It is the centre of the circle circumscribing the triangle.

circumference The distance around a circle. For each circle, the number is the product of its diameter and the number π. The formula is $C = \pi d$.

circumscribed circle A circle drawn through the vertices of a polygon. A circle can be circumscribed about any triangle but only about certain other polygons.



circumscribed polygon If the sides of a polygon are tangent to a circle, then the polygon is circumscribed about the circle. Also, see *inscribed circle*.



closed curve Intuitively, a curve is closed if you can begin at any point and trace the curve until you return to the starting point.

collinear (points) Two or more points lying on the same line.

common factor A number that is a factor of each of two or more given numbers. For example, 3 is a common factor of 6 and 9.

common multiple A number that is a multiple of each of two or more given numbers. For example, 18 is a common multiple of 2, 3, and 9.

commutative (order) principle The principle which states that the sum (or product) of any two numbers is the same regardless of the order in which they are added (or multipled):
$a + b = b + a$ or $a \cdot b = b \cdot a$

complementary angles Two angles whose measures have a sum of 90°.

composite number Any whole number greater than 1 which is not prime. Composite numbers have more than two factors.

complex fraction A fraction that has a fraction for the numerator or the denominator, or both.

concave figure See *convex figure*.

cone A space figure formed by a closed plane curve and all line segments from a point not in the plane of the curve to all the points of the curve.

congruent angles Angles which have the same measure.

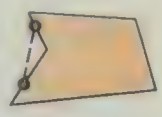
congruent figures Two figures are congruent if one of the figures is the image of the other by one of the *rigid motions* (reflection, rotation, translation) or by any combination of motions.

congruent segments Segments whose endpoints are equally far apart; segments which have the same length.

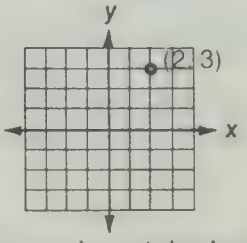
convex figure A figure is convex if every segment joining any two points on the boundary of the figure contains only points on the figure or in its interior.

convex figure

non-convex or **concave** figure



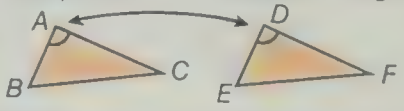
co-ordinate axes Two intersecting perpendicular number lines used for graphing ordered number pairs (*co-ordinates*).



co-ordinates An ordered number pair matched with a point in the co-ordinate plane. See the figure for *co-ordinate axes*.

correspondence A matching of elements in one set with those of another set.

corresponding angles Two angles matched by any one-to-one correspondence between angles of two triangles. For example, $\angle A$ and $\angle D$.

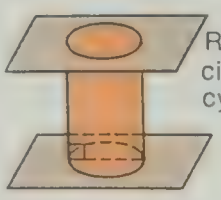


corresponding points Two points matched by any one-to-one correspondence between points of two triangles. For example, point C corresponds to point F above.

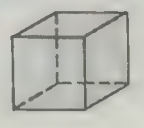
corresponding segments Two segments matched by any one-to-one correspondence between segments of two figures. For example, $\overline{AB}$ and $\overline{DE}$ are corresponding segments above.

cross section The intersection of a space figure and a plane.

cylinder The space figure formed by two congruent curves in parallel planes and the parallel segments connecting corresponding points of the curves.


Right circular cylinder

cube (geometry) A regular polyhedron each of whose six faces are squares.



cube (numeration) A number raised to the third power. 8 is the cube of 2 because $2^3 = 8$. Also, to raise a number to the third power.

deca- A prefix meaning *ten*.

decagon A 10-sided polygon.

decametre A unit of length that is 10 metres.

deci- A prefix meaning *one tenth*.

decimetre A unit of length that is $\frac{1}{10}$ of a metre, or 10 centimetres.

decimal (numeral) Any base-ten numeral written using a decimal point.

decimal point The dot that is used in a decimal numeral.

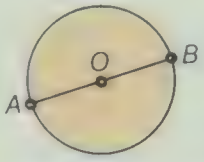
decimal system of numeration A system of numeration in which powers of ten and place value is used.

degree A unit angle that is $\frac{1}{90}$ of a right angle.

denominator For each fraction $\frac{a}{b}$, b is the denominator.

diagonal A segment connecting two nonconsecutive vertices of a polygon.

diameter Any chord that contains the centre of a circle.



difference The number resulting from subtraction.

digits The basic symbols used in a place-value system of numeration. In base ten, the symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

distributive principle The fundamental principle connecting addition and multiplication.
$a \cdot (b + c) = a \cdot b + a \cdot c$.

dividend The number to be divided in a division problem.

$$\begin{array}{r} 3 \leftarrow \text{Quotient} \\ 5\overline{)17} \leftarrow \text{Dividend} \\ \text{Divisor} \uparrow\ 15 \\ \hline 2 \leftarrow \text{Remainder} \end{array}$$

divisible A given number is divisible by a second number if the remainder is zero. If $a = b \cdot c$ with $b \neq 0$ and $c \neq 0$, then a is divisible by either b or c

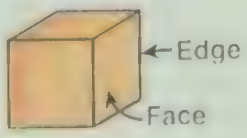
division The inverse of the operation of multiplication.

$$\begin{array}{ccc} F & F & P \\ 3 & \cdot\ 4 & 12 \end{array} \quad \begin{array}{l} P \quad F \quad F \\ 12 \div 4 = 3 \\ 12 \div 3 = 4 \end{array}$$

divisor See *dividend*.

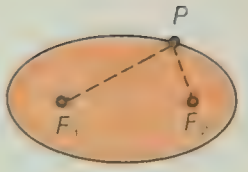
dodecahedron A polyhedron which has 12 faces. A regular dodecahedron has 12 congruent pentagonal faces

edge A segment that is a side of any face of a space figure. A cube has 12 edges.



element A member of a set

ellipse A closed plane curve generated by a point (P) moving in such a way that the sum of the distances from two fixed points (F_1 and F_2) is constant.



empty set The set containing no elements. It is denoted by "{ }" or the symbol ϕ.

equal sets Sets containing the same elements. They may be listed in different order.

equality The relation of identity. A statement of equality asserts that two symbols name the same object or number.

equally likely outcomes Outcomes or results that have the same chance of occurring.

equation A mathematical sentence using the equality symbol (=). $7 + n = 9$ is an equation.

equiangular triangle A triangle all of whose angles have the same measure.

equilateral triangle A triangle having all sides the same length.

equivalent decimals Decimals which represent the same number.

equivalent fractions Fractions which represent the same number such as $\frac{1}{2}$, $\frac{5}{10}$, or $\frac{50}{100}$.

equivalent ratios Two ratios which represent the same fractional number. For example, the ratio 5:10 is the same as 1:2.

equivalent sets Sets having the same number of elements.

estimate An approximation for a given number. Often used in the sense of a rough calculation.

even number A whole-number multiple of 2. The set of even numbers is {0, 2, 4, 6, . . .}

expanded notation A representation of a number as a sum of powers of ten such as:
$3425 = 3 \cdot 10^3 + 4 \cdot 10^2 + 2 \cdot 10^1 + 5 \cdot 10^0$.

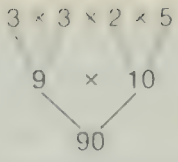
exponent A numeral written above and to the right of a mathematical expression to indicate how many times a number is to be used as a factor. For example, in the expression 5^4, the 4 indicates that 5 is to be used as a factor 4 times.

extremes The first and last terms in a proportion written in the form $a:b = c:d$. The numbers a and d are extremes and b and c are called the *means*

face Any one of the bounding polygonal regions of a space figure. See the figure for *edge*

factor Any one member of a set of numbers to be multiplied. In the equation $5 \cdot 7 = 35$, 5 and 7 are factors

factor tree A diagram suggestive of a tree showing the prime factorization of a number.



flow chart A diagram which gives instructions in a logical order.

focal points See *ellipse*.

formula A general fact or rule expressed by using symbols. For example, the area any parallelogram with base b and height h is given by $b \cdot h$. The formula is usually presented with the equation $A = b \cdot h$.

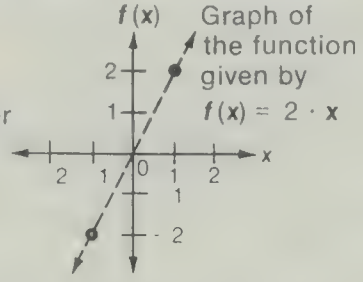
fraction A symbol for a fractional number of the form $\frac{a}{b}$ such that $b \neq 0$.

fractional number The set of non-negative rational numbers.

function A correspondence that associates each element in a first set with a unique element of a second set. Different elements in a first set may possibly be matched with the same element of the second set.

gram A unit of mass. The mass of one cubic centimetre of water at 4°C.

graph A set of points associated with a given set of numbers or number pairs showing a relation or function.

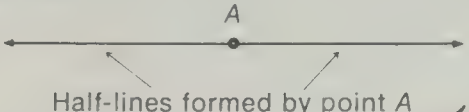


graph (statistical) A picture used to illustrate a given collection of data. It may be in the form of a bar graph, a circle graph, a line-segment graph, or a pictograph.

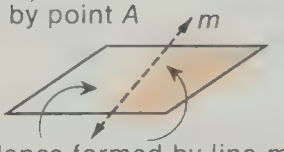
greatest common factor (GCF) The largest, or greatest, number that is a factor of each of a set of numbers.

greatest possible error Half of the basic unit in which a measurement is given. For example, if a length is given as 18 centimetres to the nearest centimetre, then the basic unit is the centimetre and the greatest possible error is 0.5 centimetre.

half-line Each of two sides of a point on a line.



half-plane The region on each of two sides of a line in a plane.



hectare A unit of area that is 10 000 square metres.

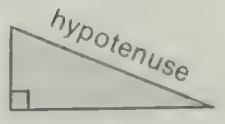
hecto- A prefix meaning *one hundred*.

height (of a prism) The perpendicular distance between the planes of the parallel faces.

heptagon A 7-sided polygon.

hexagon A 6-sided polygon.

hypotenuse The side of a right triangle opposite the right angle.



icosahedron A polyhedron of 20 faces. A regular icosahedron has equilateral triangles for its faces.

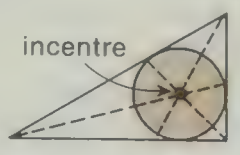
identity element For addition, zero is the identity element since $a + 0 = a$ for any number a. Similarly, for multiplication, 1 is the identity element since $a \cdot 1 = a$.

identity principles For any number a, $a + 0 = a$ and $a \cdot 1 = a$.

image The figure obtained after a rigid motion on a given figure. From a functional viewpoint, y is the image of x if there is a function f such that y is the output when x is the input; that is, $f(x) = y$.

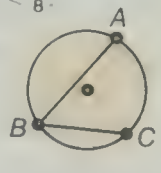
improper fraction A fraction whose numerator is greater than or equal to its denominator.

incentre The point of intersection of the angle bisectors of each angle of a triangle. The centre of the circle inscribed in a triangle.



inequality A mathematical statement using either of the symbols $<$ or $>$ such as $\frac{3}{4} < \frac{7}{8}$.

inscribed angle An angle that contains three points of the circle and has its vertex on the circle.



inscribed polygon See *circumscribed circle*.

integers The set consisting of the natural numbers. $\{1, 2, 3, \ldots\}$, the negatives of the natural numbers $\{-1, -2, -3, \ldots\}$, and zero.

intersection The intersection of two sets is the set of all elements common to both sets.

inverse (operation) Operations which are opposite in effect such as addition and subtraction or multiplication and division.

isosceles triangle A triangle with at least two congruent sides.

kilo- A prefix meaning *one thousand*.

kilogram The basic unit of mass in the metric system, 1000 grams.

kilometre A unit of length that is 1000 metres.

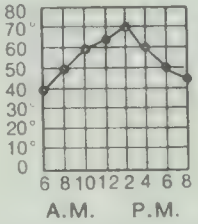
least common denominator The least common multiple of the denominators of two or more fractions. For example, the l.c.d. of $\frac{5}{6}$ and $\frac{3}{4}$ is 12.

least common multiple (LCM) The smallest non-zero number that is a multiple of each of two or more given numbers. The LCM of 4 and 6 is 12.

legs (of a right triangle) The perpendicular sides of a right triangle.

length The measure of a segment (or curve) in terms of a chosen unit.

line-segment graph A diagram used to show statistical information, particularly that of a continuous nature.



litre The unit of capacity in the metric system. 1000 cubic centimetres.

lowest-terms (simplest) fraction A fraction is in lowest terms if the numerator and denominator of the fraction have no common factor other than 1.

mean See *arithmetic mean.*

means The second and third terms in a proportion written in the form $a:b = c:d$. See *extremes.*

median (of a set) The middle number of a set containing an odd number of elements that are arranged in order. If a set contains an even number of elements, the average of the two middle numbers is the median.

median (of a triangle) The segment from the vertex to the midpoint of the opposite side of a triangle.

metre The basic unit of length in the metric system approximately 1650763.73 times the wave length of the orange-red spectral emission line of krypton-86.

metric system A system of measurement in which the metre, the litre, and the kilogram are the basic units of length, capacity, and mass.

midpoint (middle point) A point which divides a segment into two congruent segments.

milli- A prefix meaning *one thousandth.*

minute A unit of angular measure that is $\frac{1}{60}$ of a degree.

mixed decimal numeral A combination of a decimal and a fraction such as $0.4\frac{1}{3}$ and read as "four and one-third tenths."

mixed numeral A numeral such as $4\frac{2}{3}$ indicating the sum of a whole number and a fractional number; $4\frac{2}{3} = 4 + \frac{2}{3}$

mode The most frequently occurring element of a set. There may be more than one mode of a set.

multiple A first number is a multiple of a second number if there is a whole number that multiplies by the second number to give the first number. For example, 24 is a multiple of 4 and 6 since $4 \times 6 = 24$.

multiplication An operation that combines a first number (*factor*) and a second number (*factor*) to give another number (this result is called the *product* of the two factors).

multiplicative inverse See *reciprocal.*

negative integers The set of numbers $\{-1, -2, -3, \ldots\}$.

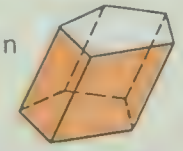
nonagon A 9-sided polygon.

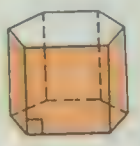
number pair An ordered pair of numbers such as (3, 4) often co-ordinates of a point in a plane.

numeral A symbol for a number.

numerator For each fraction $\frac{a}{b}$, a is the numerator.

oblique prism A prism in which the base is not perpendicular to its lateral edges.

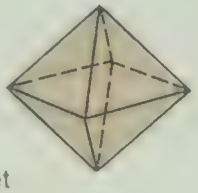

Oblique prism


Right prism

obtuse angle An angle greater than a right angle and smaller than a straight angle.

octagon An 8-sided polygon.

octahedron A polyhedron having eight faces. Each face of a regular octahedron is an equilateral triangle.



odd number Any number in the set $\{1, 3, 5, 7, \ldots\}$.

one principle For any number a, $a \cdot 1 = a$. See *identity principle* for multiplication.

one-to-one correspondence Two sets S and T are in one-to-one correspondence if there is a matching so that each element in S is matched with exactly one element in T, and vice versa.

operation A binary operation like "multiplication" associates each order paired of numbers with one number. Usually we speak of the four fundamental operations: addition, subtraction, multiplication, and division.

opposites principle For any integer a, $a + -a = 0$.

origin The intersection of the co-ordinate axes; the point associated with the number pair (0,0).

outcome A possible result in a probability experiment.

parallel lines Lines in the same plane which do not intersect.

parallelogram A quadrilateral whose opposite sides are parallel.

pentagon A 5-sided polygon.

percent Literally means "per hundred". A symbol such as 3% is an abbreviation for "3 per 100" or $\frac{3}{100}$.

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